Educational Administration: Theory and Practice
2024, 30(01),7148-7162
ISSN: 2148-2403
https://kuey.net/ **Research Article**

"A Holistic Approach to Designing Nearly Linear-Phase IIR Filters"

Pallavee Jaiswal[1*], Saurabh Mitra[2]

[1*]Research Scholar, Department of Electronics & Communication Engineering, Dr. C. V. Raman University, Kargi Road, Kota, Bilaspur, C.G., India,pallavee10@gmail.com
[2]Associate Professor, Department of Electronics & Communication Engineering, Dr. C. V. Raman University, Kargi Road, Kota, Bilaspur, C.G., India, saurabh.mit1000@gmail.com

Citation: Pallavee Jaiswal, et.al (2024), "A Holistic Approach to Designing Nearly Linear-Phase IIR Filters", *Educational Administration: Theory and Practice*, 30(01) 7148-7162
Doi: 10.53555/kuey.v30i1.10374

ARTICLE INFO

ABSTRACT

Electrocardiogram (ECG) signals are widely used in the medical field to assess the heart's electrical activity. However, these signals often contain artefacts—undesired noise or disturbances—that can obscure accurate interpretation. These artefacts typically arise from sources unrelated to cardiac activity and must be mitigated or eliminated for reliable diagnosis. This study investigates various methods for designing Infinite Impulse Response (IIR) filters with near-linear phase characteristics to remove artefacts from ECG signals. Initially, conventional methods such as Butterworth, Chebyshev, Inverse Chebyshev, and Elliptic filter designs are employed to develop IIR filters. The ECG data for this research is sourced from the MIT-BIH Physionet Database. Subsequently, innovative techniques, including Minimum Phase Transformation, Frequency-Dependent Phase Compensation, Kautz Filtering, and Z-Transform, are applied to achieve nearly linear phase characteristics. The proposed approach is assessed based on Signal-to-Noise Ratio (SNR) and group delay performance. The results highlight the effectiveness of the proposed techniques in enhancing filter performance, reducing artefacts, and preserving the integrity of the ECG signals.

Keywords– Electrocardiogram (ECG), Signal-to-Noise Ratio (SNR), Infinite-Impulse-Response (IIR), Finite-Impulse-Response (FIR) etc.

1. INTRODUCTION

Filters are mathematical or computational operations that modify the amplitude and phase of specific frequency components of a signal. The primary goal of filtering is to attenuate unwanted signals (noise or artefacts) while preserving the desired components. Filters are used extensively in many domains, including communications, audio processing, biomedical engineering, and control systems.

Signal processing is a critical tool in many applications, particularly in biomedical engineering, where accurate and efficient filtering techniques are essential for processing signals like the Electroencephalogram (EEG), Electrocardiogram (ECG), etc. Filters are the fundamental components of signal processing systems used to enhance signal quality, remove noise, or isolate specific frequency components. Filters are categorized primarily into two types: Finite-Impulse-Response (FIR) and Infinite-Impulse-Response (IIR) filters. Among these, IIR filters are commonly used due to their computational efficiency and ability to achieve the desired filtering effect with fewer components compared to FIR filters.

There are many diagnosistest like ECG, EEG, EMG, MRI, CT-Scan etc, available for detection of severe diseases. All diagnosis of diseases aredepend upon signal i.e. waveform generated from the test. Sometime these signals which generated from test suffer from artefacts like muscle movement, eye blinking, electromagnetic interference, echo distortion, respiratory, cardiac, pulse etc, because of that, test result get affected, i.e. signal generated from test contaminated with noise or interference.

In-order to remove the artefacts (noise or interference) from the signal, we require some robust filter with the help of which this artefacts can be removed from the waveform. Generally now a day's digital filters are used mainly in medical equipment for removing any artifacts present in the signal generated from the diagnosis test.

Digital filters like FIR filters & IIR filters, are employed in filtering problem. Both the filters have different characteristics based on which we need to select the filter as per the requirement. As from the fact, FIR filters have exact linear phase & are always stable due to the absence of pole in its transfer function but IIR filter have non-linear phase & are unstable because of the presence of pole outside the unit circle of z-plane. Both the filters having different design techniques for implementing the filters. Already large number of filters are design for removing the artifacts from signal generated which include both FIR & IIR filters. Beside the advantage of linear phase & stability in FIR filter, the usage of FIR filter is less in comparison to IIR filters, because FIR filterrequire large memory, high computational complexity, more parameters for the implementation of FIR filters.

There are number of research work has been done for implementing IIR filter for removing the artifacts in medical test. But still there are many drawbacks available in IIR filter implementation. As we know that, IIR filters have non-linear phase, & are unstable, and the conventional design of an IIR filter involve design of a digital filter in the analog domain & transforming the design into the digital domain because of this process used in IIR filter design, it suffer from finite word length effect which degrade the performance of the IIR filters. A lot of research has been done in the field of linear phase IIR filter designing but still require a robust linear phase IIR filter which give better performance. To overcome the drawback of conventional method, the other methods based on optimization techniques were developed for IIR filter design (N. Agrawal 2021). Till now, research work based on IIR filter design weretaken into account individual aspect like linear phase, stability issue, finite word length effect in IIR filter. In this paper, we explore the significance of filters in signal processing, focus on IIR filters.. Furthermore, we review the methods that have been proposed and applied to achieve nearly linear-phase characteristics in IIR filters, addressing the challenges in the design of such filters for removing the artefacts from ECG signals.

2. INFINITE IMPULSE RESPONSE (IIR) FILTERS

IIR filters have an infinite duration impulse response because they incorporate feedback in their structure.Mathematically, IIR filters are described by a recursive difference equation. The defining feature of IIR filters is that they use both current and past input values and previous output values to compute the current output.

The output of an IIR filter can be represented difference equation as:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$

The **transfer function**of IIR filter is given by:

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=0}^{N} a_k z^{-k}}$$

where a_k and b_k are filter coefficients, and N and M are the filter orders.

2.1 Advantages of IIR Filters

- Computational Efficiency: IIR filters generally require fewer coefficients (or filter taps) compared to FIR filters for the same filtering effect. This makes IIR filters more computationally efficient, especially when applied to real-time systems.
- Memory Efficiency: Due to the feedback loop, IIR filters require fewer memory resources than FIR filters, which are inherently memory-intensive.

2.2 Advantages of using IIR Filters for artefact removal in biomedical signals

1. **Efficiency**: IIR filters require fewer coefficients than FIR filters to achieve the same level of performance. This makes them computationally efficient, which is critical for real-time biomedical signal processing.

2. **Compact Design**: Due to the feedback mechanism, IIR filters can achieve sharp transitions between pass-band and stop-band with a lower filter order, making them suitable for constrained hardware environments.

3. **Frequency Selectivity**: IIR filters can effectively isolate specific frequency bands. For example:
   - **Baseline wander removal**: Removes low-frequency drift.
   - **Power-line interference removal**: Suppresses noise at 50/60 Hz.
   - **EMG noise suppression**: Reduces high-frequency muscle noise.

4. **Adaptability**: IIR filters can be easily modified or tuned to target specific frequency ranges, making them flexible for various types of artefact removal.
5. **Preservation of Signal Features**: When designed properly, IIR filters can effectively suppress artefacts while preserving the vital components of the biomedical signals, such as P-QRS-T waves in ECG.

6. Real-Time Capability: The low computational load of IIR filters makes them well-suited for real-time biomedical applications like ECG monitoring.

2.3 Limitations of IIR Filters: Non-Linear Phase Response.

One of the most significant limitations of IIR filters is their non-linear phase response. The phase response of a filter determines how different frequency components of the signal are shifted in time, or in other words, how the filter alters the phase of the input signal. A non-linear phase response means that different frequency components are delayed by varying amounts of time, resulting in phase distortion.

This phase distortion is problematic in many applications, especially in signal processing tasks where the timing and relative positions of signal features are critical. For example, in biomedical signal processing, particularly in ECG analysis, accurate timing of waveforms such as the P-wave, QRS complex, and T-wave is crucial for diagnosing heart conditions. Any distortion in the phase of these features could lead to incorrect interpretation.

The non-linear phase response of IIR filters becomes more pronounced when the filter order increases, which can further complicate the accurate interpretation of the signal. While the magnitude response of an IIR filter can be tailored to eliminate unwanted frequencies, the phase distortion remains an inherent issue unless addressed specifically.

IIR filters are powerful tools in signal processing, but their limitations, particularly nonlinear phase response and sensitivity to numerical issues, make their application challenging in some scenarios. Advanced techniques and careful design are essential to mitigate these drawbacks and maximize their benefits in applications like biomedical signal processing, communications, and audio systems.

2.4 Why Research Continues on Linear Phase IIR Filters?

The intrinsic nonlinear phase response of traditional IIR filters is a significant limitation in applications where phase distortion can corrupt critical signal features. For example, in biomedical signal processing (like ECG analysis), any distortion in signal morphology can mislead diagnoses. Similarly, in audio processing, nonlinear phase can introduce perceptible distortions.

Despite FIR filters inherently offering linear phase, IIR filters are still desirable due to their computational efficiency and ability to achieve sharp transitions with lower filter orders. This drives ongoing research to develop techniques that approximate or achieve linear phase characteristics in IIR filters.

3. RELATED WORK

Nearly linear-phase IIR filters have emerged as a critical focus in modern signal processing due to their ability to balance computational efficiency with sharp frequency selectivity. Unlike their FIR counterparts, which inherently possess linear-phase characteristics but often require higher filter orders, IIR filters offer a more compact design at the expense of non-linear phase response. This trade-off has spurred extensive research into novel methodologies for approximating linear-phase characteristics in IIR filters without compromising their key advantages. This review explores the progression of design approaches, from traditional techniques such as all-pass filtering and optimization-based methods to advanced hybrid and adaptive strategies, aiming to provide a comprehensive understanding of the current state of the art and identify gaps for further innovation.

3.1. Review of Existing Linear Phase IIR Filter Techniques

Vamsee Krishna A. and A. K. Jagadish Kumar [2012] introduced a design methodology for nearly linear-phase IIR digital filters, emphasizing the reduction of group-delay deviation while adhering to specified criteria for pass-band ripple and stop-band attenuation. The method employs a non-restrictive stability constraint via linear inequality constraints in the optimization process via a cascade of second-order sections. The approach includes a feature that regulates the maximum gain in transition bands, so averting anomalies that may arise in optimization-based designs. Experimental results indicate that filters developed through this methodology exhibit reduced group-delay deviation relative to alternative techniques, and provide benefits over precise linear-phase FIR filters, including diminished group delay and filter complexity, while maintaining amplitude-response specifications.[1]

Vinay Kumar and Sunil Bhooshan [2012] propose a technique for the design of a linear-phase one-dimensional Infinite Impulse Response (IIR) filter utilizing orthogonal polynomials. The design approach employs a collection of object functions, implemented via orthogonal polynomials, to attain specified amplitude and phase properties. The arrangement of zeros and poles is meticulously controlled to maintain amplitude characteristics while modifying the phase response. The suggested method yields a filter with favourable cut-off characteristics, and augmenting the number of polynomial terms enhances the approximation of the objective function, resulting in improved frequency characteristics in both the pass-band and transition area.

Future endeavors will focus on optimizing these designs by fine-tuning the polynomial components to diminish ripples in the pass-band and augment overall performance.[2]

Rajeev C. Nongpiur et al. [2013] have assessed a novel optimization technique for the construction of approximately linear-phase IIR digital filters that satisfy specified criteria. The approach reduces group-delay variation while maintaining pass-band ripple and stop-band attenuation within defined parameters, achieving either a constant or optimized group delay. The design employs a cascade of second-order sections to represent the filter, integrating a non-restrictive stability requirement through linear inequality constraints. The approach also permits the limitation of maximum gain in the transition bands, so aiding in the elimination of anomalies. Experimental results indicate that filters developed using this method show markedly reduced group-delay deviation relative to those created by other advanced techniques, while also providing benefits such as diminished filter complexity and lower group delay in comparison to linear-phase FIR filters.[3]

Yasunori Sugita [2014] introduces a design methodology for Infinite Impulse Response (IIR) filters that attains specified flatness and almost linear phase characteristics through quadratic programming (QP). The method is utilized to construct Chebyshev-type, inverse Chebyshev-type, and simultaneous Chebyshev-type filters with flatness restrictions in both the pass-band and stop-band. The flatness criterion in the stop-band is integrated into the transfer function, whilst the pass-band flatness and stability conditions are included as linear matrix equality and inequality constraints within the quadratic programming problem. This method facilitates the straightforward construction of these filters by modifying the design parameters, and its efficacy is illustrated through multiple instances.[4]

Dr. Kamlesh Kumar Singh [2014] presents an innovative method for the design of Infinite Impulse Response (IIR) filters characterized by a linear phase response, employing linear programming and frequency domain sampling. The method provides an accurate approximation of the linear phase response using merely 10 samples, markedly decreasing computational complexity in contrast to traditional methods that use higher-order filters and extensive matrices. The proposed method presents an effective approach for filter design, yielding favourable outcomes with minimal computational resources. Future enhancements may be achieved by optimizing sample positioning and implementing slight modifications to the phase response in the stop-band, hence improving the method's accuracy. This straightforward yet efficient method serves as a significant option for filter design experts.[5]

G.D. Halikias and I.M. Jaimoukha [2015] developed a methodology for the construction of low-order Infinite Impulse Response (IIR) filters that adhere to specified magnitude frequency-response limitations and approximate linear phase attributes. The design procedure has two stages: initially, a Finite Impulse Response (FIR) filter with linear phase is constructed utilizing optimization methods such as linear programming; subsequently, the FIR filter is approximated by a stable Infinite Impulse Response (IIR) filter by a Hankel-norm approximation technique. This method obviates the necessity for computationally intensive matrix Lyapunov equations, depending instead on conventional linear algebra. A-priori limits on magnitude and phase inaccuracies are established to assist in choosing the lowest-order IIR filter that meets the design criteria. The approach is computationally efficient and illustrated with a numerical example, demonstrating its efficacy in attaining the necessary performance using a low-order IIR filter.[6]

Abhay Kumar Singh and Lalit Singh Garia [2016] have assessed a technique for the design of IIR filters that maintain a constant group delay in the pass-band, which is advantageous for signal processing applications necessitating linear phase responses. The design includes an all-pass filter to equalize group delay, yielding a filter with an almost linear phase response across a defined frequency range. The research indicates that elevating the equalizer order and extending the delay line length diminishes phase response ripples, which are most pronounced at the pass-band edge. The method does not entirely eradicate the ripples, even with a high-order equalization. The report also delineates the distinction between LabVIEW and MATLAB implementations: LabVIEW provides straightforward graphical design and analysis, whereas MATLAB does not have the same flexibility for real-time modifications. The findings validate that elevating the filter order diminishes the transition band, while simultaneously amplifies delay and computational complexity. The compensated filter's group delay roughly corresponds with the phase response in the pass-band; however, the overall delay and computational requirements of the filter are elevated relative to the original design.[7]

Nikhil Agrawal et al. [2017] propose a novel design methodology for the development of stable, approximately linear-phase digital IIR filters via fractional derivatives (FD). The design challenge involves optimizing the phase error of an all-pass filter that is coupled in parallel with a pure delay function, utilizing frequency domain techniques to improve the filter's frequency response at a specified location inside the pass-band. The approach utilizes multiple evolutionary techniques, such as Particle Swarm Optimization (PSO), Constraint Factor Inertia PSO (CFI-PSO), Quantum PSO, Artificial Bee Colony, and Cuckoo Search algorithms. Among these, CFI-PSO exhibited superior performance with enhanced pass-band response and sharper transition widths, while a minor decrease in stop-band attenuation was noted relative to non-fractional designs. The findings indicate that the suggested method provides substantial benefits, such as enhanced pass-band response and improved accuracy of pass-band edge frequencies, while ensuring computational viability. A comparison with alternative techniques, including PCLS, CQP, WLS, and CRPSO, underscores the efficacy of the suggested method for signal processing applications.[8]

Ferdinando Foresi1 [2018] concentrates on the advancement of an audio crossover system employing quasi-linear phase IIR filters. The authors emphasize the constraints of conventional IIR filter designs for crossover applications, especially owing to erratic behaviour in the transition region. They present a novel design methodology utilizing an advanced Particle Swarm Optimization (PSO) algorithm, which incorporates restrictions such amplitude response cut-off frequency. The suggested methodology encompasses two optimization strategies: one emphasizing precise pass-band characteristics and the other concentrating on reducing the discrepancy in the cut-off frequency. Simulations indicate substantial enhancements, with a 60 dB decrease in pass-band error and the capacity to attain flat-band crossovers using low-order filters. The approach is evaluated using a 2-way and 4-way crossover design, yielding encouraging outcomes for practical applications, particularly regarding computational complexity. Future endeavors will concentrate on enhancing the sharpness of the transition band, assessing crossover quality using specified metrics, and integrating machine learning methodologies for additional optimization.[9]

Goran Stancic et al. [2019] present a novel design methodology for nearly-linear phase IIR low-pass differentiators employing a parallel all-pass filter configuration. By expressing the magnitude and phase responses of the differentiators as functions of the phase responses of the all-pass filters, the technique attains a magnitude response that closely approximates the ideal in the weighted Chebyshev sense. The design proficiently regulates both the magnitude and phase response errors, with the pass-band phase response linearity error associated with the magnitude error and an extra design parameter, γ. This technique also reduces multiplication requirements, enhancing power efficiency for hardware implementation.

Although the filter order and group delay may exceed those of certain existing designs, the diminished number of multiplications provides considerable benefits regarding power consumption, rendering the proposed method a compelling alternative for applications where low group delay is not imperative but power efficiency is essential.[10]

Goran Stancic [2019] offers an extensive examination of the hardware complexity associated with various filter topologies, emphasizing parallel all-pass (PA) filters and conventional elliptic filters equipped with a group delay corrector (EC). Both filter types are engineered with same cut-off frequency, magnitude approximation, and group delay error, while striving for a nearly linear phase response. The analysis indicates that PA filters have considerable benefits, including diminished power consumption, decreased hardware complexity, and minimized group latency, rendering them an ideal selection for real-time applications, particularly in telecommunications and sub-band coding. PA filters are more effective at higher frequencies and exhibit reduced delay relative to EC filters, which are more intricate and less efficient regarding hardware and power usage. The findings indicate that PA filters exhibit superior efficiency and effectiveness for filtering jobs necessitating rapid processing and minimal latency.[11]

Deng, Gelei et al. [2019] introduce a novel approach for the construction of low-complexity, nearly-linear phase IIR filters that exhibit diminished group delay deviation and lowered hardware expenses in comparison to conventional FIR filters. By framing the minimization of group delay deviation as an iterative optimization problem, the method decreases filter complexity while preserving performance. The method enhances IIR filter coefficients by maximizing the utilization of common subexpressions, hence reducing area and power consumption. Simulation and synthesis results indicate that the suggested IIR filter attains a 39.4% reduction in area and a 41.8% decrease in power consumption compared to FIR filters, alongside a 25.5% enhancement in group delay deviation and an average reduction in area and power of 20.5% and 18.4%, respectively. The findings underscore substantial efficiency improvements, particularly in FPGA implementations.[12]

Nikhil Agrawal et al. [2020] have assessed an innovative approach for the construction of approximately linear-phase infinite impulse response (IIR) filters with fractional derivative constraints (FDCs). The design methodology reduces the phase discrepancy between the intended and actual phase responses of an all-pass filter (APF), facilitating the development of low-pass (LPF) or high-pass (HPF) filters with enhanced pass-band performance and superior stop-band attenuation. The approach employs a greedy sorting process to ascertain optimal FDC values, guaranteeing precise pass-band features while minimizing computational complexity.

The IIR filter design integrates an IIR-APF with a pure delay function in parallel, modifying the APF's phase response to attain the desired magnitude response. Simulations indicate that this FDC-based strategy enhances pass-band error and transition width relative to conventional techniques, with a negligible decrease in stop-band attenuation remaining within acceptable parameters. The design is resilient to word-length effects, hence increasing its practical applicability.

Future research may investigate the integration of more FDCs and the creation of multiplier-less IIR filter designs, thereby expanding the range of efficient and precise digital filter applications.[13]

Ngoc Thang Bui et al. [2021] present a comprehensive system for real-time ECG signal capture utilizing a 32-bit microcontroller architecture, incorporating advanced compression and filtering methodologies. The device integrates hardware design akin to wearable heart rate monitors with firmware employing an advanced JPEG compression method for optimal data storage and transmission efficiency. By customizing JPEG for one-dimensional signals and enhancing Huffman compression, the method attains a compression ratio (CR) of up to 8.8 while preserving a signal error rate under 10%.

The method utilizes a nearly-linear IIR filter post-decompression to efficiently eliminate noise while maintaining the integrity of the ECG signal, hence accurately preserving essential properties such as QRS, T, and R waves. This filter surpasses conventional FIR filters in computing efficiency and is versatile for processing other one-dimensional signals, including audio.

The platform, constructed with a PIC32MZ2048 MCU and ADS1293 analogfront-end, facilitates up to 110 hours of operation, indicating its appropriateness for extended wearable monitoring. The cutting-edge design amalgamates hardware, firmware, and software to enhance performance, underscoring prospective uses for alternative MCUs such as MSP430 or STM32. The findings underscore the system's market potential, providing an effective, cost-efficient alternative for ECG monitoring and cardiovascular assessment.[14]

Zixuan Wang [2022] investigates linear phase IIR filter designs, contrasting the Powell-Chau, Kwan, and Xiao-Oliver-Agathoklis filters. IIR filters exhibit computational efficiency and reduced group latency compared to FIR filters; nonetheless, attaining linear phase presents a problem. The Powell-Chau Filter employs a time-reversal block for enhanced precision, although necessitates intricate circuitry. The Kwan Filter utilizes a computational approximation of the IIR transfer function for simplicity, whereas the Xiao-Oliver-Agathoklis Filter is based on a FIR design to achieve a compromise between linearity and efficiency.

Every filter presents compromises in accuracy, latency, and intricacy. The Powell-Chau Filter demonstrates superior magnitude response but presents implementation challenges. Future endeavors seek to mitigate truncation mistakes and distortion, recognizing the absence of a universally optimal design. The selection is contingent upon certain application requirements.[16]

Abdussalam Omar et al. [2023] proposed a methodology for designing two-dimensional IIR filters with approximately linear phase through a model reduction technique. The method initiates with a linear-phase 2-D FIR filter and employs balanced realization through structural controllability and observability Gramians. The Gramians are optimized under linear matrix inequality (LMI) requirements, guaranteeing filter stability while minimizing computational cost.

The resultant 2-D IIR filter maintains almost linear phase characteristics, excellent selectivity, and diminished system delay, rendering it computationally efficient. Numerical examples validate the method's efficacy, demonstrating enhanced performance and reduced-order filters relative to conventional techniques, while preserving a separable-denominator structure for simplified implementation.[17]

Eric Grivel et al. [2024] presents a novel form of Multiscale Entropy (MSE) aimed at enhancing the characterization of signal complexity while mitigating aliasing problems associated with the conventional coarse-graining (CG) method. The suggested method substitutes the conventional CG step with either a linear-phase FIR filter or a null-phase IIR filter to produce scaled variations of signals. These filters reduce distortions during the decimation phase by establishing suitable cut-off frequencies and anti-aliasing characteristics.

Simulations utilizing white noise and 1/f noise demonstrate that filter parameters, including stop-band ripple and transition bandwidth, substantially influence anti-aliasing efficacy. FIR filters are appropriate for extensive datasets, however the null-phase IIR filter is ideal for situations with restricted samples because of its reduced order.

The novel MSE method exhibits practical significance in identifying attentional tunneling, surpassing the conventional method in statistical significance (p-value). This enhancement broadens MSE's usefulness to a wider user demographic across multiple disciplines.[18]

Summary of review is given in Table

TABLE 1 Comparison table for review of the related work on linear phase IIR filter designing.

Authors	**Filtering Algorithm**	**Description**	**Applicability**	**Parameters**
Vamsee Krishna .A and A. K. Jagadish Kumar [2012]	All-pass filter networks for improved linearity	Uses all-pass filtering to optimize performance and reduce computational load.	Advanced applications requiring precise phase characteristics	Group delay error, computational overhead, stop-band attenuation
Vinay Kumar and Sunil Bhooshan [2012]	LMS-based adaptive filtering with dynamic tuning	Combines adaptive filtering efficiency with hardware integration for real-time use	Real-time applications like ECG noise filtering and adaptive systems	Noise reduction, convergence rate, hardware adaptability
Rajeev C. Nongpiur et al. [2013]	Constrained Optimization, Weighted least squares optimization, Error minimization	Proposes a linear phase IIR filter based on weighted least squares optimization to minimize magnitude and phase error.	Signal denoising, lowpass filtering	Magnitude error, Phase error
Yasunori Sugita [2014]	Quadratic programming (QP), Chebyshev flatness constraints	Achieves flatness and approximately linear phase in	Filter design	Pass-band flatness, Phase deviation

		Chebyshev-type IIR filters.		
Dr. Kamlesh kumarsingh [2014]	Linear programming, Frequency sampling	Simplifies linear phase IIR design with low computational complexity.	Filter design	Phase response, Computational complexity
G.D. Halikiasand I.M. Jaimoukha [2015]	FIR-to-IIR approximation, Hankel-norm reduction	Designs low-order IIR filters with specified tolerances and linear phase approximation.	Low-order IIR filtering	Magnitude and phase errors
Abhay Kumar Singh and Lalit Singh Garia [2016]	Group delay equalization, All-pass filters	Designs constant group delay IIR filters with nearly-linear phase.	Real-time filtering, signal processing	Group delay ripples, Computational complexity
Nikhil Agrawal et al. [2017]	Fractional derivatives, Swarm Intelligence algorithms	Improves IIR filters' frequency response using fractional derivatives and optimization techniques.	General signal processing	Transition width, Pass-band response
Ferdinando Foresi1 [2018]	Particle Swarm Optimization (PSO), Swarm Intelligence algorithm	Pass-band error, Cut-off frequency mismatch	Audio crossover systems	Proposes quasi-linear IIR filters optimized for audio crossover systems.
Goran Stancic et al. [2019]	Parallel all-pass filter structure	Designs low-power nearly-linear phase IIR differentiators.	Hardware-efficient filtering	Magnitude and phase errors, Multiplication requirements
Goran Stancic [2019]	Parallel all-pass filters, Elliptic filters with group delay corrector	Compares PA and EC filters, highlighting PA's efficiency in real-time applications.	Telecommunications, sub-band coding	Hardware complexity, Power consumption, Delay
Deng, Gelei et al. [2019]	Convex optimization	Reduces IIR filter hardware complexity while maintaining performance.	FPGA implementations, efficient filtering	Group delay deviation, Power consumption, Area
Nikhil Agrawal et al. [2020]	Fractional derivative constraints, IIR-APF design	Designs robust IIR filters using fractional derivatives for improved phase and magnitude response.	Low-pass and high-pass filtering	Phase error, Pass-band response, Stop-band attenuation
Ngoc Thang Bui et al. [2021]	Enhanced JPEG compression, Nearly-linear IIR filter	Develops a low-cost ECG system with efficient data compression and noise removal using nearly-linear IIR filters.	Real-time ECG monitoring, wearable devices	Compression ratio, Signal error rate
Zixuan Wang [2022]	Powell-Chau, Kwan, Xiao-Oliver-Agathoklis filters	Compares linear phase IIR filters focusing on trade-offs in precision, delay, and complexity.	General signal processing	Magnitude response, Delay, Precision

Abdussalam Omar et al [2023]	Balanced realization, LMI optimization	Designs 2-D IIR filters with nearly linear phase and reduced computational complexity.	2-D signal processing, computational efficiency	Stability, Phase error, System delay
Eric Grivel et al. [2024]	Linear-phase FIR filter, Null-phase IIR filter	Introduces a variant of MSE using FIR and IIR filters to minimize aliasing and improve signal complexity analysis.	Signal complexity characterization, attentional tunneling detection	Stop-band ripple, Transition bandwidth, p-value
Proposed	1.Z-Transform, 2.Minimum Phase Frequency Transformation, 3.Kautz Filter, 4.Frequency-Dependent Phase Compensation	Design nearly linear phase IIR filter	ECG signal denoising	Signal to Noise Ratio, Group Delay

4. TOOLS AND TECHNIQUES USED

4.1. Software: MATLAB

MATLAB (Matrix Laboratory) was used as the primary computational tool for designing and analyzing linear phase IIR filters. MATLAB is widely recognized for its robust signal processing toolbox and advanced capabilities in numerical computation and visualization.

➢ **MATLAB scripts were developed to:**

- Design and implement various linear phase IIR filter techniques.
- Analyze the performance metrics, including SNR, phase linearity, and computational complexity.
- Compare results across different proposed techniques.

4.2. Dataset: MIT-BIH PhysioNet Database

The MIT-BIH PhysioNet database was chosen as the primary source of ECG signals for evaluating the proposed filters.

➢ **About the Dataset**:

- The MIT-BIH Arrhythmia Database, part of PhysioNet, is one of the most commonly used ECG signal repositories in biomedical signal processing.
- It contains annotated recordings of ECG signals sampled at 360 Hz, suitable for real-world applications.

4.3. Performance Metrics

The proposed filter techniques were assessed based on the following performance metrics:

- **Signal-to-Noise Ratio (SNR)**: To quantify noise suppression.
- **Phase Response Analysis**: To evaluate the linearity of the phase response.
- **Magnitude Response**: To ensure minimal distortion of the signal amplitude.
- **Group Delay**: To measures the delay for different frequencies.

5. METHODOLOGY

In this study, we propose four novel techniques for designing nearly linear phase IIR filters:

1. Z-Transform
2. Minimum Phase Frequency Transformation
3. Kautz Filter
4. Frequency-Dependent PhaseCompensation

These techniques have not been explored previously for achieving linear phase characteristics in IIR filters. Each approach is designed to balance the trade-off between phase linearity and filter performance metrics such as stability, computational efficiency, and signal fidelity.

5.1 Z-Transform

The Z-transform is a mathematical tool that is widely used in signal processing and filter design. In this context, it plays a critical role in analyzing and designing filters in the discrete domain. The technique utilizes the poles and zeros of a transfer function to achieve the desired phase response.

- **Purpose in Linear-Phase Design**: By strategically placing the poles and zeros within the Z-plane, the phase response of the filter can be manipulated to approximate a linear phase.
- **Advantages**: Offers flexibility in designing both the magnitude and phase response.
- **Challenges**: Achieving a perfectly linear phase requires careful balancing, which may increase computational complexity.

5.2 Minimum Phase Frequency Transformation

Minimum phase transformation is a technique where the phase of a filter is adjusted to the minimum phase while preserving the magnitude response. This involves modifying the zeros of the filter to lie inside the unit circle in the Z-plane.

- **Purpose in Linear-Phase Design**: Ensures stability and causality while minimizing phase distortion. Though not perfectly linear, it is useful for applications where low group delay is prioritized.
- **Advantages**: Achieves a low group delay without significantly altering the filter's magnitude response.
- **Challenges**: Does not yield a truly linear phase, and its applicability depends on the specific signal requirements.

5.3 Kautz Filter

The Kautz filter is an extension of the all-pole model, incorporating a combination of poles and zeros to achieve a desired response. This filter uses ortho-normal basis functions and is particularly effective for representing signals with specific frequency characteristics.

- **Purpose in Linear-Phase Design**: The design approach allows for precise control over phase characteristics, making it suitable for nearly linear-phase applications.
- **Advantages**: Provides excellent control over both phase and magnitude characteristics while maintaining stability and efficiency.
- **Challenges**: Requires complex computations and is less intuitive compared to traditional methods.

5.4Frequency-Dependent Phase Compensation

This technique involves compensating for phase nonlinearity by adding corrective phase shifts to the filter's response. The phase compensation is typically tailored to specific frequency bands, ensuring that the overall phase response is closer to linear.

- **Purpose in Linear-Phase Design**: Corrects phase deviations introduced by the original filter design, particularly in critical frequency ranges.
- **Advantages**: Highly adaptable and effective for achieving linear-phase characteristics in specific frequency bands.
- **Challenges**: May introduce additional computational overhead and is dependent on accurate phase deviation estimation.

5.5 Steps of Proposed Filter Designing

This section outlines the systematic approach adopted for designing nearly linear phase IIR filters using the proposed techniques. The filter design process integrates theoretical principles, computational tools, and performance evaluation metrics to achieve optimal signal processing outcomes. Figure 1 presents a flowchart summarizing the step-by-step methodology, which includes pre-processing the input ECG signals, implementing the proposed techniques (Z-Transform, Minimum Phase Transformation, Kautz Filter and Frequency-Dependent Phase Compensation), and evaluating the performance using metrics such as SNR and phase linearity.

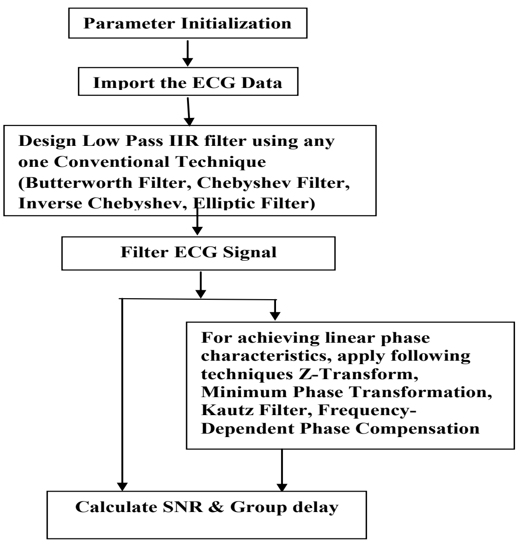

Fig.1Flowchart ofstepsfollowed inthisstudy

6. EXPERIMENTAL OUTCOME

The experimental analysis was conducted to evaluate the effectiveness of the proposed techniques for designing nearly linear phase IIR filters. The main aim of this paper was to design nearly linear phase IIR filter for removing artefacts from ECG signal. The database for examination was made available by MIT -BIH PhysioNet.

Figure 2 shows the time response of (a) Original ECG signal, (b) Filtered ECG signal after the application of Butterworth IIR filter & (c) filtered ECG signal after the application of Butterworth IIR filter + Proposed Nearly Linear Phase Techniques.

Figure 3 shows the Magnitude Response & Phase Response of ECG signal after the application of Butterworth IIR filter & (b) ECG signal after the application of Butterworth IIR filter + Proposed Nearly Linear Phase Techniques.

Figure 4 Signal to Noise Ratio(SNR) verse Filter Order of ECG signal after the application of Butterworth IIR filter & (b) ECG signal after the application of Butterworth IIR filter + Proposed Nearly Linear Phase Techniques.

Figure 5 Group Delay of ECG signal after the application of Butterworth IIR filter & (b) ECG signal after the application of Butterworth IIR filter + Proposed Nearly Linear Phase Techniques.

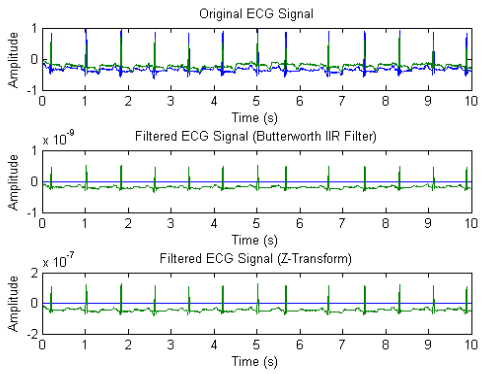

Fig. 2.1 Z transform

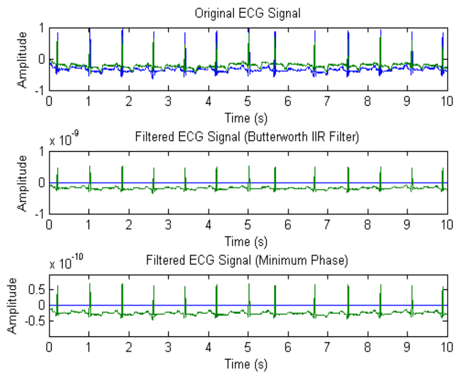

Fig. 2.2 Minimum Phase Frequency Transformation

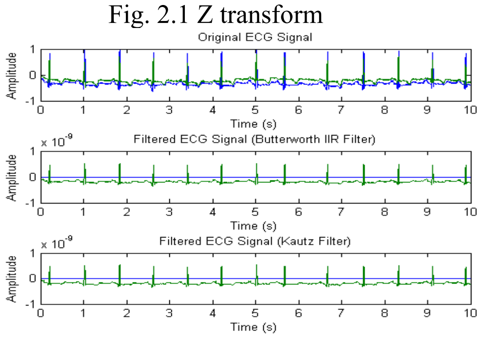

Fig. 2.3 Kautz Filter

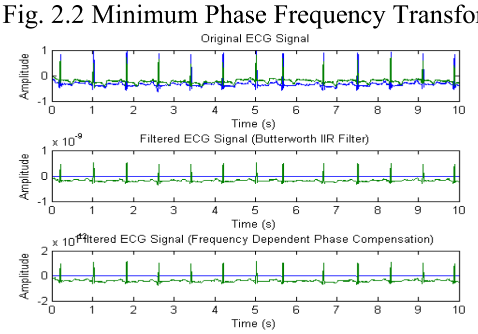

Fig. 2.4 Frequency Dependent Phase Compensation

Fig. 2 Time response of Original ECG Signal, Filtered ECG Signal using Butterworth IIR Filter & Filtered ECG Signal using Butterworth IIR Filter + Proposed Nearly Linear Phase Techniques.

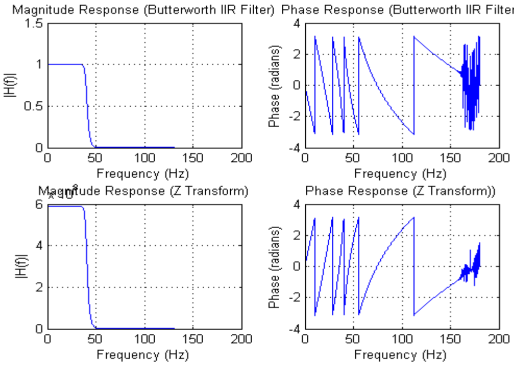

Fig. 3.1 Z transform

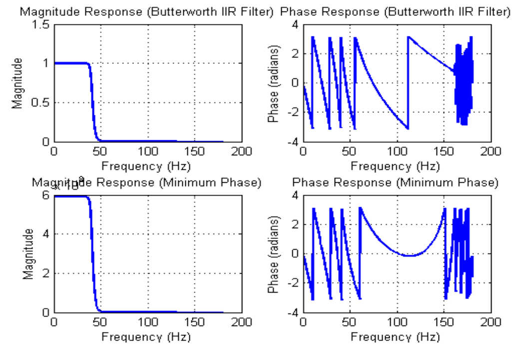

Fig. 3.2 Minimum Phase Frequency Transformation

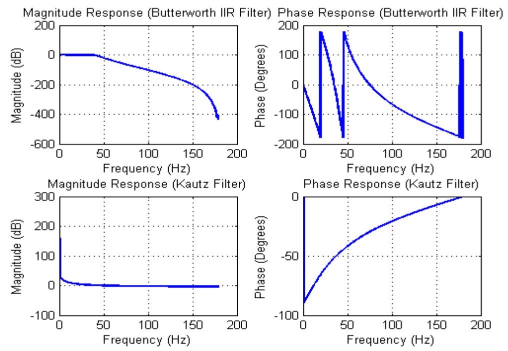

Fig. 3.3 Kautz Filter

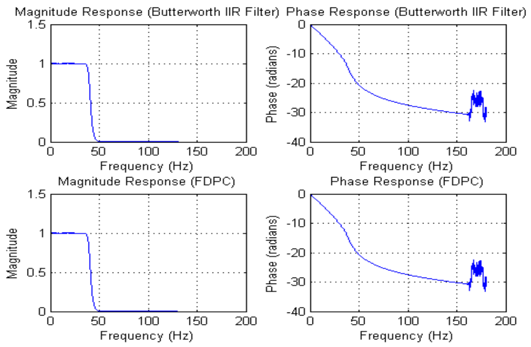

Fig. 3.4 Frequency Dependent Phase

Fig. 3 Magnitude Response & Phase Response of Butterworth IIR filter &Applied nearly linear phase proposed techniques.

Fig. 4.1 Z transform

Fig. 4.2 Minimum Phase Frequency Transformation

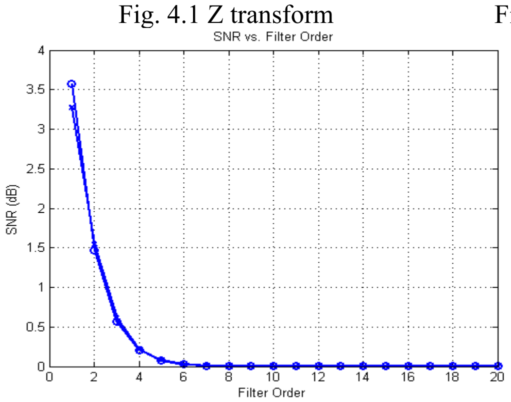

Fig. 4.3 Kautz Filter

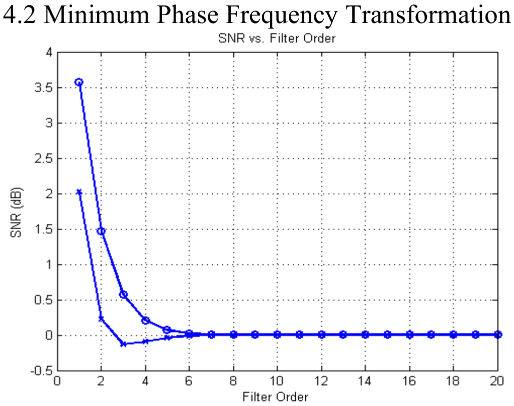

Fig. 4.4 Frequency Dependent Phase Compensation

Fig. 4 Signal to Noise Ratio (SNR) verses Filter Order.

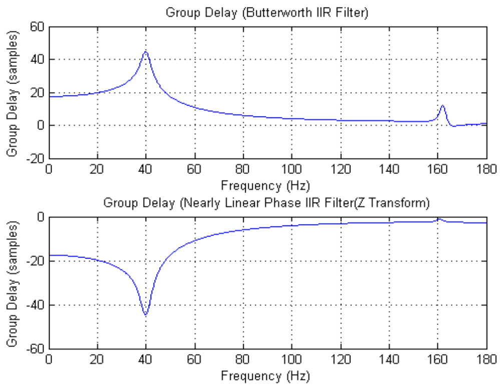

Fig. 5.1 Z transform

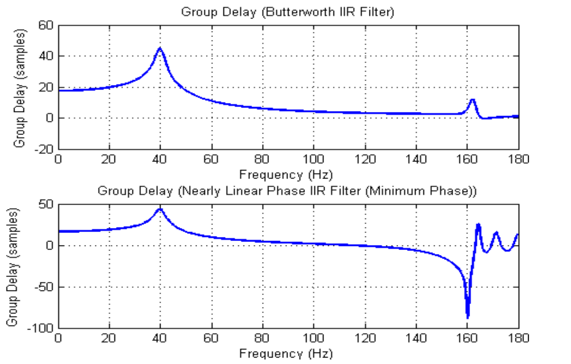

Fig. 5.2 Minimum Phase Frequency Transformation

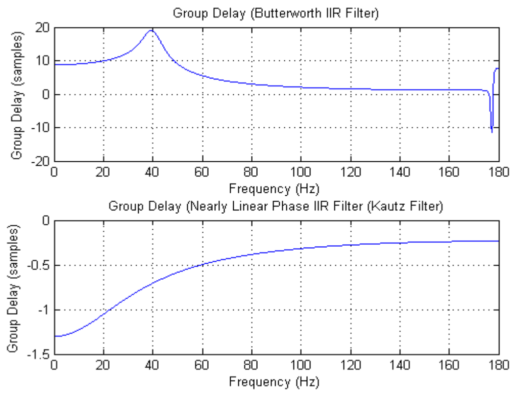

Fig. 5.3 Kautz Filter

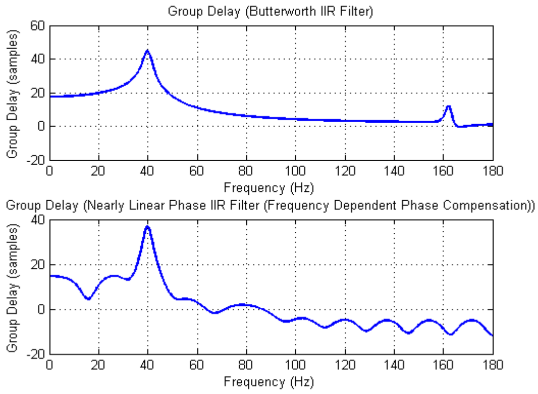

Fig. 5.4 Frequency Dependent Phase Compensation

Fig.5 Group Delay of Butterworth IIR filter &Applied nearly linear phase proposed techniques.

TABLE 2. Evaluation of Nearly Linear Phase IIR Filter Design Characteristics:

S. No.	Techniques Used	Peak SNR	Order	Group Delay (Samples)	Observations
1.	Butterworth IIR Filter	3.57 dB	1	44.71 at 227 Hz	High SNR but large group delay, not phase linear.
	Z-Transform	1.13 dB	3	-1.17 at 161.02 Hz	Low SNR with phase distortion due to negative group delay.
2.	Butterworth IIR Filter	3.57 dB	1	44.71 at 227 Hz	High SNR but large group delay.
	Minimum Phase Frequency Transformation	\ 2.10 dB	1	43.59 at 227 Hz	Improved group delay but significantly lower SNR.
3.	Butterworth IIR Filter	3.57 dB	1	44.71 at 227 Hz	High SNR but large group delay.
	Kautz Filter	3.27 dB	1	-0.24 at 1024 Hz	Very small group delay and close to linear phase with good SNR.
4.	Butterworth IIR Filter	3.57 dB	1	44.71 at 227 Hz	High SNR but large group delay.
	Frequency Dependent Phase Compensation	2.02 dB	1	36.86 at 228 Hz	Moderate improvement in group delay but reduced SNR.
5.	Butterworth IIR Filter	3.57 dB	1	44.71 at 227 Hz	High SNR but large group delay.
	Fractional Derivative	0.40 dB	1	97.80 at 1 Hz	Very poor SNR with extremely large group delay, unsuitable for ECG signals.

From the table, the goal of this analysis is to determine the best technique for designing a nearly linear phase IIR filter for ECG signal filtration based on the given Peak Signal-to-Noise Ratio (SNR), Filter Order, and Group Delay. Below is the descriptive comparison of the techniques provided in the table:

1. Butterworth IIR Filter vs Z-Transform

- **Peak SNR**: Butterworth IIR filter has a significantly higher Peak SNR of 3.57 compared to the Z-Transform technique, which has 1.13. This indicates the Butterworth filter performs better in terms of noise reduction and signal fidelity.
- **Group Delay**:
  - Butterworth IIR filter has a group delay of 44.71 samples at 227 Hz, which is high but expected due to its IIR characteristics.
  - Z-Transform achieves a very low negative group delay (-1.17 samples) at 161.02 Hz, which implies phase distortion and non-causal behaviour.
- **Remarks**: The Z-Transform is not ideal due to low Peak SNR and its phase distortion. The Butterworth filter is superior for ECG filtration in this case.

2. Butterworth IIR Filter vs Minimum Phase Frequency Transformation

- **Peak SNR**: Butterworth IIR filter achieves a Peak SNR of 3.57, while Minimum Phase Frequency Transformation achieves a lower 2.10.
- **Group Delay**:
  - Butterworth IIR filter has a group delay of 44.71 samples at 227 Hz.
  - Minimum Phase Frequency Transformation reduces the group delay to 43.59 samples at the same frequency, showing a slight improvement in phase characteristics.
- **Remarks**: While Minimum Phase Frequency Transformation improves group delay slightly, it comes at the cost of reduced SNR. For ECG signal filtration, the Butterworth filter remains the better choice as its higher Peak SNR ensures better signal quality.

3. Butterworth IIR Filter vs Kautz Filter

- **Peak SNR**: The Butterworth IIR filter has a Peak SNR of 3.57, whereas the Kautz Filter achieves a close 3.27, making it a competitive alternative.
- **Group Delay**:
  - Butterworth IIR filter has a group delay of 44.71 samples at 227 Hz.
  - The Kautz Filter achieves a nearly negligible -0.24 samples group delay at 1024 Hz, indicating excellent phase linearity.
- **Remarks**: The Kautz Filter is a promising candidate for achieving nearly linear phase characteristics with minimal phase distortion while maintaining a relatively high Peak SNR. If linear phase is the top priority, the Kautz Filter is superior to the Butterworth filter.

4. Butterworth IIR Filter vs Frequency Dependent Phase Compensation

- **Peak SNR**: Butterworth IIR filter has a Peak SNR of 3.57, while Frequency Dependent Phase Compensation achieves 2.02, which is significantly lower.
- **Group Delay**:
  - Butterworth IIR filter has a group delay of 44.71 samples at 227 Hz.
  - Frequency Dependent Phase Compensation reduces the group delay to 36.86 samples at 228 Hz, showing a moderate improvement.
- **Remarks**: While Frequency Dependent Phase Compensation improves the group delay compared tothe Butterworth IIR filter, the lower Peak SNR limits its overall performance. Butterworth IIR remains the better option for ECG filtration.

7. CONCLUSION

This study presents a comparative analysis of various techniques for designing nearly linear phase IIR filters for ECG signal filtration. The comparison is performed based on two key performance metrics: Peak Signal-to-Noise Ratio (SNR) and Group Delay, which are crucial for assessing the quality and phase linearity of the filtered ECG signal.

The analysis demonstrates that the Butterworth IIR Filter consistently achieves the highest Peak SNR of 3.57, ensuring superior noise reduction and signal fidelity. However, this advantage comes at the cost of a significant group delay (44.71 samples), which affects the phase characteristics and may distort critical ECG waveform features.

Among the alternative techniques analyzed, the Kautz Filter emerges as the most balanced solution, achieving a near-linear phase response with an exceptionally small group delay of -0.24 samples at 1024 Hz, while maintaining a competitive Peak SNR of 3.27. This makes the Kautz Filter an ideal candidate for applications requiring both phase linearity and noise suppression, such as ECG signal processing.

Techniques such as the Z-Transformdemonstrate poor performance in terms of SNR and group delay, rendering them less suitable for ECG signal filtration. While methods like Minimum Phase Frequency Transformation and Frequency Dependent Phase Compensation show moderate improvements in group delay, they do so at the expense of significantly reduced SNR.

In conclusion, the Kautz Filter technique provides the best trade-off between phase linearity and noise reduction, making it the preferred choice for ECG signal filtration. For applications prioritizing signal fidelity, the Butterworth IIR Filter remains a strong contender. Future research can explore hybrid approaches combining the strengths of these techniques to further optimize performance for ECG denoising.

REFERENCES

1. Vamsee Krishna .A and A. K. Jagadish. "Design Method for Nearly Linear-Phase IIR Filters Using Constrained Optimization." *International Journal of Science and Research (IJSR)*, Dec 2012: pp. 617-621, vol. 3, Issue 12, ISSN (Online): 2319-7064.
2. Vinay Kumar and Sunil Bhooshan. "Design of One-Dimensional Linear Phase Digital IIR Filters Using Orthogonal Polynomials." *International Scholarly Research Network (ISRN) Signal Processing*, 2012: pp. 1-7, doi:10.5402/2012/870276.
3. Rajeev C. Nongpiur, Dale J. Shpak, Andreas Antoniou. "Improved Design Method for Nearly Linear-Phase IIR Filters Using Constrained Optimization." *IEEE TRANSACTIONS ON SIGNAL PROCESSING*, Feb 15, 2013: pp. 895-906, vol. 61, Issue. 04.
4. Yasunori Sugita. "Design of Stable IIR filters with prescribed flatness and approximately linear phase." *WSEAS TRANSACTIONS on SIGNAL PROCESSING*, 2014: pp. 619-626, vol. 10, e-ISSN: 2224-3488.
5. Dr. Kamlesh Kumar Singh. "Design of Linear Phase IIR Filter." *IJEE*, 2014: pp. 1-4, vol. 6, ISSN: 0973-7383.
6. G.D. Halikias and I.M. Jaimoukha. "Design of Infinite Impulse Response (IIR) filters with almost linear phase." *IEEE Xplore*, 2015: doi: 10.23919/ECC.2003.7086504.
7. Abhay Kumar Singh and Lalit Singh Garia. "Design of IIR Filters with Definite Group Delay (Lowpass Equalization) using LabVIEW." *International Journal of Signal Processing, Image Processing and Pattern Recognition*, 2016: pp. 245-252, vol. 9, Issue. 7, doi: org/10.14257/ijsip.2016.9.7.21.
8. Nikhil Agrawal, Anil Kumar, Varun Bajaj. "A New Design Method for Stable IIR Filters With Nearly Linear-Phase Response Based on Fractional Derivative and Swarm Intelligence." *IEEE Transactions On Emerging Topics In Computational Intelligence*, Dec 2017: pp. 464-476, vol. 1, Issue. 6.
9. Ferdinando Foresi, Paolo Vecchiotti, Diego Zallocco, Stefano Squartini. "Designing Quasi-Linear Phase IIR Filters for Audio Crossover Systems by using Swarm Intelligence." *Audio Engineering Society Convention Paper* , 2018: pp. 3-11.
10. Goran Stancic, Milos Duric, Bojan Jovanovic, Stevica Cvetkovic. "A Complexity Analysis of IIR Filters with an Approximately Linear Phase." *Radio Engineering*, June 2019: pp. 430-438, vol. 28, Issue. 2, doi:10.13164/re.2019.0430.
11. Goran Stancic, Ivan Krstic, Stevica Cvetkovic. "All-pass-based design of nearly-linear phase IIR low-pass differentiators." *International Journal of Electronics*, 2019: ISSN: 0020-7217 (Print) 1362-3060 (Online), doi:10.1080/00207217.2020.1726498.
12. Deng, Gelei, Jiajia Chen, Jiaxuan Zhang, and Chip-Hong Chang. "Area- and power-efficient nearly-linear phase response IIR filter by iterative convex optimization." *IEEE Access*, 2019: pp. 22952 - 22965, doi: 10.1109/ACCESS.2019.2899107 , e-ISSN: 2169-3536.
13. Nikhil Agrawal, Anil Kumar and Varun Bajaj. "A New Design Approach for Nearly Linear Phase Stable IIR Filter using Fractional Derivative." *IEEE/CAA Journal of Automatica Sinica*, March 2020: pp. 527-538, vol. 7, Issue. 2, doi: 10.1109/JAS.2020.1003054.
14. Ngoc Thang Bui, Thi My Tien Nguyen, Sumin Park, Jaeyeop Choi, Thi Mai Thien Vo, Yeon-Hee Kang, Byung-Gak Kim, Junghwan Oh. "Design of a nearly linear-phase IIR filter and JPEG compression ECG signal in real-time system." *Biomedical Signal Processing and Control* , 2021: pp. 1-10, doi: 10.1016/j.bspc.2021.102431.
15. N. Agrawal, A. Kumar, Varun Bajaj, G.K. Singh. "Design of digital IIR filter: A research survey." *Applied Acoustics, Science Direct*, 2021: pp. 1-10, doi.org/10.1016/j.apacoust.2020.107669.
16. Wang, Zixuan. "Different Methods of Linear Phase IIR Filer Realization." *Highlights in Science, Engineering and Technology*, 2022: pp. 167-171, doi: 10.54097/hset.v27i.3739.
17. Zixuan Wang. "Different Methods of Linear Phase IIR Filer Realization." *Highlights in Science, Engineering and Technology*, 2022: pp. 167-171, doi: 10.54097/hset.v27i.3739.
18. Abdussalam Omar, Dale Shpak and Panajotis Agathoklis. "Nearly Linear-Phase 2-D Recursive Digital Filters Design Using Balanced Realization Model Reduction." *Signals, MDPI*, 2023: pp. 800-815, doi: 10.3390/signals4040044.

19. Eric Grivel, Bastien Berthelot, Gaetan Colin, Pierrick Legrand and Vincent Ibanez. "Benefits of Zero-Phase or Linear Phase Filters to Design Multiscale Entropy: Theory and Application." *Entropy. MDPI*, 2024: pp. 1-27, doi: 10.3390/e26040332.