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**Research Article**

# A Quantitative Analysis of Multi-Asset Portfolio Selection Using Risk-Return and Efficient Frontier(2020-2022)

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**ABSTRACT**

Diversification remains a fundamental principle and a critical task for portfolio managers to mitigate investment risk. Modern Portfolio Theory (MPT), developed by Harry Markowitz, provides a robust framework for constructing feasible sets of portfolios and identifying the optimal portfolio among them. A key result in the context of theory is the construction of the Efficient Frontier and identifying the optimum portfolio among the different sets of portfolios. Efficient Frontier is a graphical representation of the ideal trade-off between the risk of the portfolio and the expected return of the portfolio.

However, the practical effectiveness of the Efficient Frontier depends largely on the careful selection of securities, appropriate assignment of weightage, and the inclusion of assets with low or negative correlations. This study aims to bridge the gap between theoretical portfolio principles and practical construction by emphasizing correlation-driven security selection within the Indian financial market.

In this study, the researcher focuses on constructing an efficient frontier for various portfolio combinations and identifying an optimal portfolio comprising securities with low or negative correlations, based on their historical return patterns. This approach is consistent with the core principle of diversification, illustrating how the strategic combination of such assets can enhance diversification benefits and reduce overall portfolio risk without materially affecting expected returns.

To evaluate the effectiveness of diversification in line with Markowitz's theory, the researcher has selected a set of five diversified securities representing various sectors of the Indian financial system, including private and public sector banks, the IT sector, the bond market, and the commodity market. It comprises HDFC Bank, SBI Bank, HCL Technologies Ltd., SBI Gold ETF, and Bharat Bond ETF. These securities were chosen to ensure broad sectoral representation, as they include large-cap stocks from the banking, IT, and FMCG sectors, along with commodity-based and fixed-income exchange-traded funds (ETFs). To assess the effectiveness of the Markowitz model, 104 weeks of data have been utilized. Return series, correlation coefficients, and mean-variance optimization calculations have been employed to construct various portfolio combinations.

The resulting Efficient Frontier is analyzed to identify the Minimum Variance Portfolio and the Optimal Portfolio. The findings demonstrate that deliberate, correlation-conscious asset selection significantly enhances diversification opportunities and efficiency of the portfolio. The study reinforces the practical relevance of MPT by highlighting the effect of risk-return trade-off between various portfolios and how incorporating negatively or weakly correlated securities effectively reduces overall portfolio risk and increases diversification opportunities.

These insights hold practical significance for investors and portfolio managers, particularly in emerging markets like India, where asset correlations can vary considerably across sectors. By strengthening the connection between correlation analysis and portfolio efficiency, this research contributes to both academic literature and practical investment strategies, offering a foundation for constructing more resilient and efficient portfolios.

***Keywords:*** *Modern Portfolio Theory, Portfolio Diversification, Minimum Variance Portfolio, Optimal Portfolio, Mean-Variance Optimization, Efficient Frontier, Indian Financial Market*

## 1. INTRODUCTION

In the domain of modern finance, investment decision-making is driven by two major needs. First, to maximize returns but also to manage and mitigate associated risks. The second is the selection of a portfolio, which involves selecting the best portfolio from the various choices of a feasible set of portfolios. With a wider range of investment options available to investors, the task of constructing a portfolio in such a way that balances risk and return has gained critical importance. The foundation of this analytical approach to portfolio construction was laid by Harry Markowitz in 1952 through his seminal work on Modern Portfolio Theory (MPT). Markowitz introduced a quantitative model that emphasized diversification and the mathematical association between asset returns and risk, revolutionizing portfolio management. Markowitz was the first to identify and mathematically demonstrate how diversification can reduce risk.

A fundamental element of Markowitz's theory is the concept of the Efficient Frontier. It is a graphical representation of a set of ideal portfolios offering the maximum possible expected return for a given level of risk, on the other hand, the minimum risk for a given level of expected return. This principle empowers investors to make rational and informed decisions regarding asset allocation, depending on their individual risk appetites and return expectations. The Efficient Frontier serves not just as a theoretical construct, but in addition, it also serves as a practical guide for constructing portfolios that are well-diversified and risk efficient. Constructing the Efficient Frontier requires a detailed analysis of historical return data, standard deviations (as a proxy for risk), and the correlation between selected assets. Through the combination of assets with differing risk-return characteristics and interrelationships, investors can achieve significant risk reduction via diversification. Particularly noteworthy is the inclusion of assets that are not perfectly correlated, or ideally negatively correlated, as these combinations allow for the smoothing out of volatility in overall portfolio returns.

In the context of the Indian financial market, the application of the Efficient Frontier becomes especially relevant due to the diversity of available asset classes, such as equities, commodities (like gold), and fixed-income instruments (such as bond ETFs). With markets becoming increasingly volatile and globally interconnected, the relevance of structured portfolio construction methodologies like the Markowitz model has never been greater.

This research examines how smart investors can take benefit by simply “Do not put all eggs in one basket,” i.e., diversification. This study also helps investors construct the best risky portfolio with the help of effective diversification. For the said objectives, this study leverages weekly return data from April 2022 to March 2024 for a selected group of five diversified assets from various sectors of the Indian Financial System viz. HDFC Bank Ltd. (HSFC.NS: Private Sector Banking), State Bank of India (SBIN.NS: Public Sector Banking), HCL Technologies Ltd. (HCLTECH.NS: IT services), SBI Gold ETF (SETFGOLD.NS: Commodity), and REC Bond ETF(RECLTD.NS: Fixed Income Bond). These assets span across different sectors and exhibit a range of correlations—some positive, some near-zero, and some negative—providing an ideal dataset for constructing a robust Efficient Frontier for various combinations of assets.

The objective of this research is to apply the Markowitz Mean-Variance Optimization model to construct the Efficient Frontier and identify the optimal portfolio from within the universe of available combinations. By calculating expected returns, standard deviations, and pairwise correlations for the chosen assets, the study aims to demonstrate the practical utility of diversification and optimization in real-world investment scenarios. The negative correlation observed between the Gold ETF and equity assets like Infosys further enhances the diversification potential of the portfolio.

Through this study, the usefulness of the Efficient Frontier as a strategic tool for risk-adjusted investment decision-making is highlighted. It reaffirms the importance of careful asset selection, correlation analysis, and optimization techniques in achieving investment objectives. Ultimately, the study contributes to the broader academic and practical understanding of how quantitative models like Markowitz's can be effectively employed in contemporary portfolio management within the Indian market context.

## 2. LITERATURE REVIEW

### Markowitz's Foundational Work

Markowitz (1952) introduced the Modern Portfolio Theory (MPT), laying the foundation for optimal portfolio construction through mean-variance optimization. He emphasized the importance of diversification and demonstrated that combining assets with imperfect correlations reduces portfolio risk more effectively than investing in individual assets. The concept of the Efficient Frontier — portfolios that maximize return for a given level of risk — revolutionized the approach to asset allocation and remains central to contemporary portfolio management.

*Reference:* Markowitz, H. (1952). Portfolio selection. The Journal of Finance, 7(1), 77–91. https://doi.org/10.2307/2975974

### 2. Empirical Application in Indian Markets

Chaudhary and Bhatia (2020) applied Markowitz's model to the Indian stock market and demonstrated the practical benefits of constructing diversified portfolios. Their findings showed that including assets like gold and bonds in equity-dominant portfolios led to lower risk and better performance, especially during market volatility. The study validated MPT in the context of emerging economies and emphasized its relevance in dynamic market conditions.

*Reference:* Chaudhary, A., & Bhatia, B. S. (2020). Portfolio optimization using Markowitz model: Evidence from Indian stock market. International Journal of Finance and Economics, 25(3), 421–433. https://doi.org/10.1002/ijfe.1765

### Role of Gold in Diversification

Verma and Kumar (2018) explored the role of gold ETFs in improving portfolio efficiency under MPT. Their research showed that gold had a negative correlation with equity assets, especially during periods of economic uncertainty. As a result, the inclusion of gold in portfolios significantly reduced risk and enhanced diversification benefits, aligning with Markowitz's assertion that low-correlation assets are key to optimal portfolios.

*Reference:* Verma, R., & Kumar, A. (2018). Gold as a diversification tool in Indian investment portfolios: An application of modern portfolio theory. Journal of Indian Business Research, 10(2), 152–168. https://doi.org/10.1108/JIBR-03-2017-0039

### Sectoral Diversification & Efficient Frontier

Sen and Mehtab (2021) conducted a comparative study of portfolios created using the Markowitz model with Indian stocks from various sectors. They observed that portfolios built using assets from uncorrelated sectors yielded better risk-adjusted returns. Their study supports the theory that effective sectoral diversification helps in forming portfolios closer to the Efficient Frontier, particularly in emerging markets like India.

*Reference:* Sen, R., & Mehtab, M. (2021). Sectoral diversification and portfolio optimization using Markowitz theory in India. Asian Journal of Research in Banking and Finance, 11(2), 18–32. https://doi.org/10.5958/2249-7323.2021.00010.1

### Risk and Return Trade-Off Using ETFs

Gupta and Singh (2019) evaluated the impact of including bond ETFs in equity portfolios under MPT. Their study showed that while bond ETFs are not negatively correlated with equities, their lower volatility contributes to risk minimization. They concluded that ETF-based diversification offers a practical approach to portfolio efficiency, especially for retail investors seeking stable returns.

*Reference:* Gupta, V., & Singh, S. (2019). Evaluating bond ETFs for portfolio diversification in India: A modern portfolio theory perspective. Global Business Review, 20(1), 134–147. https://doi.org/10.1177/0972150918813376

## 3. RESEARCH METHODOLOGY

The fundamental objective of every rational investor is to achieve maximum returns while maintaining a low to moderate level of risk. This principle underlies the strategy of diversification, which seeks to optimize returns through the reduction of unsystematic risk, ultimately leading to the formulation of well-structured investment portfolios. The present study aims to construct portfolios that deliver the highest possible returns for the lowest associated risk. A quantitative research methodology has been adopted, grounded in the application of Harry Markowitz's Modern Portfolio Theory (MPT), with particular emphasis on the construction and analysis of the Efficient Frontier. The research involves the selection of five securities representing distinct sectors within the Indian financial system: HDFC Bank Ltd., State Bank of India (SBI), HCL Technologies Ltd., SBI Gold ETF, and REC Ltd. These securities were purposefully selected to ensure sectoral diversity encompassing Banking, Information Technology, Commodities, and Fixed-Income sectors, which further enhances the potential for effective portfolio diversification.

To evaluate the effectiveness of the Markowitz Model, this study utilizes historical weekly closing prices of selected assets over a period of 104 weeks, spanning from April 2020 to March 2022, i.e., two years. Weekly return percentages were computed using the Natural Logarithmic (log) returns rather than the simple returns. The use of log returns is justified by their additive properties over time and across assets, which facilitates more accurate portfolio-level return analysis. Furthermore, Modern Portfolio Theory (MPT) presumes that asset returns are together follows normal distribution, and this condition is more closely met by log returns due to their tendency to exhibit statistical normality than the simple return. This methodological choice aligns with the theoretical assumptions underpinning MPT, thereby enhancing the robustness of the analysis.

$$Return\ of\ Security = Ln\left(\frac{Return\ of\ The\ Week}{Return\ of\ the\ Previous\ Week}\right)$$

Later, for the formation of efficient frontiers, it is required to convert weekly returns and weekly standard deviations of return of a security into annual returns and annual standard deviation. This can be derived from this data using the formula:

$Annual\ Return(\%) = (\sum Weekely\ Returns\ )$ x $\frac{52}{n}$

$Annual\ Standard\ Deviation\ (\%) = Standard\ Deviation\ of\ Weekly\ Returns$ x $\sqrt{52}$

where '*n*' represents the number of weeks used in the study.

Subsequently, descriptive statistics such as return, standard deviation and the Karl Pearson correlation coefficient were computed for each pair of assets to assess the degree of correlation.

Return of Portfolio = $\sum w1\ x\ R1 + w2\ x\ R2$

Standard Deviation of Portfolio = $w1^2\ x\ 61^2\ +\ w2^2\ x\ 62^2\ +\ 2\ x\ w1\ x\ 61\ x\ w2\ x\ 62\ x\ r\ 12$

Here, w1and w2 = weightage of 1st and 2nd asset.

61 and 62 = standard deviation of returns of 1st and 2nd asset.

r12 = Correlation Coefficient between returns of 1st and 2nd asset.

Using Excel-based matrix computations, all possible two-asset combinations were generated by the researcher. The Efficient Frontier was constructed by plotting expected returns against the corresponding portfolio standard deviations. The Minimum Variance Portfolio (MVP) and the Optimal Portfolio based on the maximum Sharpe Ratio were identified from the frontier.

Sharpe Ratio = $\frac{Rp-Rf}{6}$

Here, Rp = Return of Portfolio.

Rf = Risk-free rate, which is 6.50% p.a.

6 = Standard Deviation of portfolio.

This methodology enables the empirical testing of MPT's core proposition, that portfolios composed of assets with low or negative correlations will lie closer to the Efficient Frontier and offer superior risk-adjusted returns. The study intentionally emphasizes correlation-driven security selection, thereby bridging the theoretical underpinnings of diversification with practical implementation strategies relevant to the Indian investment landscape.

## 4. OBJECTIVES

**Objectives of the Study**

1. **To construct an Efficient Frontier** using the Markowitz Mean-Variance Optimization Model by analyzing historical return data, risk metrics, and correlations of selected securities from diverse sectors of the Indian financial market.
2. **To evaluate the effectiveness of diversification** by selecting and combining assets with low, zero, or negative correlations, thereby minimizing portfolio risk without significantly compromising expected returns.
3. **To identify the Minimum Variance Portfolio and Optimal Portfolio** based on risk-return trade-offs and Sharpe Ratio analysis among the various portfolio combinations generated.
4. **To assess the practical applicability of Modern Portfolio Theory (MPT)** in building risk-efficient investment portfolios within the context of an emerging market like India.

## 5. DATA ANALYSIS & INTERPRETATION

**Table 1: Summary of Weekly Returns, Weekly Standard Deviation, Annual Returns and Annual Standard Deviation of all assets.**

| *Assets* | *HDFC Bank Ltd* | *SBI Bank* | *HCL Technologies Ltd.* | *SBI GOLD ETF* | *REC Limited (RECLTD.NS)* |
|---|---|---|---|---|---|
| *Weekly Return (%)* | *60%* | *105%* | *110%* | *9%* | *54%* |
| *Weekly S.D* | *4%* | *6%* | *4%* | *2%* | *5%* |
| *Annual Return(%)* | *30%* | *53%* | *55%* | *5%* | *27%* |
| *Annual S.D* | *29%* | *41%* | *31%* | *14%* | *34%* |

*(Source: Prepared by the researcher based on closing price of securities obtained from Yahoo Finance )*

**Table 2: Summary of Correlation Coefficient between returns of security.**

| *Assets* | *HDFC Bank Ltd* | *SBI Bank* | *HCL Technologies Ltd.* | *SBI GOLD ETF* | *REC Ltd.* |
|---|---|---|---|---|---|
| *HDFC Bank Ltd* | 1.000 | | | | |
| *SBI Bank* | 0.097 | 1.000 | | | |
| *HCL Technologies Ltd.* | -0.076 | 0.131 | 1.000 | | |
| *SBI GOLD ETF* | -0.090 | -0.081 | 0.122 | 1.000 | |
| *REC Ltd.* | 0.00 | 0.541 | 0.213 | 0.013 | 1.000 |

*(Source: Prepared by the researcher based on weekly returns of securities)*

## Data Analysis:

Table 1 comprises five assets exhibiting diverse return-risk profiles. HCL Technologies Ltd. and SBI Bank offer the highest annual returns of 55% and 53%, respectively, albeit with elevated risks of 31% and 41%, positioning them as aggressive growth assets and suitable for risk-taking investors. HDFC Bank Ltd. presents a balanced profile with a 30% return and 29% risk, potentially suitable for moderate risk-appetite investors. In contrast, SBI GOLD ETF, with a 5% return and 14% standard deviation, acts as a low-risk hedge. REC Limited BOND is a fixed income security, despite its bond classification, which shows a relatively high return of 27% and risk of 34%, indicating characteristics akin to hybrid or high-yield instruments. This variation facilitates optimal diversification under the Markowitz framework.

Table 2 is the correlation matrix reveals low to moderate inter-asset association, indicating strong diversification potential. Notably, HCL Technologies shows a low and negative correlation with most assets, enhancing its role in reducing portfolio risk. SBI Bank and REC Limited exhibit a moderate positive correlation (0.541), suggesting some common risk exposure. Meanwhile, SBI GOLD ETF maintains weak or negative correlations with all other assets, confirming its utility as a non-correlated hedge. REC Ltd. and HDFC Banks Ltd. exhibit a 0 correlation, indicating there is no linear association between the returns of both securities. No or zero correlation between them signifies that the returns of these two assets move **independently** of each other, and their price movements are **uncorrelated**.

In terms of portfolio theory, this provides a **significant diversification benefit**. The overall weak to moderate interdependence across assets supports the application of Modern Portfolio Theory to construct efficient portfolios with reduced unsystematic risk.

**1st Portfolio: HDFC Bank Ltd. and SBI Bank (Banking + Banking)**

**Table 3 Summary of 1st Portfolio: HDFC Bank Ltd. & SBI Bank**

| | *Return* | *Risk* | *Correlation* |
|---|---|---|---|
| *HDFC Bank Ltd.* | 30.00% | 29.00% | 0.097 |
| *SBI Bank* | 53.00% | 41.00% | |
| *Risk-Free Rate* | 6.50% | | |

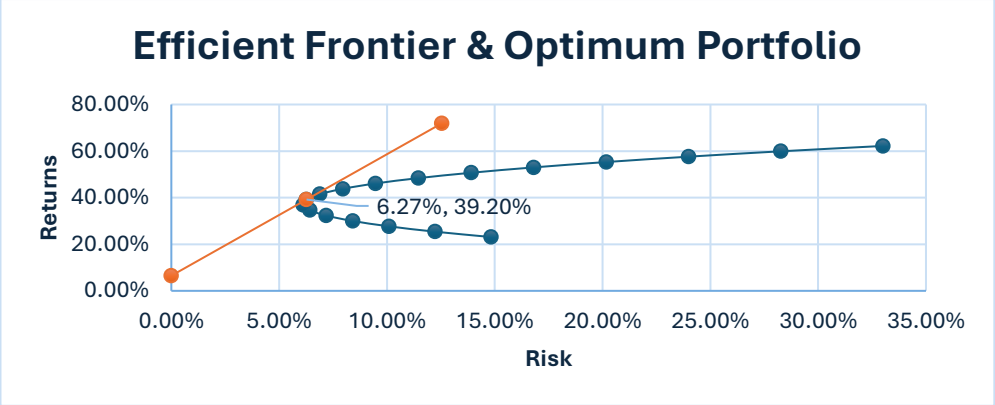
*(Source: Prepared by the researcher)*

**Table 4: Risk-Return Matrix.**

| *HDFC Bank Ltd.* | *SBI Bank* | *Rp* | *σp* | *Sharpe Ratio* | |
|---|---|---|---|---|---|
| *130%* | *-30%* | *23.10%* | *14.83%* | *1.1196* | |
| *120%* | *-20%* | *25.40%* | *12.23%* | *1.5455* | |
| *110%* | *-10%* | *27.70%* | *10.09%* | *2.1010* | |
| *100%* | *0%* | *30.00%* | *8.41%* | *2.7943* | |
| *90%* | *10%* | *32.30%* | *7.19%* | *3.5894* | |
| *80%* | *20%* | *34.60%* | *6.42%* | *4.3743* | |
| ***70%*** | ***30%*** | ***36.90%*** | ***6.12%*** | ***4.9688*** | **MVP** |
| ***60%*** | ***40%*** | ***39.20%*** | ***6.27%*** | ***5.2146*** | **Optimum Portfolio** |
| *50%* | *50%* | *41.50%* | *6.88%* | *5.0860* | |
| *40%* | *60%* | *43.80%* | *7.95%* | *4.6914* | |
| *30%* | *70%* | *46.10%* | *9.48%* | *4.1780* | |
| *20%* | *80%* | *48.40%* | *11.46%* | *3.6550* | |
| *10%* | *90%* | *50.70%* | *13.91%* | *3.1781* | |
| *0%* | *100%* | *53.00%* | *16.81%* | *2.7662* | |
| *-10%* | *110%* | *55.30%* | *20.17%* | *2.4194* | |
| *-20%* | *120%* | *57.60%* | *23.99%* | *2.1301* | |
| *-30%* | *130%* | *59.90%* | *28.27%* | *1.8892* | |
| *-40%* | *140%* | *62.20%* | *33.00%* | *1.6878* | |

*(Source: Prepared by the researcher)*

**Graph 1: Efficient Frontier for 1st Portfolio- HDFC Bank & SBI Bank**




*(Source: Prepared by the researcher)*

**Interpretation:**
The analysis of the efficient frontier constructed using HDFC Bank Ltd. and SBI Bank reveals significant benefits of diversification, even between assets from the same sector. HDFC Bank offers a moderate return of 30% at a risk level of 29%, while SBI Bank yields a higher return of 53% with greater volatility 41%. Despite their industry similarity, the correlation coefficient between the two is low, 0.097, but positive, allowing for effective risk reduction when combined in a portfolio. The simulation of various portfolio combinations identified the Minimum Variance Portfolio (MVP) at a 70:30 allocation (HDFC: SBI), yielding a return of 36.90% at a minimal risk of 6.12%. More importantly, the optimum portfolio—based on the highest Sharpe ratio of 5.2146—is achieved with a 60:40 allocation, providing a return of 39.20% and risk of 6.27%. This optimum point, which lies on the Capital Market Line, represents the most efficient risk-return trade-off possible when a risk-free rate of 6.5% is included. The results confirm the core principle of Modern Portfolio Theory: combining assets with imperfect correlation can lead to portfolios with superior risk-adjusted performance compared to individual securities.

**Second Portfolio: HDFC Bank Ltd. and HCL Technologies Ltd. (Banking + IT Sector)**

**Table 5: Summary of 2nd Portfolio: HDFC Bank Ltd. & HCL Technologies Ltd.**

| | *Return* | *Risk* | *Correlation* |
|---|---|---|---|
| *HDFC Bank Ltd.* | 30.00% | 29.00% | -0.076 |
| *HCL Technologies* | 55.00% | 31.00% | |
| *Risk-Free Rate* | 6.50% | | |

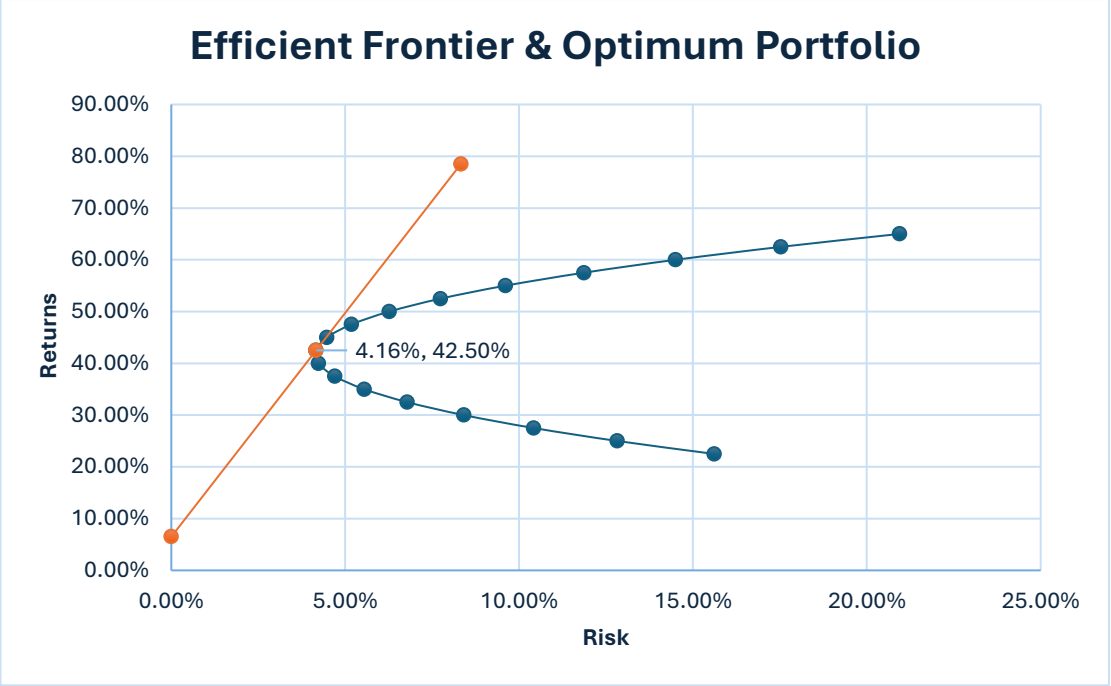
*(Prepared by the researcher)*

**Table 6: Risk-Return Matrix.**

| *HDFC Bank Ltd.* | *HCL Technologies Ltd.* | *Rp* | *σp* | *Sharpe Ratio* | |
|---|---|---|---|---|---|
| *130%* | *-30%* | *22.50%* | *15.61%* | *1.02* | |
| *120%* | *-20%* | *25.00%* | *12.82%* | *1.44* | |
| *110%* | *-10%* | *27.50%* | *10.42%* | *2.01* | |
| *100%* | *0%* | *30.00%* | *8.41%* | *2.79* | |
| *90%* | *10%* | *32.50%* | *6.79%* | *3.83* | |
| *80%* | *20%* | *35.00%* | *5.55%* | *5.14* | |
| *70%* | *30%* | *37.50%* | *4.70%* | *6.60* | |
| *60%* | *40%* | *40.00%* | *4.24%* | *7.91* | |
| *50%* | *50%* | *42.50%* | *4.16%* | *8.65* | (MVP) (Optimum Portfolio) |
| *40%* | *60%* | *45.00%* | *4.48%* | *8.60* | |
| *30%* | *70%* | *47.50%* | *5.18%* | *7.92* | |
| *20%* | *80%* | *50.00%* | *6.27%* | *6.94* | |
| *10%* | *90%* | *52.50%* | *7.75%* | *5.94* | |
| *0%* | *100%* | *55.00%* | *9.61%* | *5.05* | |
| *-10%* | *110%* | *57.50%* | *11.86%* | *4.30* | |
| *-20%* | *120%* | *60.00%* | *14.50%* | *3.69* | |
| *-30%* | *130%* | *62.50%* | *17.53%* | *3.19* | |
| *-40%* | *140%* | *65.00%* | *20.95%* | *2.79* | |

*(Source: Prepared by the researcher)*

**Graph 2: Efficient Frontier for 2nd Portfolio- HDFC Bank Ltd. & HCL Technologies Ltd.**




*(Source: Prepared by the researcher)*

**Interpretation:**
The analysis of the efficient frontier using HDFC Bank Ltd. and HCL Technologies Ltd. demonstrates a superior diversification outcome due to the presence of a slightly negative correlation (-0.076) between the two assets. HDFC Bank offers a return of 30% with a risk of 29%, while HCL Technologies provides a higher return of 55% at a marginally higher risk of 31%. This near-zero and negative correlation facilitates meaningful risk reduction when the assets are combined in varying proportions. Simulation of portfolio weights reveals that the optimum portfolio achieving the highest Sharpe ratio of 8.65 is obtained with an equal allocation of 50% to both assets, delivering a portfolio return of 42.50% at a minimal risk of 4.16%. The graphical representation illustrates this point on the efficient frontier where the tangency portfolio touches the Capital Market Line originating from the risk-free rate of 6.5%. This result strongly validates the core principle of Modern Portfolio Theory: by combining assets with negatively correlated movements, investors can construct portfolios that significantly outperform individual securities on a risk-adjusted basis. The dramatic improvement in the Sharpe ratio, along with a minimal increase in portfolio risk, underscores the critical role of asset selection and allocation in constructing optimal portfolios.

**3rd Portfolio: HDFC Bank Ltd. and REC Ltd. (Banking + Fixed Income)**

**Table 7: Summary of 3rd Portfolio Combination: HDFC Bank Ltd. & REC Ltd.**

| | *Return* | *Risk* | *Correlation* |
|---|---|---|---|
| *HDFC Bank Ltd.* | *30.00%* | *29.00%* | *0* |
| *REC Ltd. Bond* | *27.00%* | *34.00%* | |
| *Risk-Free Rate* | *6.50%* | | |

*(Source: Prepared by the researcher)*

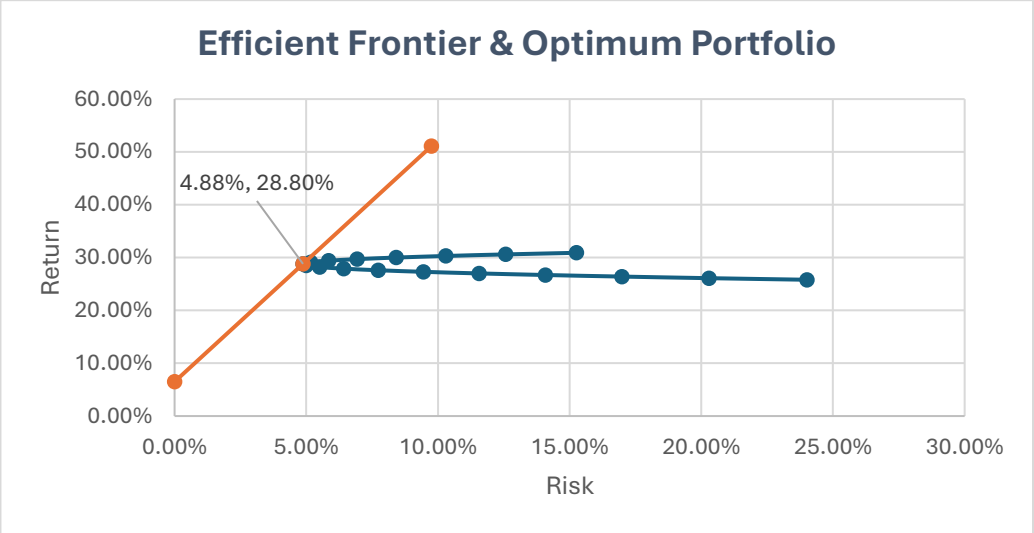
**Table 8: Risk-Return Matrix.**

| *HDFC Bank Ltd.* | *REC Ltd.* | *Rp* | *бp* | *Sharpe Ratio* | |
|---|---|---|---|---|---|
| *130%* | *-30%* | *30.90%* | *15.25%* | *1.60* | |
| *120%* | *-20%* | *30.60%* | *12.57%* | *1.92* | |
| *110%* | *-10%* | *30.30%* | *10.29%* | *2.31* | |
| *100%* | *0%* | *30.00%* | *8.41%* | *2.79* | |
| *90%* | *10%* | *29.70%* | *6.93%* | *3.35* | |
| *80%* | *20%* | *29.40%* | *5.84%* | *3.92* | |
| *70%* | *30%* | *29.10%* | *5.16%* | *4.38* | |
| *60%* | *40%* | *28.80%* | *4.88%* | *4.57* | (MVP) (Optimum Portfolio) |
| *50%* | *50%* | *28.50%* | *4.99%* | *4.41* | |
| *40%* | *60%* | *28.20%* | *5.51%* | *3.94* | |
| *30%* | *70%* | *27.90%* | *6.42%* | *3.33* | |

| | | | | |
|---|---|---|---|---|
| *20%* | *80%* | *27.60%* | *7.73%* | *2.73* |
| *10%* | *90%* | *27.30%* | *9.45%* | *2.20* |
| *0%* | *100%* | *27.00%* | *11.56%* | *1.77* |
| *-10%* | *110%* | *26.70%* | *14.07%* | *1.44* |
| *-20%* | *120%* | *26.40%* | *16.98%* | *1.17* |
| *-30%* | *130%* | *26.10%* | *20.29%* | *0.97* |
| *-40%* | *140%* | *25.80%* | *24.00%* | *0.80* |

*(Prepared by the researcher)*

**Graph 3: Efficient Frontier for 3rd Portfolio- HDFC Bank & REC Ltd.**




*(Prepared by the researcher)*

**Interpretation:**

The portfolio analysis involving HDFC Bank Ltd. and REC Ltd. Bond presents a nuanced view of risk-return trade-offs when combining a moderate-return equity with a relatively high-risk fixed-income instrument. HDFC Bank yields a 30% annual return at 29% risk, while REC Ltd. Bond, despite being labelled as a debt instrument, offers a 27% return with a notably higher risk of 34%, behaving more like a hybrid or high-yield bond. The correlation between the two assets is zero, indicating no linear relationship, which is ideal for diversification under the Markowitz framework. This lack of correlation allows investors to benefit from variance reduction without sacrificing much in expected return. However, due to the comparable return levels and relatively high individual risks, the efficiency gain from this pair would be modest compared to combinations involving more diversified or less volatile assets. The efficient frontier analysis between HDFC Bank Ltd. and REC Ltd. Bond, both exhibiting moderate return profiles with uncorrelated movements (correlation = 0), illustrates a relatively constrained yet meaningful opportunity for portfolio optimization. HDFC Bank offers a return of 30% at a 29% risk, while REC Ltd. Bond yields a slightly lower return of 27% but at a higher volatility of 34%, defying traditional fixed-income behavior. Through simulation of various weight combinations, the analysis identifies the optimum portfolio—which also coincides with the minimum variance portfolio (MVP)—at a 60:40 allocation in favor of HDFC Bank, delivering an expected return of 28.80%, a risk of 4.88%, and the highest Sharpe ratio of 4.57.

In classical portfolio theory, a zero-correlation coefficient ($\rho = 0$) between two assets suggests the possibility of moderate diversification, as their returns are independent. However, in the present case involving HDFC Bank Ltd. and REC Ltd., the observed diversification benefit — as indicated by the gradual decline in portfolio risk — is relatively nominal rather than substantial. This can be attributed to the high and nearly comparable standard deviations (risks) of both assets (29% for HDFC Bank and 34% for REC Ltd.) and their similar levels of expected return (30% and 27%, respectively). When two assets have similar return profiles but high and symmetric levels of volatility, the risk-offsetting effect becomes less pronounced, even in the absence of correlation. Additionally, the shape of the portfolio variance function is largely influenced by the relative weights and risk asymmetry of the constituent assets. In this dataset, the marginal benefit of introducing REC Ltd. into a portfolio dominated by HDFC Bank Ltd. increases only slightly as the weight of REC increases. While the Sharpe ratio does improve and reaches an optimum at 60% HDFC Bank and 40% REC Ltd., the overall risk reduction (from 8.41% to a minimum of 4.88%) is relatively limited given the initially high individual volatilities. Hence, diversification is achieved, but its magnitude is constrained by the comparable high risks and moderately close returns of the two uncorrelated assets.

**4th Portfolio: SBI Bank and REC Ltd. (Banking + Fixed Income)**

**Table 9: Summary of Second Portfolio: SBI Bank Ltd. & REC Ltd.**

| | *Return* | *Risk* | *Correlation* |
|---|---|---|---|
| *SBI Bank* | 53.00% | 41.00% | 0.54 |
| *REC Ltd Bond* | 27.00% | 34.00% | |
| *Risk-Free Rate* | 6.50% | | |

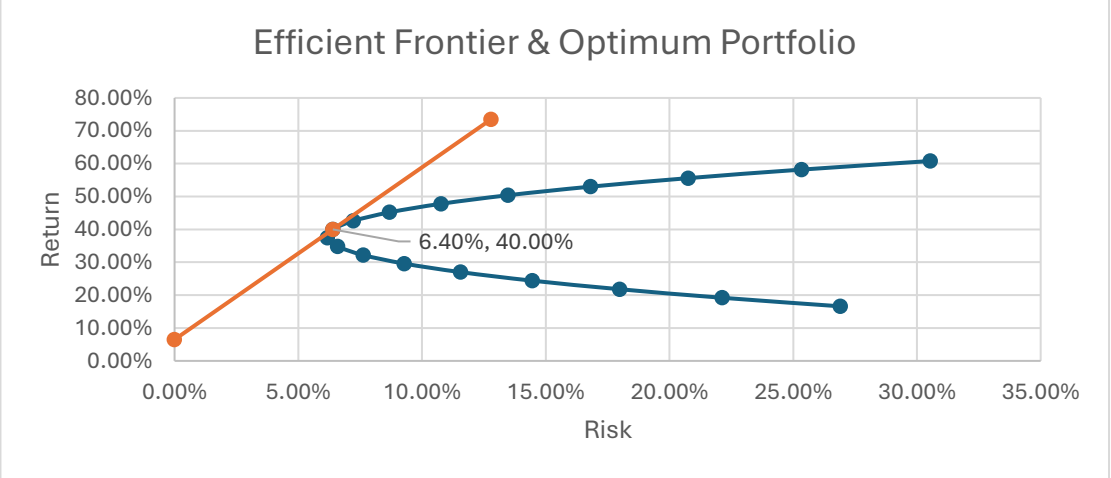
*(Source: Prepared by the researcher)*

**Table 10: Risk-Return Matrix.**

| *SBI Bank* | *REC Ltd.* | *Rp* | *бp* | *Sharpe Ratio* | |
|---|---|---|---|---|---|
| 130% | -30% | 60.80% | 30.54% | 1.778 | |
| 120% | -20% | 58.20% | 25.34% | 2.040 | |
| 110% | -10% | 55.60% | 20.76% | 2.365 | |
| 100% | 0% | 53.00% | 16.81% | 2.766 | |
| 90% | 10% | 50.40% | 13.48% | 3.256 | |
| 80% | 20% | 47.80% | 10.77% | 3.833 | |
| 70% | 30% | 45.20% | 8.69% | 4.452 | |
| 60% | 40% | 42.60% | 7.23% | 4.992 | |
| 50% | 50% | 40.00% | 6.40% | 5.238 | Optimum Portfolio |
| 40% | 60% | 37.40% | 6.18% | 4.998 | MVP |
| 30% | 70% | 34.80% | 6.59% | 4.293 | |
| 20% | 80% | 32.20% | 7.62% | 3.371 | |
| 10% | 90% | 29.60% | 9.28% | 2.489 | |
| 0% | 100% | 27.00% | 11.56% | 1.773 | |
| -10% | 110% | 24.40% | 14.46% | 1.238 | |
| -20% | 120% | 21.80% | 17.99% | 0.851 | |
| -30% | 130% | 19.20% | 22.14% | 0.574 | |
| *-40%* | *140%* | *16.60%* | *26.91%* | *0.375* | |

*(Source: Prepared by the researcher)*

**Graph 4: Efficient Frontier for 4th Portfolio- SBI Bank & REC Ltd.**




*(Prepared by the researcher)*

**Interpretation:**

The portfolio optimization analysis involving SBI Bank and REC Ltd. Bond presents a compelling case of diversification within assets that exhibits a moderately strong positive correlation (0.54). SBI Bank offers a high annual return of 53% with a significant risk of 41%, while REC Ltd. Bond provides a comparatively lower return of 27% with a moderately substantial risk of 34%, positioning itself as a high-yield but volatile fixed-income instrument. Despite the positive correlation, portfolio simulation across various weight combinations reveals substantial gains in risk-adjusted performance. The optimum portfolio is achieved at a 50:50 allocation, offering a return of 40.00% and a risk of 6.40%, which corresponds to the highest Sharpe ratio of 5.238, indicating exceptional risk-adjusted efficiency. Meanwhile, the Minimum Variance Portfolio (MVP) lies close to a 40:60 allocation, delivering a slightly lower return of 37.40% at a reduced risk of 6.18%. The efficient frontier graph, supported by these calculations, shows that combining these two assets meaningfully reduces

volatility while preserving a substantial portion of return, defying the limitations typically associated with positively correlated pairs. This outcome reinforces the power of portfolio construction, where even assets with shared directional tendencies can be optimized to produce portfolios that significantly outperform either asset in isolation on a risk-adjusted basis.

**5th Portfolio: HCL Technologies Ltd. & SBI Gold ETF (IT + Commodity)**

**Table 11: Summary of the fifth Portfolio Combination**

| | *Return* | *Risk* | *Correlation* |
|---|---|---|---|
| *HCI Technologies Ltd.* | 27.00% | 34.00% | 0.122 |
| *SBI GOLD ETF* | 5.00% | 14.00% | |
| *Risk-Free Rate* | 6.50% | | |

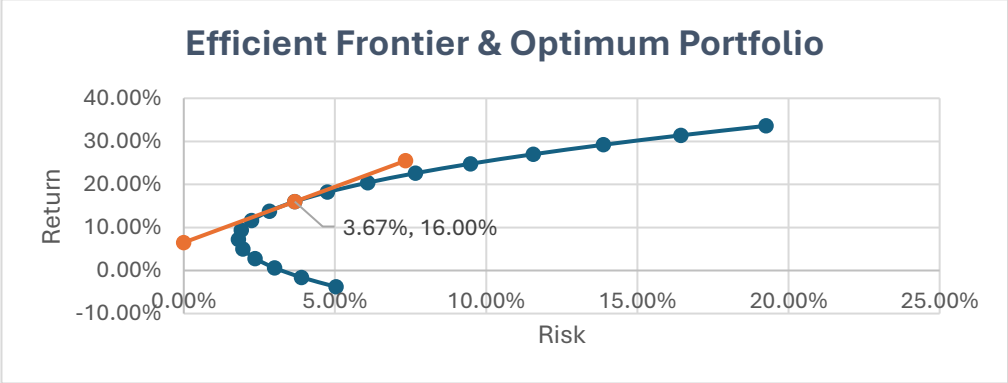
*(Source: Prepared by the researcher)*

**Table 12: Risk-Return Matrix.**

| *HCI Technologies Ltd.* | *SBI GOLD ETF* | Rp | бp | Sharpe Ratio |
|---|---|---|---|---|
| 130% | -30% | 33.60% | 19.26% | 1.4071 |
| 120% | -20% | 31.40% | 16.45% | 1.5140 |
| 110% | -10% | 29.20% | 13.88% | 1.6355 |
| 100% | 0% | 27.00% | 11.56% | 1.7734 |
| 90% | 10% | 24.80% | 9.49% | 1.9288 |
| 80% | 20% | 22.60% | 7.66% | 2.1011 |
| 70% | 30% | 20.40% | 6.08% | 2.2844 |
| 60% | 40% | 18.20% | 4.75% | 2.4611 |
| 50% | 50% | 16.00% | 3.67% | 2.5883 Optimum Portfolio |
| 40% | 60% | 13.80% | 2.83% | 2.5759 |
| 30% | 70% | 11.60% | 2.24% | 2.2720 |
| 20% | 80% | 9.40% | 1.90% | 1.5242 |
| 10% | 90% | 7.20% | 1.81% | 0.3872 MVP |
| 0% | 100% | 5.00% | 1.96% | -0.7653 |
| -10% | 110% | 2.80% | 2.36% | -1.5682 |
| -20% | 120% | 0.60% | 3.01% | -1.9627 |
| -30% | 130% | -1.60% | 3.90% | -2.0770 |
| -40% | 140% | -3.80% | 5.04% | -2.0433 |

*(Source: Prepared by the researcher)*

**Graph 5: Efficient Frontier for 5th Portfolio: HCL Technologies Ltd. & SBI Gold ETF**




*(Source: Prepared by the researcher)*

**Interpretation:**

The portfolio analysis combining HCL Technologies Ltd. and SBI GOLD ETF highlights the effectiveness of diversification between a high-return, high-risk equity and a low-return, low-risk commodity-based asset. HCL Technologies offers a return of 27% with a risk of 34%, while SBI GOLD ETF provides a modest return of 5% at a much lower risk of 14%. Despite a weak positive correlation of 0.122, the combination results in substantial portfolio efficiency gains. The optimum portfolio, offering the highest Sharpe ratio of 2.5883, is achieved at a 50:50 allocation, yielding a return of 16.00% at a reduced risk of 3.67%. Interestingly, the Minimum Variance Portfolio (MVP) occurs at a 10:90 allocation, with the lowest risk of 1.81%, though it suffers from a much lower Sharpe ratio of 0.3872. These results confirm that including even a small proportion of a volatile but high-

return asset like HCL Technologies can significantly enhance risk-adjusted returns, especially when combined with a relatively stable asset like gold. The efficient frontier graph demonstrates how the blend of equity and commodity assets, despite limited correlation, effectively shifts the portfolio toward higher returns without proportionally increasing the risk. This case clearly supports the Modern Portfolio Theory assertion that optimal diversification is achievable not only through uncorrelated assets but even when correlations are weakly positive, provided the risk-return characteristics are strategically aligned.

**6th Portfolio: REC Ltd. and SBI GOLD ETF (Fixed Income + Commodity)**

**Table 13: Summary of 6th Portfolio: REC Ltd. and SBI GOLD ETF**

| | *Return* | *Risk* | *Correlation* |
|---|---|---|---|
| *REC Ltd Bond* | *27.00%* | *34.00%* | *0.013* |
| *SBI Gold ETF* | *5.00%* | *14.00%* | |
| *Risk-Free Rate* | *6.50%* | | |

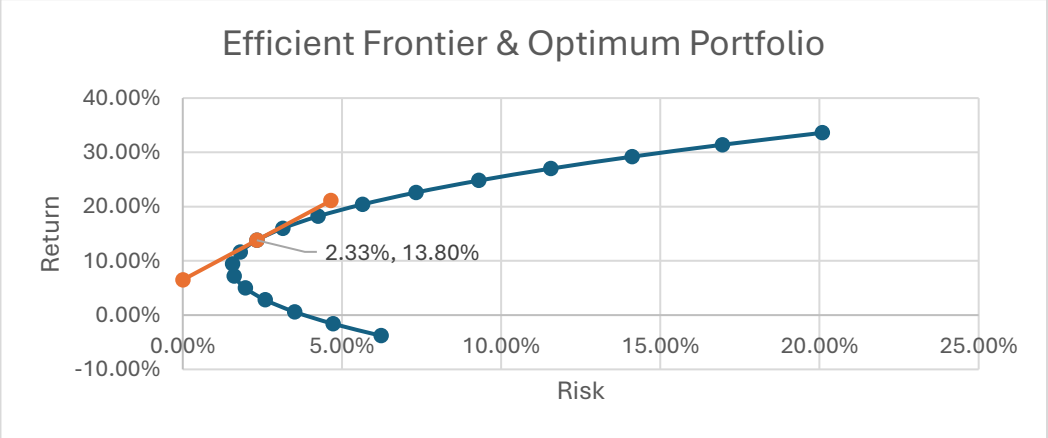
*(Source: Prepared by the researcher)*

**Table 14: Risk-Return Matrix for various combinations.**

| *SBI Bank* | *REC Ltd.* | *Rp* | *6p* | *Sharpe Ratio* | |
|---|---|---|---|---|---|
| *130%* | *-30%* | *33.60%* | *20.08%* | *1.349* | |
| *120%* | *-20%* | *31.40%* | *16.95%* | *1.469* | |
| *110%* | *-10%* | *29.20%* | *14.11%* | *1.609* | |
| *100%* | *0%* | *27.00%* | *11.56%* | *1.773* | |
| *90%* | *10%* | *24.80%* | *9.30%* | *1.968* | |
| *80%* | *20%* | *22.60%* | *7.32%* | *2.198* | |
| *70%* | *30%* | *20.40%* | *5.64%* | *2.464* | |
| *60%* | *40%* | *18.20%* | *4.25%* | *2.755* | |
| *50%* | *50%* | *16.00%* | *3.14%* | *3.024* | |
| *40%* | *60%* | *13.80%* | *2.33%* | *3.137* | Optimum Portfolio |
| *30%* | *70%* | *11.60%* | *1.80%* | *2.832* | |
| *20%* | *80%* | *9.40%* | *1.56%* | *1.854* | MVP |
| *10%* | *90%* | *7.20%* | *1.62%* | *0.433* | |
| *0%* | *100%* | *5.00%* | *1.96%* | *-0.765* | |
| *-10%* | *110%* | *2.80%* | *2.59%* | *-1.428* | |
| *-20%* | *120%* | *0.60%* | *3.51%* | *-1.679* | |
| *-30%* | *130%* | *-1.60%* | *4.72%* | *-1.715* | |
| *-40%* | *140%* | *-3.80%* | *6.22%* | *-1.655* | |

*(Source: Prepared by the researcher)*

**Graph 6: Efficient Frontier for 6th Portfolio: REC Ltd. and SBI GOLD ETF.**




*(Source: Prepared by the researcher)*

**Interpretation:**

The analysis of the portfolio comprising REC Ltd. Bond and SBI GOLD ETF illustrate a powerful risk-diversification strategy rooted in the low correlation (0.013) between the two assets. REC Ltd. Bond, despite being classified as a debt instrument, delivers a high return of 27% with substantial volatility (34%), while SBI GOLD ETF provides a safe haven with a modest return of 5% and minimal risk of 14%. The simulation of various portfolio allocations reveals that combining these assets significantly enhances the portfolio's risk-

adjusted performance. The optimum portfolio, identified at a 40:60 allocation between REC and Gold respectively, achieves the highest Sharpe ratio of 3.137, offering a return of 13.80% at a remarkably minimal risk of 2.33%. Meanwhile, the Minimum Variance Portfolio (MVP) occurs at a 20:80 allocation, delivering a lower return of 9.40% with the least risk at 1.56%. These results affirm the essence of Modern Portfolio Theory: even when dealing with a volatile high-yield bond and a stable commodity ETF, substantial portfolio efficiency can be unlocked when correlation is nearly zero. The declining Sharpe ratios beyond the optimum point further emphasize the importance of precise allocation, as overexposure to either asset dilutes the portfolio's efficiency. Thus, this pairing stands as a practical demonstration of how strategic diversification across asset classes can dramatically reduce portfolio risk while maintaining acceptable returns.

## 6. CONCLUSION.

The Markowitz Efficient Frontier analysis across various two-asset combinations—HDFC Bank Ltd., SBI Bank, HCL Technologies Ltd., REC Ltd. Bond, and SBI GOLD ETF—reveals substantial benefits of diversification driven by varied risk-return profiles and correlation structures. Combinations involving low or negative correlations consistently resulted in superior risk-adjusted returns, underscoring the foundational principle of Modern Portfolio Theory. The pairing of HCL Technologies and HDFC Bank achieved the highest Sharpe ratio of 8.65 at a 50:50 allocation, making it the most efficient portfolio overall. Similarly, the SBI Bank and REC Ltd. Bond combination delivered a Sharpe ratio of 5.24 at a balanced 50:50 weight, while the pairing of REC Bond and SBI GOLD ETF, despite including a safe asset, yielded a strong Sharpe ratio of 3.14 at a 40:60 allocation. These findings confirm that blending assets with complementary volatility and low correlation can generate portfolios that outperform individual assets both in terms of absolute return and risk efficiency.

From a practical investment perspective, for high-risk investors seeking aggressive growth, the HCL Technologies and HDFC Bank (50:50) or SBI Bank and REC Ltd. Bond (50:50) portfolios are most appropriate, offering elevated returns with optimized risk profiles. For moderate-risk investors, the HDFC Bank and REC Ltd. Bond ETF (60:40) or HCL Technologies and SBI GOLD ETF (50:50) combinations strike a balance between return enhancement and risk containment. Meanwhile, risk-averse investors or those seeking capital preservation should consider REC Ltd. Bond ETF and SBI GOLD ETF (20:80 or 40:60) portfolios, which offer low standard deviation and consistent returns with minimal exposure to market volatility. This tiered portfolio recommendation aligns asset allocation with individual risk appetites and demonstrates the practical relevance of efficient frontier modelling in real-world investment strategy formulation.

While the **Markowitz Model (Mean-Variance Optimization)** is foundational to modern portfolio theory and offers a powerful framework for constructing efficient portfolios, its **practical application** is not without limitations and challenges**.** Despite being a cornerstone of modern finance, the practical implementation of the Markowitz mean-variance model faces several significant challenges. Firstly, the model relies heavily on precise estimates of expected returns, variances, and covariances. Small errors or inaccuracies in these inputs can lead to significantly different portfolio recommendations. Secondly, for portfolios with a large number of assets, the estimation of the covariance matrix involves complicated calculations. Third, the model assumes that returns of assets are normally distributed, but in reality, financial returns often show skewness and kurtosis, which the Markowitz framework does not account for, thereby underestimating the probability of extreme events such as black swans. Due to the above difficulties associated with Markowitz Model it has found less use in practical applications of portfolio analysis. Modern approaches like the **Black-Litterman model** and **robust optimization techniques** have been developed to improve the practical usability of Markowitz's framework, which reflects the limitations of the original model in real-world applications.

## 7. REFERENCES


1. Markowitz, H. M. (1952). Portfolio selection. *The Journal of Finance, 7*(1), 77–91. https://doi.org/10.2307/2975974
2. Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance, 19*(3), 425–442. https://doi.org/10.2307/2977928
3. Bodie, Z., Kane, A., & Marcus, A. J. (2014). *Investments* (10th ed.). McGraw-Hill Education.
4. Elton, E. J., Gruber, M. J., Brown, S. J., & Goetzmann, W. N. (2011). *Modern portfolio theory and investment analysis* (9th ed.). Wiley.
5. Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics, 33*(1), 3–56. https://doi.org/10.1016/0304-405X(93)90023-5
6. Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The Review of Economics and Statistics, 47*(1), 13–37. https://doi.org/10.2307/1924119
7. Reilly, F. K., & Brown, K. C. (2012). *Investment analysis and portfolio management* (10th ed.). Cengage Learning.
8. Statman, M. (1987). How many stocks make a diversified portfolio? *Journal of Financial and Quantitative Analysis, 22*(3), 353–363. https://doi.org/10.2307/2330969

9. Bailey, J. V., Richards, T. M., & Tierney, D. E. (2007). Evaluating portfolio performance. In J. L. Maginn, D. L. Tuttle, J. E. McAleavy, & J. D. Pinto (Eds.), *Managing investment portfolios* (3rd ed., pp. 689–712). CFA Institute.
10. Levy, H., & Sarnat, M. (1970). International diversification of investment portfolios. *The American Economic Review, 60*(4), 668–675. https://www.jstor.org/stable/1817984
11. Haugen, R. A. (2001). *Modern investment theory* (5th ed.). Prentice Hall.
12. Malkiel, B. G. (1999). *A random walk down Wall Street* (7th ed.). W. W. Norton & Company.
13. Solnik, B. H. (1974). Why not diversify internationally rather than domestically? *Financial Analysts Journal, 30*(4), 48–54. https://doi.org/10.2469/faj.v30.n4.48
14. Kritzman, M. (1993). Portfolio optimization: The state of the art in theory and practice. *Financial Analysts Journal, 49*(1), 30–33. https://doi.org/10.2469/faj.v49.n1.30
15. Gupta, R., & Basu, P. (2011). Portfolio optimization in the Indian stock market: An empirical study. *International Journal of Financial Management, 1*(2), 19–32.