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Research Article

Multi-Objective Welded Beam Design Optimization By Four Valued Refined Neutrosophic Optimization Technique: A Comparative Study

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ABSTRACT

Structural design optimization in precise or imprecise environment plays an important role in civil engineering as well as in mathematics. Conventionally in most of the situation structural designs are treated as single objective optimization problem with cost as objective function. Practically there exists some real life problems where multiple and conflicting objects frequently exist.So a methodology known as Multi-objective Four Valued Refined Neutrosophic Optimization (MOFVRNO) technique has been introduced in this paper.This optimization technique is generalization of fuzzy set (FS)theory,Intuitionistic Fuzzy Set (IFO)Theory and above all Single Valued Neutrosophic Set Optimization (SVNSO)theory.In this problem cost and deflection of the beam have been considered as objective functions. Also in this welded beam design problem maximum shear stress in the weld group, maximum bending stress in the beam, buckling load of the beam have been considered as constraints. The design variables are length ,depth ,height,width in this optimization problem.Now from the mathematical point of view indeterminacy is again refined as uncertain(U),Contradiction(Truth∧Falsity,T$\wedge F$) in Four valued Refined Neutrosophic Optimization technique.The results obtained by different methods like nonlinear optimization technique,Fuzzy optimization technique ,Intuitionistic Fuzzy Optimization technique ,Single Valued Neutrosophic Optimization technique andFour Valued Refined Neutrosophic Optimization technique have been discussed in the perspective of welded beam design problem in this paper. It helps to make comparison among them to find most cost effective method.

Keywords: Neutrosophi Set, Single Valued Neutrosophic Set, Four Valued Refined Neutrosophic Set, Multi-objective Four Valued Refined Neutrosophic Optimization Technique, Multi objective Welded Beam Design.

Mathematics Subject Classifications: 90C30,90C70.

1.Introduction:

Most of the real life problem includes imprecision. In that case most of the time we have to rely on imprecise human reasoning to understand the physical process. But this imprecision carries very useful information.Until 19th century scientist and mathematicians defined this uncertainty as undesirable state and they ignored at all costs. But later on physicist observed that Newtonian Mechanics and its underlining calculus did not solve the problems at the molecular level.Then researcher replaced particulars of microscopic entities by statistical average which is based on statistical mechanics.In this way uncertainties were taken into account by developing statistical mechanics.Infact statistical mechanics is based on probability theory which can handle various uncertainties. In between 19th century to the late 20 th century probability theory was the leading theory used to solve uncertainty in scientific model. In 1960 Dempster first time introduced the concept of absence of information in his famous theory Evidence.Zadeh(1965) introduced an essential idea in logic that named fuzzy set theory.Zadeh's work influenced the concept of uncertainty and destroyed the notion of probability theory as a solitary representation of uncertainty. Zadeh challenged the binary logic of probability theory by illustrating possibility theory which is special case of fuzzy sets.Glennshafer in the year 1970's extended Dempster's work and developed a complete theory of evidence. Later on in 1980 so many researcher combined the concept of evidence theory, possibility theory with the use of fuzzy measures. From the philosophical point of view uncertainty can be considered as inverse information.Information or data about certain scientific or engineering problem might be uncertain, vague, fuzzy, ambiguous, dubois, conflicting, lacking in some other way. When this type of situation raised we need the concept of fuzzy to cope with them. Introducing fuzzy number Zadeh handled this type of imprecise data.Fuzzy set is usually consists of different objects with membership function or grade.The set theoretic notion of relation, union, intersection, complement, concavity, convexity can be defined for fuzzy sets also. To include uncertainty in the membership degree Intuitionistic Fuzzy Set was first introduced by Attanassov(1997).To deal with inexact,uncertain,vague parameters and inexact information in real world problem Smarandache(1999) introduced Neutrosophic Set Theory. Later on considering universe of discourse as real line Wang et al(2010) introduced single valued Neutrosophic set (SVNS) as a special case of NS. Later on Kandasamy (2016) ,Smarandache (2018),Zadeh(2018) introduced double refined Neutrosophic set(DRNS),Triple Refined Neutrosophic Set(TRNS) to handle incomplete and inconsistent information efficiently than SVNS.Four Valued Refined Neutrosophic Set has been used in the paper of Sajida et al(2020).On the other hand there exist some real life problem in engineering branch where nonlinear model is frequently used.Ziemmermann(1978) first introduced LPP and this work has been considered as extension of Bellman's Zadeh's theory(1970).Later on somany researcher considered optimization method in imprecise environment for LPP and nonlinear programming problem as their subject of interest.They are Tanaka Asai(1984),Chanas(1983),Verdigay (1984),Carlsonand Korhonen(1986),Campos (1989), Lavandula (1989), Sakawa and Yano (1989), Carls and Dev (2013), Guu and Wu (2019), Zhou et al (2020),Ghodousian(2019),Sahindis (2004),Chakraborty et al(2013).Wang et al (2017) and Bharati(2018a,b) and so on.Now uncertainty also has been widely addressed by Sarkar et al (2016,2017) in their paper with different type of single objective and multi-objective structural design problem.The aim of this paper is to make a comparative study of the result obtained by fuzzy, intuitionistic fuzzy ,single valued Neutrosophic optimization technique and four valued refined optimization technique.In section 2 prelimineries of four valued refined Neutrosophic set has been discussed .In section 3 a multiobjective FVRNSO technique has been studied.In section 4welded beam design has been solved by FVRNO technique.In section 5 Numerical Example has been illustrated. Lastly in section 6 we make a conclusion.

2 Preliminaries:

2.1 Fuzzy Set(2019):A fuzzy set (Zadeh 1965) is a set that can contains elements partially that is the property that an elements belong to the set under consideration be a truth with a partial degree of truth. Given a universe set X, a fuzzy set $\tilde{A}^F$ is an ordered set (Universe element ,truth degree of membership of that element) denoted mathematically as

$$\tilde{A}^F = \{x, T_{\tilde{A}^F}(x) : x \in X\}$$

Where $T_{\tilde{A}^F}(x) \in [0,1]$.

2.2 Intuitionistic Fuzzy Set: Given a Universe X, an intuitionistic fuzzy set (Attanassov 1986) is a set of triplet $\left(x, T_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x)\right)$ where $T_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x)$ represents the truth and falsity grade respectively and $0 \le T_{\tilde{A}^F}(x) + F_{\tilde{A}^F}(x) \le 1$,$T_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x) \in [0,1]$.Clearly one can obtain a fuzzy set when $T_{\tilde{A}^F}(x) + F_{\tilde{A}^F}(x) = 1$.

2.3 Neutrosophic Set: Neutrosophic Set (Smarandache 1995) is a generalized concept in which each component $x \in X$to a set $\tilde{A}^N$ has the membership degree $T_{\tilde{A}^F}(x)$, non-membership degree $F_{\tilde{A}^F}(x)$ as well as a degree of indeterminacy $I_{\tilde{A}^F}(x)$ where $T_{\tilde{A}^F}(x), I_{\tilde{A}^F}(x)$,
and $F_{\tilde{A}^F}(x)$are real slandered or nonstandard subsets of $]0^-, 1^+[$.

2.4 Single Valued Neutrosophic Set (SVNS): In single valued Neutrosophic Set (Smarandache 2010) each $x \in X$ to a set $\tilde{A}^{SN}$is characterized by$T_{\tilde{A}^F}(x), I_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x)$ belong to [0,1]
And $0 \le T_{\tilde{A}^F}(x) + I_{\tilde{A}^F}(x) + F_{\tilde{A}^F}(x) \le 3$. Thus a single valued Neutrosophic set $\tilde{A}^{SN}$ is expressed as $\tilde{A}^{SN} = \{x, T_{\tilde{A}^F}(x), I_{\tilde{A}^F}(x), F_{\tilde{A}^F}(x) : x \in X\}$.

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2.5. Four Valued Refined Neutrosophic Set(FVRNS)(G. Freen et.al. 2019): Refinement of any of T,I,F involves the extenics (Zadeh 2018).Four valued refined Neutrosophic Set can be defined in a number of ways by splitting indeterminacy in different manners. Here in the present work we only focus in the below mentioned criteria.A four valued refined Neutrosophic set is such a type of Neutrosophic set in which indeterminacy split into two parts as U=Uncertain and C=Contradiction where $C = T \wedge F$.The values of T,I,C and F belong to [0,1] and $0 \le T + U + C + F \le 4$.Thus FVRNS is represented as

$$\tilde{A}^{RN} = \{(x, T_{\tilde{A}^{RN}}, I_{\tilde{A}^{RN}}, F_{\tilde{A}^{RN}}) : x \in X\}$$

When X is continuous then

$$\tilde{A}^{RN} = \int \{x, T_{\tilde{A}^{RN}}, U_{\tilde{A}^{RN}}, C_{\tilde{A}^{RN}}, F_{\tilde{A}^{RN}}/dx : x \in X\}$$

And when X is discrete its representation will be

$$\tilde{A}^{RN} = \sum_{i=1}^{n} \{T_{\tilde{A}^{RN}}, U_{\tilde{A}^{RN}}, C_{\tilde{A}^{RN}}, F_{\tilde{A}^{RN}}/x_i : x_i \in X\}$$

The complement of four valued refined Neutrosophic Set is denoted by C_r

$T_{C_r}(x) = F_{\tilde{A}^{RN}}$,

$$U_{C_r}(x) = 1 - U_{\tilde{A}^{RN}}$$

,

$$C_{C_r}(x) = 1 - C_{\tilde{A}^{RN}}$$
$$F_{C_r}(x) = T_{\tilde{A}^{RN}}$$

For all $x \in X$.

The definition of FVRNS and the complement guarantees the following results

Th-1(G. Freen et.al. 2019):For two FVRNS $\tilde{A}^{RN}$ and$\tilde{B}^{RN}$$\tilde{A}^{RN} \subseteq \tilde{B}^{RN}$ iff

$$T_{\tilde{A}^{RN}}(x) \le T_{\tilde{B}^{RN}}(x),$$
$$U_{\tilde{A}^{RN}}(x) \le U_{\tilde{B}^{RN}}(x),$$
$$C_{\tilde{A}^{RN}}(x) \le C_{\tilde{B}^{RN}}(x)$$
$$F_{\tilde{A}^{RN}}(x) \ge F_{\tilde{B}^{RN}}(x)$$

Th-2 (G. Freen et.al. 2019)$\tilde{A}^{RN} \subseteq \tilde{B}^{RN}$ iff $C_r(\tilde{B}^{RN}) \le C_r(\tilde{A}^{RN})$.The union of two four valued refined Neutrosophic sets $\tilde{A}^{RN}$ and $\tilde{B}^{RN}$ is a four valued refined Neutrosophic Set $\tilde{C}^{RN}$ indicated as $\tilde{C}^{RN} = \tilde{A}^{RN} \cup \tilde{B}^{RN}$ whose truth membership T, uncertainty U,contradictory C and falsity membership are identified with those of $\tilde{A}^{RN}$ and $\tilde{B}^{RN}$ as

$$T_{\tilde{C}^{RN}}(x) = max\{T_{\tilde{A}^{RN}}(x), T_{\tilde{B}^{RN}}(x)\}$$
$$U_{\tilde{C}^{RN}}(x) = max\{U_{\tilde{A}^{RN}}(x), U_{\tilde{B}^{RN}}(x)\}$$
$$C_{\tilde{C}^{RN}}(x) = max\{C_{\tilde{A}^{RN}}(x), C_{\tilde{B}^{RN}}(x)\}$$
$$F_{\tilde{C}^{RN}}(x) = min\{F_{\tilde{A}^{RN}}(x), F_{\tilde{B}^{RN}}(x)\}$$

Th.3 (G. Freen et.al. 2019)$\tilde{A}^{RN} \cup \tilde{B}^{RN}$ is the smallest four valued refined Neutrosophic set containing both $\tilde{A}^{RN}, \tilde{B}^{RN}$.

Proof: The proof is obvious.

The intersection of two four valued refined Neutrosophic Set $\tilde{A}^{RN}$ and $\tilde{B}^{RN}$ is a four valued refined Neutrosophic Set $\tilde{C}^{RN}$ indicated as $\tilde{C}^{RN} = \tilde{A}^{RN} \cap \tilde{B}^{RN}$ whose truth membership T, uncertainty U,contradictory C and falsity membership are identified with those of $\tilde{A}^{RN}$ and $\tilde{B}^{RN}$ as follows

$$T_{\tilde{C}^{RN}}(x) = min\{T_{\tilde{A}^{RN}}(x), T_{\tilde{B}^{RN}}(x)\}$$
$$U_{\tilde{C}^{RN}}(x) = min\{U_{\tilde{A}^{RN}}(x), U_{\tilde{B}^{RN}}(x)\}$$
$$C_{\tilde{C}^{RN}}(x) = min\{C_{\tilde{A}^{RN}}(x), C_{\tilde{B}^{RN}}(x)\}$$
$$F_{\tilde{C}^{RN}}(x) = max\{F_{\tilde{A}^{RN}}(x), F_{\tilde{B}^{RN}}(x)\}$$

Th.4. (G. Freen et.al. 2019)$\tilde{A}^{RN} \cap \tilde{B}^{RN}$ is the largest four valued refined Neutrosophic set containing both $\tilde{A}^{RN}, \tilde{B}^{RN}$.

Proof: The proof is obvious.

The difference of two four valued refined Neutrosophic Set $\tilde{A}^{RN}$ and $\tilde{B}^{RN}$ is a four valued refined Neutrosophic Set $\tilde{C}^{RN}$ indicated as $\tilde{C}^{RN} = \tilde{A}^{RN} \backslash \tilde{B}^{RN}$ whose truth membership T, uncertainty U,contradictory C and falsity membership are identified with those of $\tilde{A}^{RN}$ and $\tilde{B}^{RN}$ as follows

$$T_{\tilde{C}^{RN}}(x) = min\{T_{\tilde{A}^{RN}}(x), F_{\tilde{B}^{RN}}(x)\}$$
$$U_{\tilde{C}^{RN}}(x) = min\{U_{\tilde{A}^{RN}}(x), 1 - U_{\tilde{B}^{RN}}(x)\}$$
$$C_{\tilde{C}^{RN}}(x) = min\{C_{\tilde{A}^{RN}}(x), 1 - C_{\tilde{B}^{RN}}(x)\}$$
$$F_{\tilde{C}^{RN}}(x) = max\{F_{\tilde{A}^{RN}}(x), T_{\tilde{B}^{RN}}(x)\}$$

It can be easily checked that four valued refined Neutrosophic Set satisfies most properties such as associativity, distributivity, idempotency absorption, involution and De Morgan's law. But it does not satisfy the principle of exclude middle like fuzzy set ,IFS and SVNS. All of the above mentioned operation can be verified by the example given below.

Example: Semantic webservice quality evaluation Zadeh(2018) is done by domain experts. Assume $[a_1, a_2, a_3]$ in which $a_1$is capability,a_2 is trustworthiness and a_3 is price where a_1, a_2, a_3 are in [0,1].From expert questionnaire the option could be a grade of "excellent service" a grade of "intermediate service " a grade of "contradictory service" a grade of "bad service and Y are four valued refined Neutrosophic Sets of A defined by

$$X = \frac{< 0.2,0.6,0.3,0.5 >}{a_1} + \frac{< 0.5,0.3,0.7,0.4 >}{a_2} + \frac{< 0.7,0.3,0.4,0.2 >}{a_3}$$
$$Y = \frac{< 0.4,0.7,0.3,0.5 >}{a_1} + \frac{< 0.2,0.8,0.3,0.5 >}{a_2} + \frac{< 0.3,0.6,0.8,0.5 >}{a_3}$$

3. Multi-objectiveFour Valued Refined Neutrosophic Optimization Technique

Consider a nonlinear multi-objective optimization problem
Minimize $[f_i(x)], i = 1,2, \dots p$ (1)
Such that $[g_i(x)] \le b_j j = 1,2. \dots q$ (2)

Where x are decision variables ,$f_i(x)$ represents here objective functions, $g_i(x)$ represents constraint functions and $p$and $q$represent the number of objective functions and constraint respectively.Now the decision set σa conjunction of four valued neutrosophic objectives and constraints is defined as

$$\tilde{D} = \left(\bigcap_{p=1}^{k} \tilde{O}_k \right) \cap \left(\bigcap_{p=1}^{k} \tilde{L}_k \right) = \left(x, T_{\tilde{D}}, U_{\tilde{D}}, C_{\tilde{D}}, F_{\tilde{D}}, \right) \quad (3)$$

Where $T_{\tilde{D}}(x) = \min\left(T_{\tilde{O}_1}(x), T_{\tilde{O}_2}(x), \dots . T_{\tilde{O}_p}(x); L_{\tilde{L}_1}(x), T_{\tilde{L}_2}(x) \dots . T_{\tilde{L}_q}(x) \right)$ (4)

$$U_{\tilde{D}}(x) = \min\left(U_{\tilde{O}_1}(x), U_{\tilde{O}_2}(x), \dots . U_{\tilde{O}_p}(x); U_{\tilde{L}_1}(x), U_{\tilde{L}_2}(x) \dots . U_{\tilde{L}_q}(x) \right) \quad (5)$$

$$C_{\tilde{D}}(x) = \min\left(C_{\tilde{O}_1}(x), C_{\tilde{O}_2}(x), \dots . C_{\tilde{O}_p}(x); C_{\tilde{L}_1}(x), C_{\tilde{L}_2}(x) \dots . C_{\tilde{L}_q}(x) \right) \quad (6)$$

$$F_{\tilde{D}}(x) = \max\left(F_{\tilde{O}_1}(x), F_{\tilde{O}_2}(x), \dots . F_{\tilde{O}_p}(x); F_{\tilde{L}_1}(x), F_{\tilde{L}_2}(x) \dots . F_{\tilde{L}_q}(x) \right) \quad (7)$$

Where $T_{\tilde{D}}(x), U_{\tilde{D}}(x), C_{\tilde{D}}(x), F_{\tilde{D}}(x)$ represent truth,uncertainty,contradictory and falsity membership of four valued refined neutrosophic decision set respectively.Now using the four valued refined neutrosophic optimization the above problem is remodeled as non linear optimization as

$Max\alpha, Max\gamma, Min\beta, Max\delta$ (8)
such that $T_{\tilde{O}_k}(x) \ge \alpha, U_{\tilde{O}_k}(x) \ge \gamma, F_{\tilde{O}_k}(x) \le \beta, C_{\tilde{O}_k}(x) \ge \delta$ (9)
$T_{\tilde{L}_{j(x)}}(x) \ge \alpha, U_{\tilde{L}_{j(x)}}(x) \ge \gamma, F_{\tilde{L}_{j(x)}}(x) \le \beta, C_{\tilde{L}_{j(x)}}(x) \ge \delta$ (10)
$\alpha \ge \beta, \alpha \ge \gamma, \alpha \ge \delta$ (11)
$\alpha + \beta + \gamma + \delta \le 4$ (12)
$\alpha, \beta, \gamma, \delta \in [0,1]$ (13)
$g_j(x) \le b_j, x \ge 0, j = 1,2 \dots q$ (14)
$x \ge 0$ (15)

Computational Algorithm:
Step-1: Solve the first objective function as single objective function taken from set of k objectives.The values of decision variables and objective function will be computed subject to the given constraints.
Step-2:Now compute the values of unresolved objectives i.e (k-1) using decision variables from step 1.
Step-3: Continue to the remaining (k-1) objective functions by going through step 1 and step 2

$$\begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \dots & f_p(x^1) \\ f_1(x^2) & f_2^*(x^2) & \dots & f_p(x^2) \\ \dots & \dots & \dots & \dots \\ f_1(x^r) & f_2(x^r) & \dots & f_p^*(x^r) \end{bmatrix} \quad (16)$$

Step-4 :Find the lower bound $\tilde{L}_p{}^T$ and the upper bound $\tilde{U}_p{}^T$ corresponding to each objective $f_k(x)$.The lower and upper bounds for truth membership of objectives are $\tilde{U}_p{}^T = max[f_2(x^r)]$ (17)
$\tilde{L}_p{}^T = min[f_2(x^r)]$where $r = 1,2\dots,p$The upper bound $\tilde{U}_p{}^F$and lower bound $\tilde{L}_p{}^F$for falsity membership of objectives are
$\tilde{U}_p{}^F = \tilde{U}_p{}^T$and $\tilde{L}_p{}^F = \tilde{L}_p{}^T + t(\tilde{U}_p{}^T - \tilde{L}_p{}^T)$. (18)

Upper bound $\tilde{U}_p{}^F$and lower bound $\tilde{L}_p{}^U$for uncertainty membership of objectives are $\tilde{L}_p{}^U = \tilde{L}_p{}^T$ (19)
$\tilde{U}_p{}^U = \tilde{L}_p{}^T + s(\tilde{U}_p{}^T - \tilde{L}_p{}^T)$ (20)
And upper bound $\tilde{U}_p{}^F$and lower bound $\tilde{L}_p{}^U$for contradictory membership of objectives are $\tilde{L}_p{}^C = \tilde{L}_p{}^T \wedge \tilde{L}_p{}^F$ (21)
$\tilde{U}_p{}^C = \tilde{L}_p{}^T \wedge \tilde{L}_p{}^F + l(\tilde{U}_p{}^T \wedge \tilde{U}_p{}^F - \tilde{L}_p{}^T \wedge \tilde{L}_p{}^F)$ (22)
Where $t, s, l \in [0,1]$
Step-5: In this step ,we will define truth,uncertainty,falsity and contradictory membership functions as follows

$$T_p(f_p(x)) = \begin{cases} 1 if f_p(x) \le \tilde{L}_p{}^T \\ \frac{\tilde{U}_p{}^T - f_P(x)}{\tilde{U}_p{}^T - \tilde{L}_p{}^T} if \tilde{L}_p{}^T \le f_P(x) \le \tilde{U}_p{}^T \\ 0 if f_P(x) \ge \tilde{U}_p{}^T \end{cases} \quad (23)$$

$$U_p(f_p(x)) = \begin{cases} 1 if f_p(x) \le \tilde{L}_p{}^U \\ \frac{\tilde{U}_p{}^U - f_P(x)}{\tilde{U}_p{}^U - \tilde{L}_p{}^U} if \tilde{L}_p{}^U \le f_P(x) \le \tilde{U}_p{}^U \\ 0 if f_P(x) \ge \tilde{U}_p{}^U \end{cases} \quad (24)$$

$$F_p(f_p(x)) = \begin{cases} 0 if f_p(x) \le \tilde{L}_p{}^F \\ \frac{f_P(x) - \tilde{L}_p{}^F}{\tilde{U}_p{}^F - \tilde{L}_p{}^F} if \tilde{L}_p{}^F \le f_P(x) \le \tilde{U}_p{}^F \\ 1 if f_P(x) \ge \tilde{U}_p{}^F \end{cases} \quad (25)$$

$$C_p(f_p(x)) = \begin{cases} 1 if f_p(x) \le \tilde{L}_p{}^C \\ \frac{\tilde{U}_p{}^C - f_P(x)}{\tilde{U}_p{}^C - \tilde{L}_p{}^C} if \tilde{L}_p{}^C \le f_P(x) \le \tilde{U}_p{}^C \\ 0 if f_P(x) \ge \tilde{U}_p{}^C \end{cases} \quad (26)$$

Step-6:Now four valued refined neutrosophic optimization method for multi-objective nonlinear programming problem gives a corresponding non-linear problem as

Max $\alpha - \beta + \gamma + \delta$ (27)

Such that $T_p(f_p(x)) \ge \alpha$ (28)

$U_p(f_p(x)) \ge \gamma$ (29)

$F_p(f_p(x)) \le \beta$ (30)

$C_p(f_p(x)) \geq \delta$ (31)

$g_j(x) \leq b_j, j = 1,2\ldots, q$ (32)

$\alpha + \beta + \gamma + \delta \leq 4$ (33)

$\alpha \geq \beta, \alpha \geq \gamma, \alpha \geq \delta$ (34)

$x \geq 0$ (35)

Where $\alpha, \beta, \gamma, \delta \in [0,1]$

4. Numerical Illustration of Multi-objective Welded Beam Design optimization using Four Valued Refined Neutrosophic Optimization Technique.

In design formulation ,a welded beam ([2017],fig.1) has to be designed at minimum cost whose constraints are shear stress in weld (τ),bending stress in the beam (σ),buckling load on the bar (P) and deflection of the beam(δ). The design variables are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} h \\ l \\ t \\ b \end{bmatrix}$ where h is the weld size, l is the length of the weld ,t is the depth of the welded beam and b is the width of the welded beam.

The single objective crisp welded beam optimization problem can be formulated as

$Minimize\ C(X) = 1.10471x_1^2x_2 + 0.04811(14 + x_2)x_3x_4$ (36)

Such that

$g_1(x) = \tau - \tau_{max} \leq 0$ (37)

$g_2(x) = \sigma(x) - \sigma_{max} \leq 0$ (38)

$g_3(x) = x_1 - x_4 \leq 0$ (39)

$g_4(x) = 1.10471x_1^2x_2 + 0.04811(14 + x_2)x_3x_4 - 5 \leq 0$ (40)

$g_5(x) = 0.125 - x_1 \leq 0$ (41)

$g_6(x) = \delta(x) - \delta_{max} \leq 0$ (42)

$g_7(x) = P - P_C(x) \leq 0$ (43)

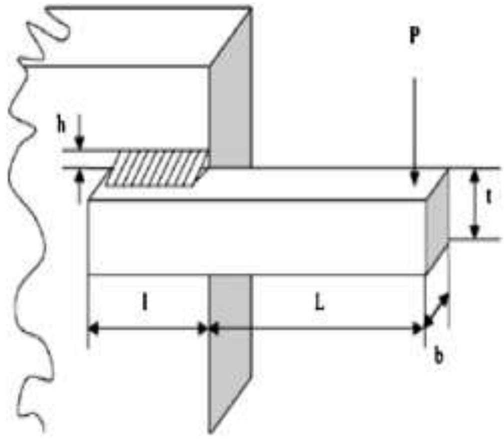

Fig.1. Design of welded beam

$x_1, x_2, x_3, x_4 \in [0,1]$ (44)

Where $\tau(x) = \sqrt{\tau_1^2 + 2\tau_1\tau_2\frac{x_2}{2R} + \tau_2^2}$;

$\tau_1 = \frac{P}{\sqrt{2}x_1x_2}; \tau_2 = \frac{MR}{J}$;$M = P\left(L + \frac{x_2}{2}\right)$;$R = \sqrt{\frac{x_2^2}{4} + \frac{(x_1+x_3)^2}{4}}$;$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \frac{(x_1+x_3)^2}{2}\right]\right\}$; $\sigma(x) = \frac{6PL}{x_4x_3^2}$; $\delta(x) = \frac{4PL^3}{Ex_4x_3^2}$; $P_C(x) = \frac{4.013\sqrt{EGx_3^6x_4^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$.

Again P =force on beam;L =beam length beyond weld;x_1 =Height of the welded beam;x_2 =length of the welded beam;x_3 =depth of the welded beam;x_4 =width of the welded beam;$\tau(x)$ =design shear stress;$\sigma(x)$ =design normal stress for beam material;M =moment of P about centre of gravity of the weld;J =polar moment of inertia of the weld group,G =shearing modulus of beam material;δ_{max} =maximum deflection;τ_1 =primary stress on weld throat;τ_2 =secondary torsional stress on weld.

Input data of welded beam design problem (eq36-44) are given in the table 1 as follows

Table:1- Input data for welded beam design problem

Applied load $P(lb)$	Beam length beyond weld L(in)	Young Modulus $E(psi)$	Value of G(psi)	Maximum allowable shear stress $\tau_{max}(psi)$	Maximum allowable normal stress $\sigma_{max}(psi)$	Maximum allowable deflection $\delta_{max}(in)$
6000	14	3×10^6	12×10^6	13600	30000 50	0.25

Solution: According to step 2 of section 3 pay of matrix can be formulated

$$\begin{array}{c} \\ X^1 \\ X^2 \end{array} \begin{array}{cc} C(X) & \delta(X) \\ \left[\begin{array}{cc} 7.700387 & 0.2451363 \\ 11.91672 & 0.1372000 \end{array}\right] \end{array}$$

(45)

$$U_{C(X)}^T = 11.91672, L_{C(X)}^T = 7.700387;$$
$$U_{C(X)}^C = 7.700387 + t4.216333; L_{C(X)}^C = 7.700387;$$
$$U_{C(X)}^F = 11.91672; L_{C(X)}^F = 7.700387 + r(4.216333);$$
$$U_{C(X)}^U = 7.700387 + s4.216333; L_{C(X)}^U = 7.700387;$$
$$U_{\delta(X)}^T = 0.2451363; L_{\delta(X)}^T = 0.1372000; U_{\delta(X)}^C = 0.1372000 + t(10.1079363);$$
$$L_{\delta(X)}^C = 0.1372000; U_{\delta(X)}^F = 0.2451363; L_{\delta(X)}^F = 0.1372000 + r(0.1079363);$$
$$U_{\delta(X)}^U = 0.1372000 + s(0.1079363); L_{\delta(X)}^U = 0.1372000;$$

where $t, s, r \in (0,1); t = 0.27; s = 0.8; r = 0.35$
Now we define the membership function for T, F, U and C as

$$T_{C(X)}(C(X)) = \begin{cases} 1 \quad if C(X) \leq 7.700387 \\ \frac{11.91672 - C(X))}{14.216333} if 7.700387 \leq C(X) \leq 11.91672 \\ 0 \quad if C(X) \geq 11.91672 \end{cases}$$

(46)

$$U_{C(X)}(C(X)) = \begin{cases} 1 if C(X) \leq 7.700387 \\ \frac{7.700387 - C(X)}{(4.216333)s} if 7.700387 \leq C(X) \leq 4.216333s + 7.700387 \\ 0 if C(X) \geq 7.700387 + s4.216333 \end{cases}$$

(47)

$$F_{C(X)}(C(X)) = \begin{cases} 0 if C(X) \leq 7.700387 + r4.216333 \\ \frac{C(X) - 7.700387}{4.216333 - r4.216333} if 7.700387 + r4.216333 \leq C(X) \leq 11.91672 \\ 1 if C(X) \geq 11.91672 \end{cases}$$

(48)

$$C_{C(X)}(C(X)) = \begin{cases} 1 if C(X) \leq 7.700387 \\ \frac{7.700387 + t4.216333 - C(X)}{t4.216333} if 7.700387 \leq C(X) \leq 7.700387 + t4.216333 \\ 0 if C(X) \geq 7.700387 + t4.216333 \end{cases} T_{\delta(X)}(\delta(X)) = \begin{cases} 1 if \delta(X) \leq 0.1372000 \\ \frac{0.2451363 - \delta(X)}{0.1079363} if 0.1372000 \leq \delta(X) \leq 0.2451363 \\ 0 if \delta(X) \geq 0.2451363 \end{cases}$$

(49)

$$U_{\delta(X)}(\delta(X)) = \begin{cases} 1 if \delta(X) \leq 0.1372000 \\ \frac{0.1372000 - \delta(X)}{s0.1079363} if 0.1372000 \leq \delta(X) \leq 0.1372000 + s0.1079363 \\ 0 if \delta(X) \geq 0.1372000 + s0.1079363 \end{cases}$$

(50)

$$F_{\delta(X)}(\delta(X)) = \begin{cases} 1 if \delta(X) \leq 0.1372000 \\ \frac{\delta(X) - 0.1372000 - s0.1079363}{0.1079363 - s0.1079363} if 0.1372000 + s0.1079363 \leq \delta(X) \leq 0.2451363 \\ 0 if \delta(X) \geq 0.2451363 \end{cases}$$

(51)

$$C_{\delta(X)}(\delta(X)) = \begin{cases} 1 if \delta(X) \leq .1372000 \\ \frac{.1372000 - t0.1079363 - \delta(X)}{t.1079363} if .1372000 \leq \delta(X) \leq .1372000 + t0.1079363 \\ 0 if \delta(X) \geq .1372000 + t0.107936 \end{cases} \quad (52)$$

Now using above mentioned truth, uncertainty, contradictory, falsity membership function(eq. 46-51) can be solved for model I and model II by Four Valued Refined Neutrosophic Optimization Technique with different values of s,t,r. The optimum design variable such as height, length, depth, width and cost of welding of welded beam are given in table 2 and the solution are compared with the other deterministic optimization method like fuzzy, intuitionisticfuzzy, Neutrosophic optimization

Table:2- A comparative result of structural weight and deflection for $s = 0.75, t = 0.27, r = 0.25$

Methods	Height $x_1(inch)$	Length $x_2(inch)$	Depth $x_3(inch)$	Width $x_4(inch)$	Welding cost $C(X)\$$	Deflection $\delta(X)$
Fuzzy Optimization (FO)	1.502140	2	.1041478	.1822257	5	.25
Intuitionistic Fuzzy Optimization(IFO)	1.093734	1.768669	.1041093	.1820911	2.351701	25
Single Valued Refined Neutrosophic Optimization (SVRNO)Technique	1.502158	2	.1038745	.1812704	5	.25
FVRNO	.2	.2	.1105070	.1740104	.02197445	.25

It can be observed that FVRNO is the best method in finding minimum cost as compare to other deterministic method.

6. Conclusion:

In this paper we have used Four Valued Refined Neutrosophic Optimization technique to minimize cost of welding and minimize deflection for multi-objective welded beam design problem.Here we made a comparative study of the results obtained from different methods.Also we have observed that using this approach this method can overcome limitations that arises due to uncertainty and imprecise data .Moreover the method used in this paper is a significant improvement over other technique described in the literature in terms of both time and computational efficiency.this research suggests that the application of FVRNO can be extended to solve various engineering design problem.

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