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Research Article

Between Closed Sets And $g\omega$-Closed Sets

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ARTICLE INFO	ABSTRACT
	Topology, as a branch of mathematics, deals extensively with the study of spaces and their properties under continuous transformations. Central to this study are concepts like closed sets, which encapsulate the notion of completeness and limit points within a space. Recently, the notion of $g\omega$-closed sets has emerged, offering a refined understanding of convergence in weak topologies. This paper explores the intricate relationship between closed sets and $g\omega$-closed sets within the framework of topology. It delves into the fundamental characteristics that distinguish these two types of sets and investigates their implications for the convergence of sequences and continuity within topological spaces. Through a comprehensive analysis, this paper elucidates the conditions under which a set can be considered both closed and $g\omega$--closed, shedding light on the subtle nuances of convergence and completeness. Additionally, it provides examples and counterexamples to illustrate the distinctions between these concepts, offering insights into the behaviour of sequences and their limit points. By bridging the gap between closed sets and $g\omega$--closed sets, this paper contributes to a deeper understanding of convergence properties in topological spaces. It serves as a valuable resource for researchers and practitioners seeking to explore the intricate relationships between different types of sets and their implications for the study of continuous transformations and spatial structures in topology

1. Introduction

In 1970, Levine [7] introduced the notion of generalized closed (g-closed) sets in topological spaces. In 1982, Hdeib [5] introduced the notion of ω-closed sets in topological spaces. Recently, many variations of g-closed sets are introduced and investigated. One among them is $g\omega$-closed sets which were introduced by Khalid Y. Al-Zoubi [6]. In 2006, Noiri and Popa [12] introduced the notion of mg*-closed sets and studied the basic properties, characterizations and preservation properties. Also, they defined several subsets which lie between closed sets and g-closed sets. In this paper, we introduce the notion of $mg\omega$-closed sets and obtain the basic properties, characterizations and preservation properties. In the last section, we define several new subsets which lie between closed sets and $g\omega$-closed sets.

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2. Preliminaries

Let (X, τ) be a topological space and A be a subset of X. The closure of A and the interior of A are denoted by cl(A) and int(A), respectively. A subset A is said to be regular open [19] if int(cl(A)) = A. The finite union of regular open sets is said to be π-open [24].

Definition 2.1.
A subset A of a topological space (X, τ) is said to be α-open [10] if A ⊂ int(cl(int(A))). The complement of an α-open set is said to be α-closed.

Note: The family of all α-open (resp. regular open, π-open) sets in X is denoted by τ^{α} (resp. RO(X), πO(X)).

Definition 2.2.
A subset A of a topological space (X, τ) is said to be g-closed [7] (resp. g^*-closed [22] or strongly g-closed [20], πg -closed [4], rg-closed [15]) if cl(A) ⊂ U whenever A ⊂ U and U is open (resp. g-open, π-open, regular open) in (X, τ).
The complements of the above closed sets are called their respective open sets. The family of all g-open sets in (X, τ) is denoted by gO(X). The g-closure (resp. α-closure) of a subset A of X, denoted by gcl (A) (resp. αcl (A)), is defined to be the intersection of all g-closed sets (resp. α-closed sets) containing A.

Definition 2.3.
A subset A of a topological space (X, τ) is said to be αg -closed [8] (resp. $g^{\#}\alpha$ -closed [14], $\pi g\alpha$ -closed [2], $r\alpha g$ -closed [11] if αcl (A) ⊂ U whenever A ⊂ U and U is open (resp. g-open, π-open, regular open) in (X, τ).
The complements of the above closed sets are called their respective open sets.

Definition 2.4
[23] Let A be a subset of a topological space (X, τ), a point p in X is called a condensation point of A if for each open set U containing p, U ∩ A is uncountable.

Definition 2.5.
[5] A subset A of a topological space (X, τ) is called ω -closed if it contains all its condensation points. The complement of an ω -closed set is called ω -open. It is well known that a subset W of a topological space (X, τ) is ω -open if and only if for each x∈W, there exists U $\in \tau$ such that x ∈ U and U − W is countable. The family of all ω -open sets, denoted by τ_{ω} , is a topology on X, which is finer than τ . The interior and closure operator in (X, τ_{ω}) are denoted by int_{ω} and cl_{ω} respectively.

Lemma 2.6.
[5] Let A be a subset of a topological space (X, τ). Then
(1) A is ω -closed in X if and only if A = cl_{ω} (A).
(2) cl_{ω} (X \ A) = X \ int_{ω} (A).
(3) cl_{ω} (A) is ω -closed in X.
(4) x ∈ cl_{ω} (A) if and only if A ∩ G ≠ ϕ for each ω -open set G containing x.
(5) cl_{ω} (A) ⊂ cl (A).
(6) int (A) ⊂ int_{ω} (A).

Definition 2.7.
Let A be a subset of a topological space (X, τ). Then A is called
(1) $g^*\omega$ -closed [18] if cl_{ω} (A) ⊂ U whenever A ⊂ U and U is g-open in (X, τ).
(2) $g\omega$ -closed [6] if cl_{ω} (A) ⊂ U whenever A ⊂ U and U is open in (X, τ).
(3) $\pi g\omega$ -closed [3] if cl_{ω} (A) ⊂ U whenever A ⊂ U and U is π-open in (X, τ).
(4) $rg\omega$ -closed [1] if cl_{ω} (A) ⊂ U whenever A ⊂ U and U is regular open in (X, τ).

Remark 2.8.
[18] For a subset of a topological space, we obtain the following implications:

closed → g^*-closed → g-closed

↓ ↓ ↓

ω-closed → $g^*\omega$-closed → $g\omega$-closed

None of the above implications is reversible.

Lemma 2.9.
[6] The open image of an ω-open set is ω-open. Throughout the present paper, (X,τ) and (Y,σ) always denote topological spaces and f: $(X,\tau) \to (Y,\sigma)$ represents a function.

3. m-structures

Definition 3.1.
A subfamily m_x of the power set $\wp(X)$ of a nonempty set X is called a minimal structure (briefly m-structure) [16] on X if $\phi \in m_X$ and $X \in m_X$. By (X, m_x), we denote a nonempty set X with a minimal structure m_x on X and call it an m-space. Each member of m_x is said to be m_x-open (or briefly m-open) and the complement of an m_x-open set is said to be m_x-closed (or briefly m-closed).

Remark 3.2.
Let (X,τ) be a topological space. Then the families τ_ω, τ^α, τ RO(X), πO(X) and gO(X) are all m-structures on X.

Definition 3.3.
Let (X, m_X) be an m-space. For a subset A of X, the m_X-closure of A and the m_X-interior of A are defined in [9] as follows:
(1) $m_X\text{-cl}(A) = \cap\{F : A \subset F, X - F \in m_X\}$,
(2) $m_X\text{-int}(A) = \cup\{U : U \subset A, U \in m_X\}$.

Remark 3.4.
Let (X,τ) be a topological space and A be a subset of X. If $m_X = \tau$ (resp. τ_ω, τ^α, gO(X)), then we have m_X-cl(A) = cl(A) (resp. cl_ω(A), αcl(A), gcl(A)).

Lemma 3.5.
[16] Let (X, m_X) be an m-space and A be a subset of X. Then $x \in m_X$-cl(A) if and only if $U \cap A \neq \phi$ for every $U \in m_X$ containing x.

Definition 3.6.
[9] An m-structure m_X on a nonempty set X is said to have property (B) if the union of any family of subsets belonging to m_X belongs to m_X.

Remark 3.7.
Let (X,τ) be a topological space. Then the families τ_ω, τ^α, τ, πO(X) and gO(X) are all m-structures with property (B).

Lemma 3.8.
[17] Let X be a nonempty set and m_X an m-structure on X satisfying property (B). For a subset A of X, the following properties hold:
(1) $A \in m_X$ if and only if m_X-int(A) = A,
(2) A is m-closed if and only if m_X-cl(A) = A,
(3) m_X-int(A) $\in m_X$ and m_X-cl(A) is m-closed.

Definition 3.9.
[12] Let (X,τ) be a topological space and m_X an m-structure on X. A subset A of X is said to be
(1) mg^*-closed if cl(A) $\subset$ U whenever A $\subset$ U and U is m_X-open,
(2) mg^*-open if its complement is mg^*-closed.

Proposition 3.10.
[12] Let $\tau \subset m_X$. Then the following implications hold:

closed $\longrightarrow$ mg*-closed $\longrightarrow$ g-closed

Proposition 3.11.
Let $\tau \subset m_X$. Then the following implications hold:

closed $\longrightarrow$ mg*-closed $\longrightarrow$ g-closed $\longrightarrow$ $g\omega$-closed

Proof. It follows from Remark 2.8.

Theorem 3.12.
[12] Let $\tau \subset m_X$ and m_X have property (B). A subset A of X is mg*-closed if and only if cl(A) \ A does not contain any nonempty m-closed set.

Theorem 3.13.
[12] Let m_X have property (B). A subset A of X is mg*-closed if and only if m_X-cl({x}) $\cap$ A $\neq \phi$ for each x $\in$ cl(A).

4. $mg\omega$-closed sets

In this section, let (X, τ) be a topological space and m_X an m-structure on X. We obtain several basic properties of $mg\omega$-closed sets.

Definition 4.1.
Let (X, τ) be a topological space and m_X an m-structure on X. A subset A of X is said to be

(1) $mg\omega$-closed if cl_ω(A) $\subset$ U whenever A $\subset$ U and U is m_X-open,

(2) $mg\omega$-open if its complement is $mg\omega$-closed.

Remark 4.2.
Let (X, τ) be a topological space and A be a subset of X. If m_X = gO(X) (resp. τ , πO(X), RO(X)) and A is $mg\omega$-closed, then A is $g^*\omega$-closed (resp. $g\omega$-closed, $\pi g\omega$-closed, $rg\omega$-closed).

Proposition 4.3.
Let $\tau \subset m_X$. Then the following implications hold:

closed $\longrightarrow$ ω-closed $\longrightarrow$ $mg\omega$-closed $\longrightarrow$ $g\omega$-closed

Proof.
It is obvious that every closed set is ω-closed [1, 5] and every ω-closed set is $mg\omega$-closed by Lemma 2.6(1). Suppose that A is an $mg\omega$-closed set. Let A $\subset$ U and U $\in \tau$. Since $\tau \subset m_X$, cl_ω (A) $\subset$ U and hence A is $g\omega$-closed.

Proposition 4.4.
If A and B are $mg\omega$-closed, then A $\cup$ B is $mg\omega$-closed.

Proof. Let A $\cup$ B $\subset$ U and U $\in m_X$. Then A $\subset$ U and B $\subset$ U. Since A and B are $mg\omega$-closed, we have cl_ω (A $\cup$ B) = cl_ω (A) $\cup$ cl_ω (B) $\subset$ U. Therefore, A$\cup$ B is $mg\omega$-closed.

Proposition 4.5.
If A is $mg\omega$-closed and m-open, then A is ω-closed.

Proof. This is obvious.

Proposition 4.6.

If A is $mg\omega$-closed and $A \subset B \subset cl_\omega(A)$, then B is $mg\omega$-closed.

Proof. Let $B \subset U$ and $U \in m_X$. Then $A \subset U$ and A is $mg\omega$-closed. Hence $cl_\omega(B) = cl_\omega(A) \subset U$ and B is $mg\omega$-closed.

Definition 4.7.
[13] Let (X, m_X) be an m-space and A be a subset of X. The m_X- frontier of A, m_X-Fr(A), is defined as follows: m_X-Fr(A) = m_X-cl(A) $\cap$ m_X-cl(X \ A).

Proposition 4.8.
If A is a $mg\omega$-closed subset of X and $A \subset U \in m_X$, then m_X-Fr(U) $\subset int_\omega(X \setminus A)$.

Proof. Let A be $mg\omega$-closed and $A \subset U \in m_X$. Then $cl_\omega(A) \subset U$. Suppose that $x \in m_X$-Fr(U). Since $U \in m_X$, m_X-Fr(U) = m_X-cl(U) $\cap$ m_X-cl(X \ U) = m_X-cl(U) $\cap$ (X \ U) = m_X-cl(U) \ U. Therefore, $x \notin U$ and $x \notin cl_\omega(A)$. This shows that $x \in int_\omega(X \setminus A)$ and hence m_X-Fr(U) $\subset int_\omega(X \setminus A)$.

Proposition 4.9.
A subset A of X is $mg\omega$-open if and only if $F \subset int_\omega(A)$ whenever $F \subset A$ and F is m-closed.

Proof. Suppose that A is $mg\omega$-open. Let $F \subset A$ and F be m-closed. Then $X \setminus A \subset X \setminus F \in m_X$ and X \ A is $mg\omega$-closed. Therefore, we have $X \setminus int_\omega(A) = cl_\omega(X \setminus A) \subset X \setminus F$ and hence $F \subset int_\omega(A)$.
Conversely, let $X \setminus A \subset G$ and $G \in m_X$. Then $X \setminus G \subset A$ and X \ G is m-closed. By the hypothesis, we have $X \setminus G \subset int_\omega(A)$ and hence $cl_\omega(X \setminus A) = X \setminus int_\omega(A) \subset G$. Therefore, X \ A is $mg\omega$-closed and A is $mg\omega$-open.

Corollary 4.10.
Let $\tau \subset m_X$. Then the following properties hold:
(1) Every open set is $mg\omega$-open and every $mg\omega$-open set is $g\omega$-open,
(2) If A and B are $mg\omega$-open, then $A \cap B$ is $mg\omega$-open,
(3) If A is $mg\omega$-open and m-closed, then A is ω-open,
(4) If A is $mg\omega$-open and $int_\omega(A) \subset B \subset A$, then B is $mg\omega$-open.

Proof. This follows from Propositions 4.3, 4.4, 4.5 and 4.6.

Proposition 4.11.
Every mg*-closed set is $mg\omega$-closed.

Proof. It follows from Lemma 2.6(5).

Proposition 4.12. Let $\tau \subset m_X$. Then every mg*-closed set is $g\omega$-closed.
Proof. It follows from Propositions 4.3 and 4.11.

Proposition 4.13. Let $\tau \subset m_X$. Then the following implications hold:

closed $\longrightarrow$ mg*-closed $\longrightarrow$ $mg\omega$-closed $\longrightarrow$ $g\omega$-closed

Proof. It follows from Propositions 3.11, 4.3 and 4.11.

5. Characterizations of $mg\omega$-closed sets

In this section, let (X, τ) be a topological space and m_X an m-structure on X. We obtain some characterizations of $mg\omega$-closed sets.

Theorem 5.1.
A subset A of X is $mg\omega$-closed if and only if $cl_\omega(A) \cap F = \phi$ whenever $A \cap F = \phi$ and F is m-closed.

Proof. Suppose that A is $mg\omega$-closed. Let $A \cap F = \phi$ and F be m-closed. Then $A \subset X \setminus F \in m_X$ and $cl_\omega(A) \subset X \setminus F$. Therefore, we have $cl_\omega(A) \cap F = \phi$.
Conversely, let $A \subset U$ and $U \in m_X$. Then $A \cap (X \setminus U) = \phi$ and $X \setminus U$ is m-closed. By the hypothesis, $cl_\omega(A) \cap (X \setminus U) = \phi$ and hence $cl_\omega(A) \subset U$. Therefore, A is $mg\omega$-closed.

Theorem 5.2.
Let $\tau_\omega \subset m_X$ and m_X have property (B). A subset A of X is $mg\omega$-closed if and only if $cl_\omega(A) \setminus A$ does not contain any nonempty m-closed set.
Proof. Suppose that A is $mg\omega$-closed. Let $F \subset cl_\omega(A) \setminus A$ and F be m-closed. Then $F \subset cl_\omega(A)$ and $A \subset X \setminus F \in m_X$. Hence $cl_\omega(A) \subset X \setminus F$. Therefore, we have $F \subset X \setminus cl_\omega(A)$. Hence $F \subset cl_\omega(A) \cap (X \setminus cl_\omega(A)) = \phi$.
Conversely, suppose that A is not $mg\omega$-closed. Then $\phi \neq cl_\omega(A) \setminus U$ for some $U \in m_X$ containing A. Since $\tau_\omega \subset m_X$ and m_X has property (B), $cl_\omega(A) \setminus U$ is m-closed. Moreover, we have $cl_\omega(A) \setminus U \subset cl_\omega(A) \setminus A$, a contradiction. Hence A is $mg\omega$-closed.

Theorem 5.3.
Let $\tau_\omega \subset m_X$ and m_X have property (B). A subset A of X is $mg\omega$-closed if and only if $cl_\omega(A) \setminus A$ is $mg\omega$-open.
Proof. Suppose that A is $mg\omega$-closed. Let $F \subset cl_\omega(A) \setminus A$ and F be m-closed. By Theorem 5.2, we have $F = \phi$ and $F \subset \text{int}_\omega(cl_\omega(A) \setminus A)$. It follows from Proposition 4.9, $cl_\omega(A) \setminus A$ is $mg\omega$-open.
Conversely, let $A \subset U$ and $U \in m_X$. Then $cl_\omega(A) \cap (X \setminus U) \subset cl_\omega(A) \setminus A$ and $cl_\omega(A) \setminus A$ is $mg\omega$-open. Since $\tau_\omega \subset m_X$ and m_X has property (B), $cl_\omega(A) \cap (X \setminus U)$ is m-closed and by Proposition 4.9, $cl_\omega(A) \cap (X \setminus U) \subset \text{int}_\omega(cl_\omega(A) \setminus A)$. Now, $\text{int}_\omega(cl_\omega(A) \setminus A) = \text{int}_\omega(cl_\omega(A)) \cap \text{int}_\omega(X \setminus A) \subset cl_\omega(A) \cap \text{int}_\omega(X \setminus A) = cl_\omega(A) \cap (X \setminus cl_\omega(A)) = \phi$. Therefore, we have $cl_\omega(A) \cap (X \setminus U) = \phi$ and hence $cl_\omega(A) \subset U$. This shows that A is $mg\omega$-closed.

Theorem 5.4.
Let m_X have property (B). A subset A of X is $mg\omega$-closed if and only if $m_X\text{-cl}(\{x\}) \cap A \neq \phi$ for each $x \in cl_\omega(A)$.
Proof. Suppose that A is $mg\omega$-closed and $m_X\text{-cl}(\{x\}) \cap A = \phi$ for some $x \in cl_\omega(A)$. By Lemma 3.8, $m_X\text{-cl}(\{x\})$ is m-closed and $A \subset X \setminus (m_X\text{-cl}(\{x\})) \in m_x$. Since A is $mg\omega$-closed, $cl_\omega(A) \subset X \setminus (m_X\text{-cl}(\{x\}) \subset X \setminus \{x\}$. This contradicts that $x \in cl_\omega(A)$.
Conversely, suppose that A is not $mg\omega$-closed. It implies A is not mg^*-closed by Proposition 4.11. Then, by Theorem 3.13, $\phi \neq cl(A) \setminus U$ for some $U \in m_X$ containing A. There exists $x \in cl(A) \setminus U$. Since $x \notin U$, by Lemma 3.5, $m_X\text{-cl}(\{x\}) \cap U = \phi$ and hence $m_X\text{-cl}(\{x\}) \cap A \subset m_X\text{-cl}(\{x\}) \cap U = \phi$. This shows that $m_X\text{-cl}(\{x\}) \cap A = \phi$ for some $x \in cl(A)$. Thus A is mg^*-closed and hence A is $mg\omega$-closed.

Corollary 5.5.
Let $\tau_\omega \subset m_X$ and m_X have property (B). For a subset A of X, the following properties are equivalent:
(1) A is $mg\omega$-open,
(2) $A \setminus \text{int}_\omega(A)$ does not contain any nonempty m-closed set,
(3) $A \setminus \text{int}_\omega(A)$ is $mg\omega$-open,
(4) $m_X\text{-cl}(\{x\}) \cap (X \setminus A) \neq \phi$ for each $x \in A \setminus \text{int}_\omega(A)$.
(5)
Proof. This follows from Proposition 4.9 and Theorem 5.2, 5.3 and 5.4.

6. Preservation theorems

Definition 6.1

[12] A function f: (X, m_X) $\rightarrow$(Y, m_Y) is said to be

(1) M-continuous if for each x $\in$ X and each V $\in$ m_Y containing f(x), there exists U $\in$ m_X containing x such that f (U) $\subset$ V.

(2) M-closed if for each m-closed set F of (X, m_X), f (F) is m-closed in (Y, m_Y).

Theorem 6.2.

[16] Let m_X be an m-structure with property (B). Let f: (X, m_X) $\rightarrow$ (Y, m_Y) be a function from a minimal space (X, m_X) into a minimal space (Y, m_Y). Then the following are equivalent:

(1) f is M-continuous,

(2) f^{-1}(V) $\in$ m_X for every V $\in$ m_Y.

Lemma 6.3.

[12] A function f: (X, m_X) $\rightarrow$ (Y, m_Y) is M-closed if and only if for each subset B of Y and each U $\in$ m_X containing f^{-1}(B), there exists V$\in$ m_Y such that B $\subset$ V and f^{-1}(V) $\subset$ U.

Theorem 6.4.

If f: (X, τ) $\rightarrow$ (Y,σ) is closed and f: (X, m_X) $\rightarrow$ (Y, m_Y) is M-continuous, where m_X has property (B), then f (A) is $mg\omega$-closed in (Y, m_Y) for each $mg\omega$-closed set A of (X, m_X).

Proof. Let A be any $mg\omega$-closed set of (X, m_X) and f (A) $\subset$ V $\in$ m_Y. Then, since m_X has property (B), A $\subset$ f^{-1}(V) $\in$ m_X by Theorem 6.2. Since A is $mg\omega$-closed, cl_ω(A) $\subset$ f^{-1}(V) and f (cl_ω(A) $\subset$ V. Since f is closed, by Lemma 2.9, cl_ω(f (A)) $\subset$ f (cl_ω(A)) $\subset$ V. Hence f (A) is $mg\omega$-closed in (Y, m_Y).

Definition 6.5.

[6] A function f: (X, τ) $\rightarrow$ (Y,σ) is called ω-irresolute if f^{-1}(B) is ω-open in (X, τ) for every ω-open set B of (Y,σ).

Theorem 6.6.

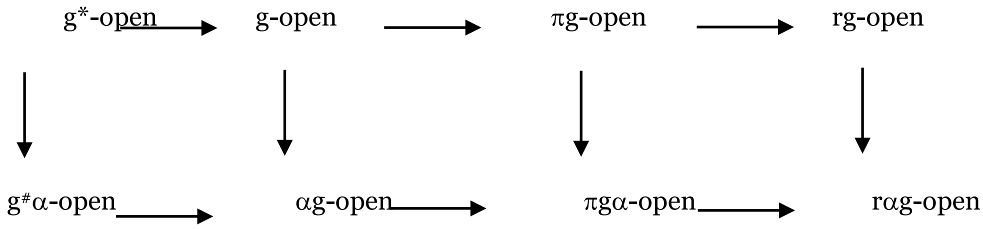
If f: (X, τ) $\rightarrow$ (Y,σ) is ω-irresolute and f: (X, m_X) $\rightarrow$ (Y, m_Y) is M-closed, then f^{-1}(B) is $mg\omega$-closed in (X, m_X) for each $mg\omega$-closed set B of (Y, m_Y).

Proof. Let B be any $mg\omega$-closed set of (Y, m_Y) and f^{-1}(B) $\subset$ U $\in$ m_X. Since f is M-closed, by Lemma 6.3, there exists V $\in$ m_Y such that B $\subset$ V and f^{-1}(V) $\subset$ U. Since B is $mg\omega$-closed in Y, cl_ω(B) $\subset$ V and since f is ω-irresolute, cl_ω(f^{-1}(B)) $\subset$ f^{-1}(cl_ω(B)) $\subset$ f^{-1}(V) $\subset$ U. Hence f^{-1}(B) is $mg\omega$-closed in (X, m_X).

7. New forms of $mg\omega$-closed sets

By gO(X) (resp. g^*O(X), πgO(X), rgO(X), $g^\#\alpha$O(X), αgO(X), πgαO(X), rαgO(X)), we denote the collection of all g-open (resp. g*-open, πg-open,
rg-open, $g^\#\alpha$-open, αg-open, πgα-open, rαg-open) sets of a topological space (X, τ). These collections are all m-structures with property (B).
By the definitions, we obtain the following diagram:

Diagram – I



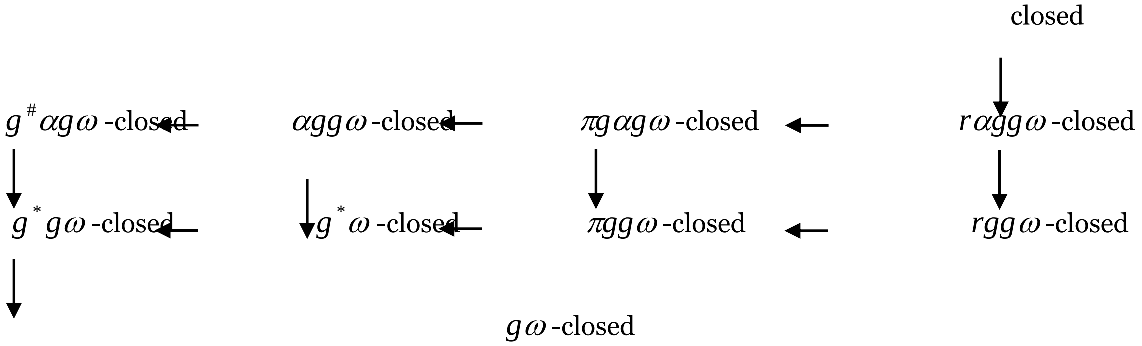
For subsets of a topological space (X, τ), we can define many new variations of $g\omega$-closed sets. For Example, in case m_X = gO(X), g*O(X), πgO(X), rgO(X), g#αO(X), αgO(X), πgαO(X), rαgO(X), we can define new types of $g\omega$-closed sets as follows:

Definition 7.1.

A subset A of a topological space (X, τ) is said to be $g\omega$-closed [6] (resp. $g^*g\omega$-closed, $g^*\omega$-closed [18]. $\pi gg\omega$-closed, $rgg\omega$-closed, $g^{\#}\alpha g\omega$-closed, $\alpha gg\omega$-closed, $\pi g\alpha g\omega$-closed, $r\alpha gg\omega$-closed) if cl_ω (A) $\subset$ U whenever A $\subset$ U and Us is open (resp. g*-open, g-open, πg -open, rg-open, g#α-open, αg-open, πgα-open, rαg-open) in (X, τ).

By Diagram I and Definition 7.1, we have the following diagram:

Diagram - II



CONCLUSION

This exploration into the relationship between closed sets and $g\omega$-closed sets within the realm of topology has provided valuable insights into the nature of convergence and continuity in topological spaces. We have uncovered the fundamental distinctions between these two types of sets and elucidated their significance in different mathematical contexts.

Through our analysis, we have demonstrated that while closed sets represent a classical notion of convergence, $g\omega$--closed sets offer a more refined understanding of convergence in the context of weak topologies. By considering the behaviour of sequences and their limit points within sets, we have discerned the subtle intricacies that distinguish closed sets from $g\omega$--closed sets, highlighting the importance of these distinctions in topology.

Furthermore, our examination of examples and counterexamples has underscored the importance of carefully considering different notions of closure when analysing topological spaces. We have seen how certain sets can be closed but not $g\omega$--closed, and vice versa, emphasizing the need for a nuanced understanding of convergence and continuity.

Overall, this study contributes to the broader understanding of topology by bridging the gap between closed sets and $g\omega$--closed sets. By elucidating the intricate relationships between these concepts, we have provided a foundation for further research and exploration into the convergence properties of topological spaces.

In the future, it would be beneficial to delve deeper into the implications of $g\omega$--closed sets in various mathematical and scientific disciplines. Additionally, exploring the connections between $g\omega$--closed sets and other topological concepts could yield further insights into the structure and behaviour of topological spaces. Overall, this investigation opens up avenues for continued exploration and development in the field of topology.

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