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Review Article

The Chaotic Features Of The TRY/USD Exchange Rate: Recurrence Plot And Non-Linear Analysis

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ABSTRACT

T The exchange rate is a vital financial variable at the international, national, macro, and micro levels. The exchange rate market is the biggest market globally with a wide range of players. An accurate analysis of a time series needs to explore its behavior. Being influenced by many economic, political, social, and natural events the exchange rate is highly capable to display chaotic behavior. In this paper, we investigated the dynamics of the Turkish Lira to United States Dollar (TRY/USD) time series from 1-January 2005 to 31-December- 2021 by employing well-known nonlinear methods of chaotic data analysis. We also utilized the Recurrence Plot to analysis the non-linear behavior of the TRY/USD exchange rate. The results indicate that the TRY/USD exchange rate is chaotic. As the exchange rate behavior is crucial for making accurate policies for a country and decreasing investment risk for traders, in modeling and forecasting the TRY/USD exchange rate, the chaotic behavior should be considered.

Keywords: Average mutual information, Chaos, False nearest neighbors, Local divergence rates, TRY/USD exchange rate

1.Introduction

An exchange rate is defined as a rate that one currency can be exchanged for another among economic zones or countries. It is implemented to specified the value of different currencies regarding each other. The exchange rate is a vital financial variable at the international, national, macro, and micro levels. The currency exchange rate has been considered by policymakers, economic agents and investors. Because the exchange rate is such an influential variable on most economic, political, trade, and international issues. Exports, imports, trade balance, foreign exchange reserves directly, and household expenses in a country are indirectly affected by exchange rate fluctuations.
The modeling and characterization of the exchange rate series have been a crucial issue, and there are exhaustive range of researches about this. The exchange rate analysis methods are divided into three categories, including technical analysis, sentiment analysis, and fundamental analysis. Most the traders to decide trading decisions heavily emphasize technical methods that use historical time series data of exchange rates [1]. After the study of Messe and Rogoff (1983), which showed the failure of structural models in determining exchange rate behavior, the exchange rate was recognized as a random variable, which was examined by technical analysis and outside of economic theories [2].
The empirical studies mostly showed a non-linear structure in exchange rate time series, and the linear model that developed to simulate and predict exchange rate did not always work best. Various parameters, including, jobless claims, announcement surprises in gross domestic product, and nonfarm payroll affect both exchange rate changes [3]. This evidence for exchange rates is categorized into two groups: (i) The exchange rate is a non-linear stochastic function of its past values. It is concluded by implementing autoregressive conditional heteroskedastic models (ARCH) [4-9], and (ii) The exchange rate is produced with a deterministic chaotic process [10]. The chaos theory is known as the "non-linear dynamics" is a mathematical concept illustrates that standard equations may give random results.
In time series analysis, nonlinear deterministic aspects of empirical data or chaotic analysis is an emerging issue. The chaos theory was first introduced and empirically used by Edward Lorenz in 1965 in meteorology. Investigation of the chaotic behavior in time series has expanded to a wide range of scientific areas, including metrology, physiology, atmospherics, the motion of astronomical systems, biology, mathematics, and even the dynamics of financial subjects [11,12]. In the economic and financial field, the first chaotic analysis is begun in

the last 30 years. The most known studies in this field are by [13] which found a strong non-linearity in financial markets.
There is no consensus on the chaotic behavior of the exchange rate. While some studies have confirmed exchange rate is chaotic [14-24], there are other studies that have ruled out the chaotic behavior of the exchange rate [25-31]. In a study, [32] examined the non-linear dependence on British Pound, German Mark, and Japanese Yen. They concluded the qualitative topological test did not support the chaotic behavior in these currencies. Using a statistical framework of moving block bootstrap methodology, [33] found a chaotic behavior in Swedish Krona exchange rate data. A chaotic behavior is also found for the Romanian ROL concerning the American Dollar (ROL/USD) by [34] using the largest Lyapunov exponent method.
Authors in [35] found a non-chaotic dynamic for the monthly data of the purchasing power parity between Canada and the United States. In a study, [18] investigated the potential chaotic behavior of the several foreign exchange rates versus the Iranian Rial daily time series during 2002 to 2007. They used the Attractor Dimension Test, Test of Lyapanuv's Biggest Exponent, and Henon Map and found a complex, chaotic behavior with a big degree of freedom. Authors in [12] investigated the possible chaos behavior for the TRY/USD exchange rate during 2000-2005 and found deterministic chaos in the log-returns series of this financial series. In a study [36] found non-chaotic dynamics for six exchange rates and six stock indexes.
Authors in [37] has investigated the possible chaotic behavior of the Moroccan exchange rates using a combining Lyapunov exponent and wavelet transform method. The findings show a chaotic structure for the exchange rates, non-chaotic behavior for currencies return, and different chaotic pattern for some exchange rates in the short and long term. In a study, [38] investigated the non-linear and chaotic pattern in the USD/TRY exchange rate using three famous tests, including the neural network Lyapunov exponent test, BDS test, and Bispectrum test. This study found a non-linear and non-chaotic structure for the TRY/USD exchange rate from 1995 to 2000.
Authors in [39] evaluated the chaos theory for the G7 stock market using monthly data for 100 years and confirmed the chaos behavior for the currencies of all countries. In a study [40] evaluated the chaotic behavior of the monthly data of the BRICS stock market during 1812 to 2017, using the Lyapunov exponents test. They found that except for the Russian ruble, the chaotic behavior is confirmed for other currencies before the dissolution of the Bretton Woods system. They also concluded that the free-floating exchange rate system caused diminishing the chaotic behavior of the exchange rates. Authors in [41] analyzed the chaotic behavior of the Philippine exchange rate using the Largest Lyapunov Exponent (LLE) Test and 0-1 tests and concluded that the Philippine exchange rate is not chaotic.
The exchange rate market is the largest market in the world. This market is composed of a wide range of participants. The exchange rate is affected by many economic, political, and natural events at national and international levels, which create growing instability in the exchange rate markets. Such reasons increase the probability of the chaotic structure of the exchange rates. There are some popular tests for determining chaotic behavior, including BDS, close return, and Lyapunov exponent tests first are used by researchers [42-44]. For the first time, [45,46] implemented the BDS test to evaluate the chaotic behavior in the financial market. The graphs of the stochastic and chaotic systems are similar, and it is difficult to distinguish among them even with the substantial test such as BDS. Hence, these tests are not more efficient for the financial market [47]. This evidence guides to develop of efficient tools for identifying the chaotic behavior of the exchange rates. So, in this paper, we used chaotic behavior determining methodologies in detail to evaluate the chaotic pattern of the daily TRY/USD exchange rate time series from 2005 to 2021.
This study is structured as follows: In the next section, the Materials and Methodology are described in detail. The results and discussions of the models are presented in section 3, and lastly, section presents the conclusions are presented in section 4.

2. Methods and Material

2.1.Data

In the present paper, to evaluate the non-linearity and chaotic behavior in the financial market, we used the daily data for the TRY/USD from 1-January-2005 to 31-December-2021. The time series of the daily TRY/USD exchange rate is shown in Fig 1. It can be seen that the TRY/USD had a significant downward trend. In recent years, this downward trend is continuing with a gentle slope. The monetary policy of the Central Bank of Turkey, such as the sharp decline in interest rate, has been one of the reasons for the devaluation of the Turkish lira.

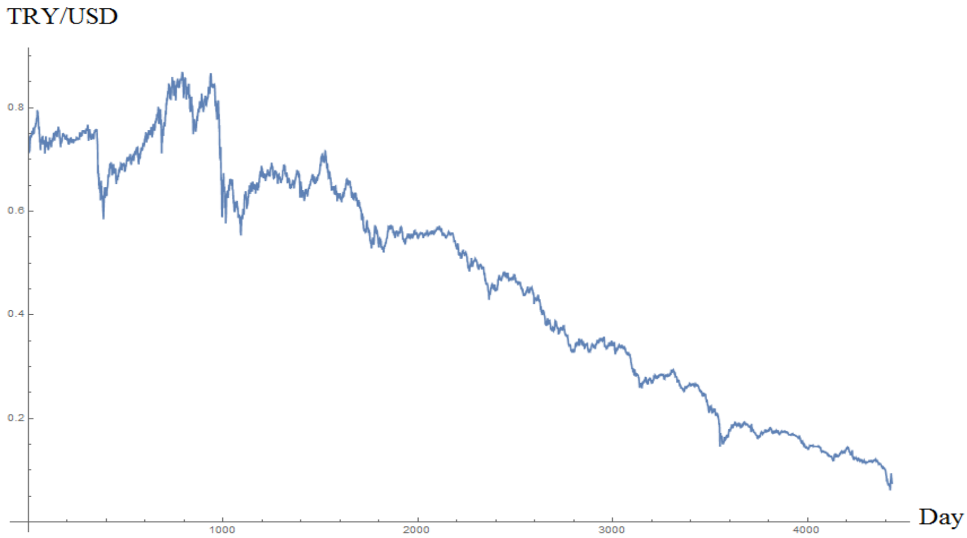

Figure 1. Daily TRY/USD exchange rate (2005-2021)

2.2. Chaotic systems

We review the methods of the chaotic systems in this section. Chaos Theory is a branch of mathematics that studies chaotic dynamic systems; Systems whose disarray and order are seemingly random but in fact follow definite patterns and rules that are highly sensitive to initial conditions. For the first time, a meteorologist named Edward Lorentz (1960) came across the chaotic problem. Chaos theory is an interdisciplinary knowledge based on which seemingly random complex systems, patterns, interconnections have feedback loops, fractals, self-similarity, repetition, and self-organization. The butterfly effect is the basis of chaos theory, and describes the phenomenon of how very small changes in the initial conditions of a deterministic nonlinear system (such as changes due to rounding of numbers in calculations or errors in measurements) can lead to large changes in the system's response; It means a sensitive dependence on initial conditions.

2.2.1. Phase space reconstruction: time delay embedding

For a chaotic time-series, phase space reconstruction (PSR) is a base and used to reconstruct the univariate time series. Generally, in a phase space, a state vector (z) shows a dynamic deterministic stationary system. The reconstruction of this vector is not possible considering the one-dimensionality of the time series. Takens (1981) showed that a univariate time series could represent the total information of a dynamic system. A state of the dynamic system is shown by a point of PSR, and based on different initial conditions, the time evaluation of the system is represented by trajectory of PSR. Considering a time series $(x_1, x_2, \dots, x_T)$, a time delay embedding can form by constructing vectors $\mathbf{x}_t = (x_{t-(m-1)\tau}, x_{t-(m-2)\tau}, \dots, x_{t-2\tau}, x_{t-\tau}, x_t)$, where $t = 1 + (m-1)\tau, \dots, T$, and using two integer values of τ (delay time), and m (embedding dimension).

Delay time: the method of average mutual information

In the Takens (1981) theory, to reconstruct the phase space, there is not any suggestion about selecting the values of τ and m, and based on Abarbanel (1996) and Kantz and Schreiber (2004), there is not any regular method for determining an optimal value of τ. Also, the features of the optimal value are not determined yet. In PSR, the value of τ should produce the vectors x_t and $x_{t+\tau}$ independent enough to be beneficial but dependent enough to maintain their correlation (Abarbanel 1996). The value of τ should not be too small or too large. To select the delay time, there are different methods. For linear time series the Auto-Correlation Function (ACF) and for non-linear time series of average mutual information (AMI) are implemented (Shah et al., 2019). In this study, we used method of AMI, which based on researcher findings it is an applicable method to choose delay time (Kantz and Schreiber, 2004).
The general dependence between two variables is measured in AMI. In PSR, for various values,
the AMI of x_t and $x_{t+\tau}$, is calculated. It is clear that, when τ was smaller, the AMI will be larger. The AMI has several maxima and minima. The first local minimum of AMI is could be a suitable choice for τ. As this τ value adds the most information to our knowledge of x_t. In prediction, τ should be selected, by trial and error, for optimizing the performance of the prediction.
The AMI among x_t and $x_{t+\tau}$, for a constant τ is computed according to the data histogram with the following formula:

$$I(\tau) = \sum_i \sum_j p_{i,j}(\tau) \log_2 \frac{p_{i,j}(\tau)}{p_i p_j} \qquad (1)$$

where, p_i presents the relative frequency of the ith bin of the histogram (the approximate probability that an observation is inside the ith bi), and $p_{i,j}(\tau)$ is the approximate probability that x_t is in the ith bin and $x_{t+\tau}$ is in the jth bin.

2.3. Embedding dimension: the method of false nearest neighbors

For determining m, the method of false nearest neighbors (FNN) is a useful technique (Kennel et al., 1992; Kantz and Schreiber, 2004).

The FNN method is based on the premise that if the embedding space is too small, some of observations in the phase space will be close to each other, which these neighbors are named FNN. In this method, m is gently increased. If there is not more FNN for some m, it will be the accurate embedding dimension (Abarbanel and Parlitz 2006).

If $\mathbf{x}_t = (x_{t-(m-1)\tau}, x_{t-(m-2)\tau}, \dots, x_{t-2\tau}, x_{t-\tau}, x_t)$ was a point in dimension m, and $\mathbf{y}_t = (y_{t-(m-1)\tau}, y_{t-(m-2)\tau}, \dots, y_{t-2\tau}, y_{t-\tau}, y_t, y_{t+\tau})$ was a point in dimension m+1, the square of the Euclidian distance among the these point , in dimensions m and m+1 are calculated as $R_m^2 = \sum_{i=0}^{m-1}(x_{t-i\tau} - y_{t-i\tau})^2$ and $R_{m+1}^2 = \sum_{i=0}^{m-1}(x_{t-i\tau} - y_{t-i\tau})^2 + (x_{t+\tau} - y_{t+\tau})^2$, respectively. Hence, by movement to higher dimension, the distance among two points enhances, which if this enhance is small, $\mathbf{y}_t$ is notified as an accurate nearest neighbor to $\mathbf{x}_t$. For comparing the amount of increase of distance between points, the dimensionless as $\frac{|x_{t+\tau}-y_{t+\tau}|}{R_m}$ could be calculated. If this value is less than a threshold value, $\mathbf{y}_t$ is an accurate nearest neighbor to $\mathbf{x}_t$,, but $\mathbf{y}_t$ will be a FNN to $\mathbf{x}_t$, if the value is larger than the threshold. There is not a unique idea about the threshold value among researches and it is between 10 to 30 (Abarbanel, 1996; Kennel and Abarbanel, 2002; Small, 2005).

2.4. Correlation dimension: the method of correlation exponent

The method correlation exponent is a useful method to determine white noise time series from chaotic ones. This method gives valuable information about the dynamics of time-series and is used to test special cases of non-linearity. If the correlation dimension shows a fractional dimension, the time series is chaotic.

The method of correlation exponent is applied to calculate the correlation dimension. If $x_1, x_2, \dots, x_T$ was a time series, $\mathbf{x}_t = (x_{t-(m-1)\tau}, x_{t-(m-2)\tau}, \dots, x_{t-2\tau}, x_{t-\tau}, x_t)$ prepared as the delay coordinates, and N was the number of delayed points. If n(r) was the number of pairs of (x_i, x_j), and C(r) is the corresponding relative frequency and calculated as:

$$C(r) = \frac{n(r)}{\binom{N}{2}} \qquad (2)$$

C(r) defined as a scale of the spatial correlation among data pairs and when N approaches infinity, is called the correlation integral. For small values of r and large values of N, C(r) is approximately a power function like $C(r) \approx \alpha r^{\gamma}$. Where, γ is named the correlation exponent.

The process is begun by defining m = 2, for a r value set, estimating C(r), for $\log C(r)$ vs. $log r$, fitting a line, and calculating γ. For higher values of m, the procedure is repeated. The system is chaotic if γ approaches a saturation value (Galka, 2000). If with increasing m the correlation exponent enhances without bound, the system is stochastic.

Based on the Grassberger- Procassia algorithm (1983), the data is i) deterministic, if the plot of correlation dimension vs. the embedding dimension be a straight line; ii) stochastic, if the plot of correlation dimension vs. the embedding dimension be in the form of a 45 degrees line between the X-axis and the Y- axis; and iii) chaotic, if the correlation exponent first increments but after acceding a particular embedding dimension finally saturates (Ghorbani et al., 2016). These statues are shown in Figure 2.

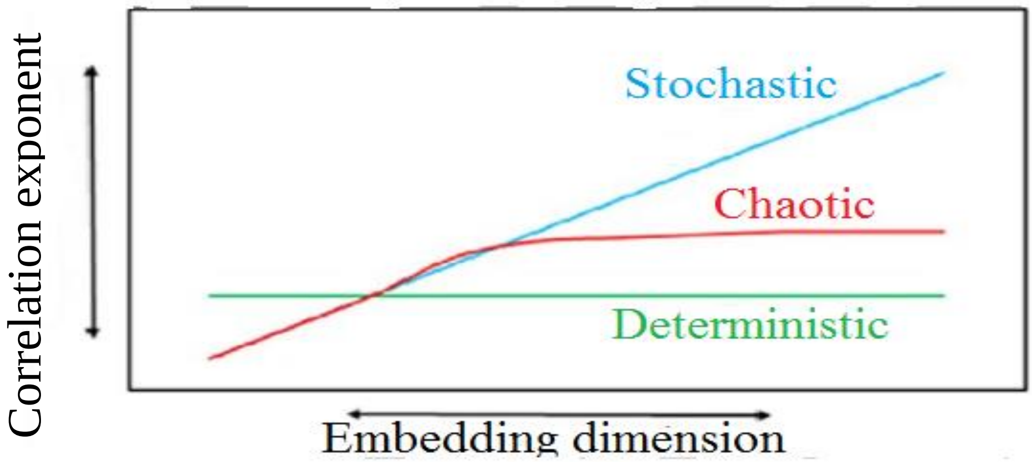

Figure 2. The differentiating deterministic, chaotic, and stochastic systems (Ghorbani et al., 2016)

2.5. Maximal Lyapunov exponent: the method of local divergence rates

The maximal Lyapunov exponent is defined as the average rate that predictability of a system is losed (Abarbanel 1996). The maximal Lyapunov exponent quantifies the strength of chaos. In a chaotic system close trajectories drastically diverge at a fast speed.

The method of local divergence rates is introduced by Kantz (1994) to calculate the maximal Lyapunov exponent. This method is based on calculating divergence rates of the close trajectories. Suppose a time series $x_1, x_2, \dots, x_T$. Now, we should select a value for ϵ (often between 0 and 20). For a t in the set $\mathcal{T}_\delta = \{1 + (m-1)\tau, \dots, T - \delta\}$, all points $\mathbf{x}_i$ in the ϵ-neighborhood of $\mathbf{x}_t$ should be find (say $\mathcal{U}_t$). After the relative time δ, for all i in $\mathcal{U}_t$, estimate the distance among $\mathbf{x}_t$ and $\mathbf{x}_i$, as below:

$$dist(\mathbf{x}_t, \mathbf{x}_i; \delta) = |x_{t+\delta} - x_{i+\delta}| \quad (3)$$

For all t in $\mathcal{T}_\delta$, the logarithm of the average of those distances should be calculated and then the avarage of the calculated values should be estimated as follow (S(δ)):

$$S(\delta) = \underset{t\in\mathcal{T}_\delta}{mean}\left(ln\left\{\underset{i\in\mathcal{U}_t}{mean}[dist(\mathbf{x}_t, \mathbf{x}_i; \delta)]\right\}\right) \quad (4)$$

The $S(\delta)$ for value of δ from 0 to 20 is calculated and S values vs. δ could be plotted. In a chaotic system, for small values of δ, some irregularities are seen in the plot. For intermediate δ values in a scaling region, S (δ) treats linearly (the slope is an estimate of the maximal Lyapunov exponent). In a chaotic system, for larger δ, S (δ) approaches a constant.

Recurrence Plot (RP)

To analyze the characteristics of the non-linear time series, the Recurrence plot (RP) is used. RP, which first introduced by Eckmann et al. (1987) is a powerful graphical tool for evaluating the qualitative information of the time series of the dynamical systems. RP is a symmetrical matrix graph and provides the times at which a dynamic system reaches to a state (Fragkou et al., 2022). A chaotic system in a phase space is characterized by strange and complex attractions and occurs repeatedly in a periodic orbit. The basic idea is that a chaotic system passes through a neighboring point again after a random time. The difference among two points of a time series (i and j) can be shown as (ω):

$$\omega = |X_i - X_j| \quad (5)$$

In the phase space at t=1, for a system which is near to state (t=j), the essential condition is $\omega = |X_i - X_j| \to 0$. We can draw ω diagonally. The diagonal line is repeatedly drawn for i=j, too. The plot could be drawn base equation (6):

$$\omega = |X_i - X_j| \quad \begin{cases} \text{Black point} & \text{if } \omega < r \\ \text{White point} & \text{otherwise} \end{cases} \quad (6)$$

where r shows the radius at an arbitrary point, and calculated as

$r = (0.01{\sim}0.1) \times |X_{max} - X_{min}|$. X_{min} *and* X_{max} presents the minimum and maximum values of time series, respectively (Lee et al., 2022). Figure 3 presents three cases of the behavior of the non-linear time series. Figure 3(a) presents an example of uncorrelated stochastic data or white noise. If a time series have periodic characteristic, we can see repeated appearance of regular horizontal line segments (Figure 3b). The recurrence plot of the Henon map which has irregular line segments in the uniform pattern indicates a typical example of chaotic behavior (Figure 3c). For random characteristics, random dots appear without a regular trajectory, in the form of random distribution (Figure 3d).

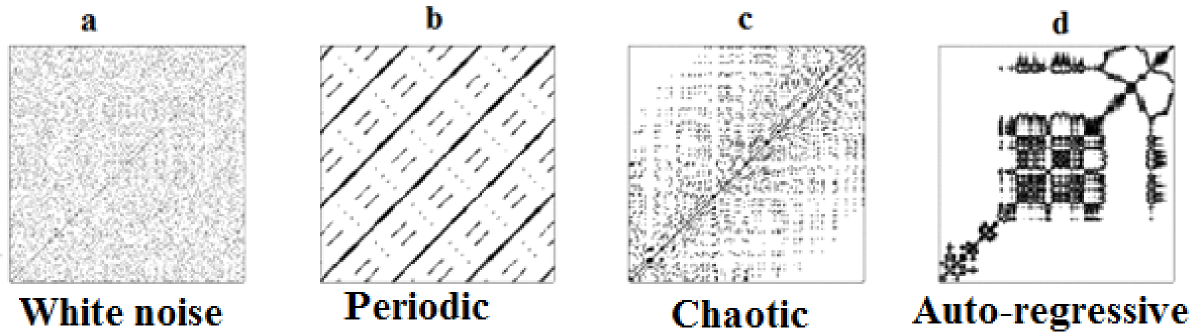

Figure 3. Recurrence plot for: a) white noise, b) harmonic oscillation with two frequencies, c) chaotic data (logistic map) with linear trend, d) and data from an auto-regressive process (Marvan et al., 2007).

2.6. Strengths

The strengths of the data are also calculated in this study. If x_t *was* the reconstructed vectors in the phase-space, m and τ were embedding dimension and delay time. Estimate the distances d_{ij} from node i to all other nodes j for all i and show the whole number of those distances by N_d. The following formula calculate the strength of $\boldsymbol{x}_i$:

$$S_i = \left(\frac{1}{N_d}\sum_{\substack{j=1\\ j\neq i}}^{N_d} d_{ij}\right)^{-1} \tag{7}$$

2. Results

To study, in more detail, the changes of the dynamics and patterns of the TRY/USD exchange rate, a temporal map of this currency is plotted as Fig. 4. This plot demonstrates considerable variation in TRY/USD exchange rate during the period of the study. After a relative increase in the exchange rate in 2007 and 2008 (from 0.7 to 0.8), the exchange rate trend has been declining, so that in 2019 this figure has decreased to 0.07. The maximum and minimum temperatures are 0.8694 and 0.0609, respectively. The median 0.5205 and mean is 0.4737. The skewness is -0.1588 and the standard deviation is 0.2267.

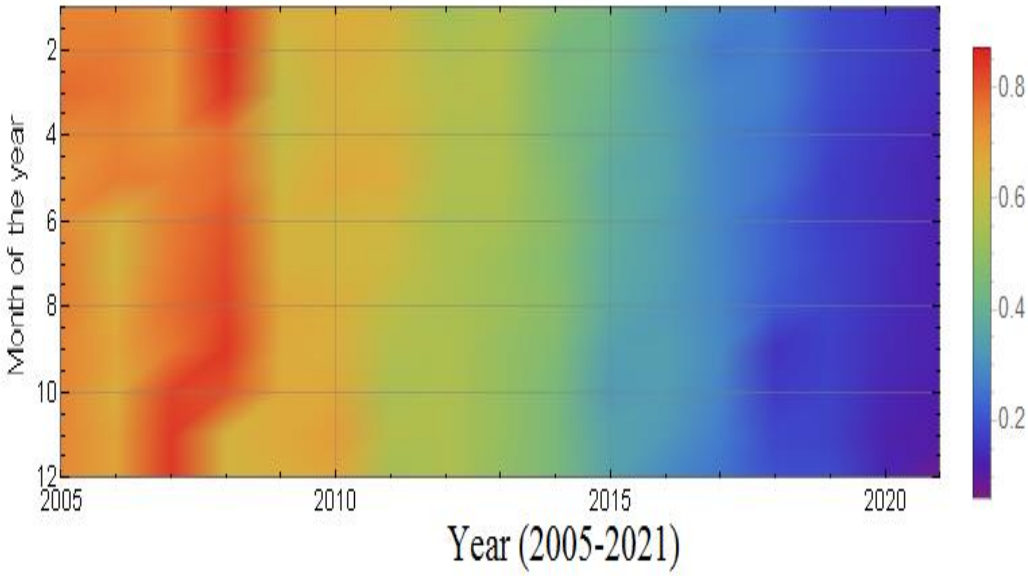

Figure 4. Temporal map of the TRY/USD change during 2005-2021

2.6. Delay time

Figure 5 presents the renovation of the state space in two dimensions for the TRY/USD. The delay time 150 days (five months), that is, the plot presents the points $(x_t, x_{t+\tau})$ for all t. After a moment, it can be seen that 150 days is accurate selection for the delay time τ.

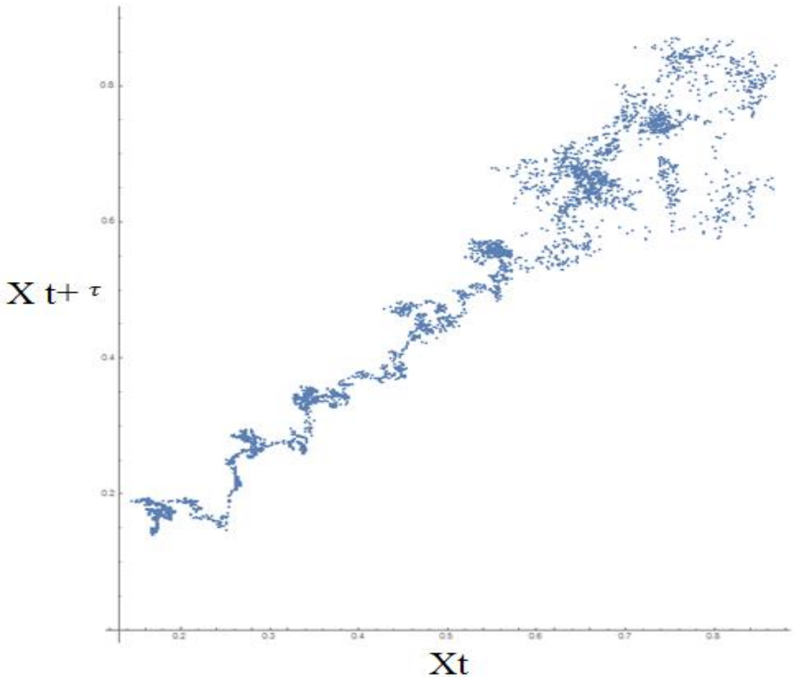

Figure 5. Reconstructed phase space with the delay time 150 days

To detect a suitable delay time for the daily TRY/USD time series, the AMI method is used. In AMI, the first local minimum is chosen as the value of τ. Variations of AMI against delay time are plotted in Figure 6. We see that the first local minimum is approximately at τ = 150 days. These values are the best selection for the delay time, because this τ value maximizes our additional information knowledge from $x_{t};$. But, if τ will be larger or smaller, the AMI be larger and will added less new knowledge.

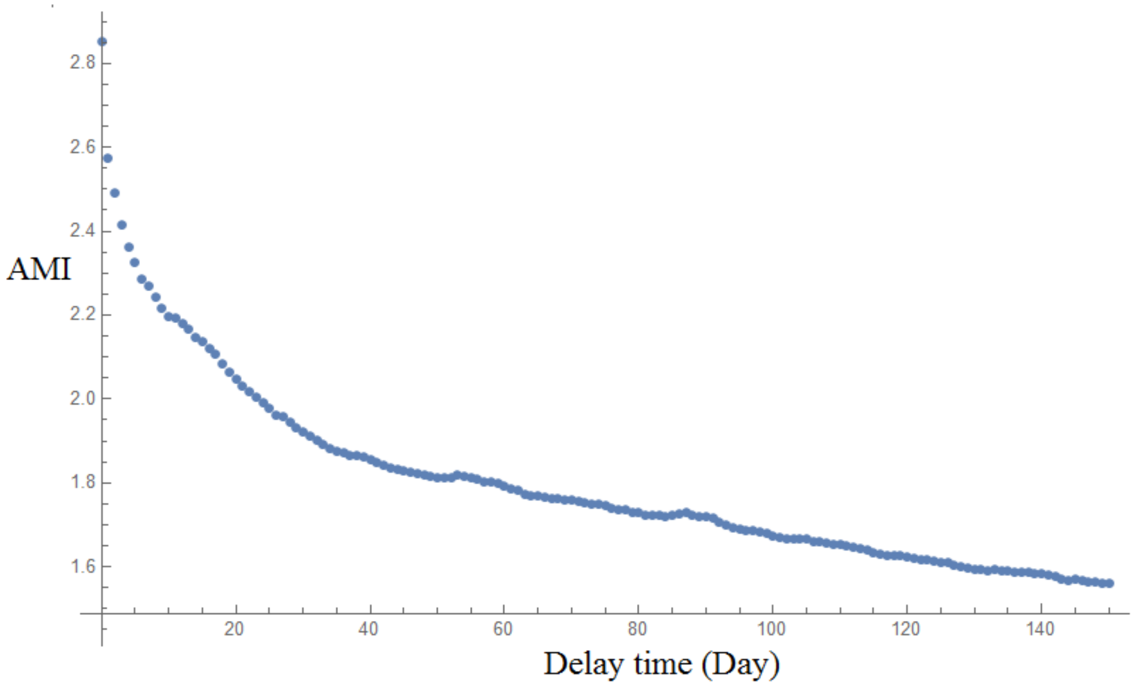

Figure 6. The AMI. The first local minimum is approximately at τ = 150

2.7. Embedding dimension

Knowing the embedding dimension and reconstructing state space is critical. The method of FNN is used to select a proper embedding dimension. Figure 7 presents the FNN percentages for embedding dimensions m = 1, 2, ..., 15. The FNN percentage is zero at m = 6. We have 3685 of the delay coordinates.

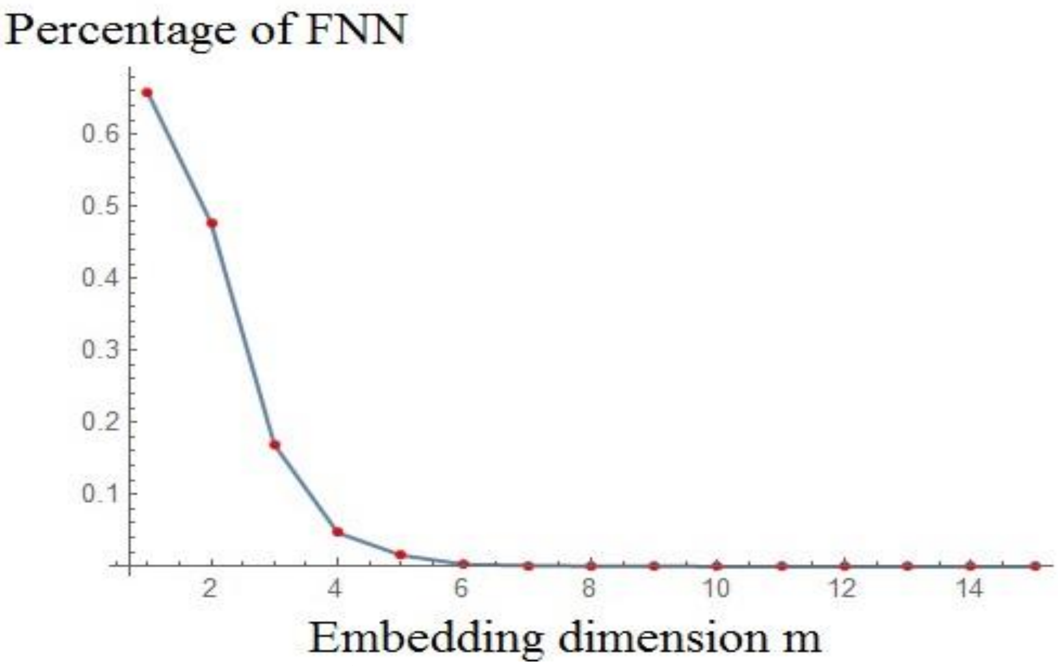

Figure 7. Percentage of FNN. The percentage is approximately zero at m = 6.

2.8. Correlation dimension

In the next step, we calculate the correlation dimension. Fig. 8(a) presents, the C(r) as a function of r for m= 1, 2, ..., 15 in logarithmic scales. The scaling region is shown among the blue vertical lines. Fig. 8(b) shows the correlation exponent (or slopes of the C(r) curves) in the scaling region, for m = 1, ..., 15. It can be seen that for m = 2, ..., 9, this slope is quite constant. The average slope (as a calculation of the correlation dimension) is approximately 1.264. Hence, the dimension of the attractor is 1.264. Because with increasing m, the correlation exponent saturates, system is considered to be chaotic. Hence, the TRY/USD exchange rate at two times instances only differs by a small amount, the difference in this currency at a later time instance may be significant. The sensitivity also causes that, while prediction of the TRY/USD exchange rate may succeed well for a short time horizon, for long horizons accurate prediction becomes impossible. Besides, the positive value of maximal Lyapunov exponent supporting the chaoticity of the data.

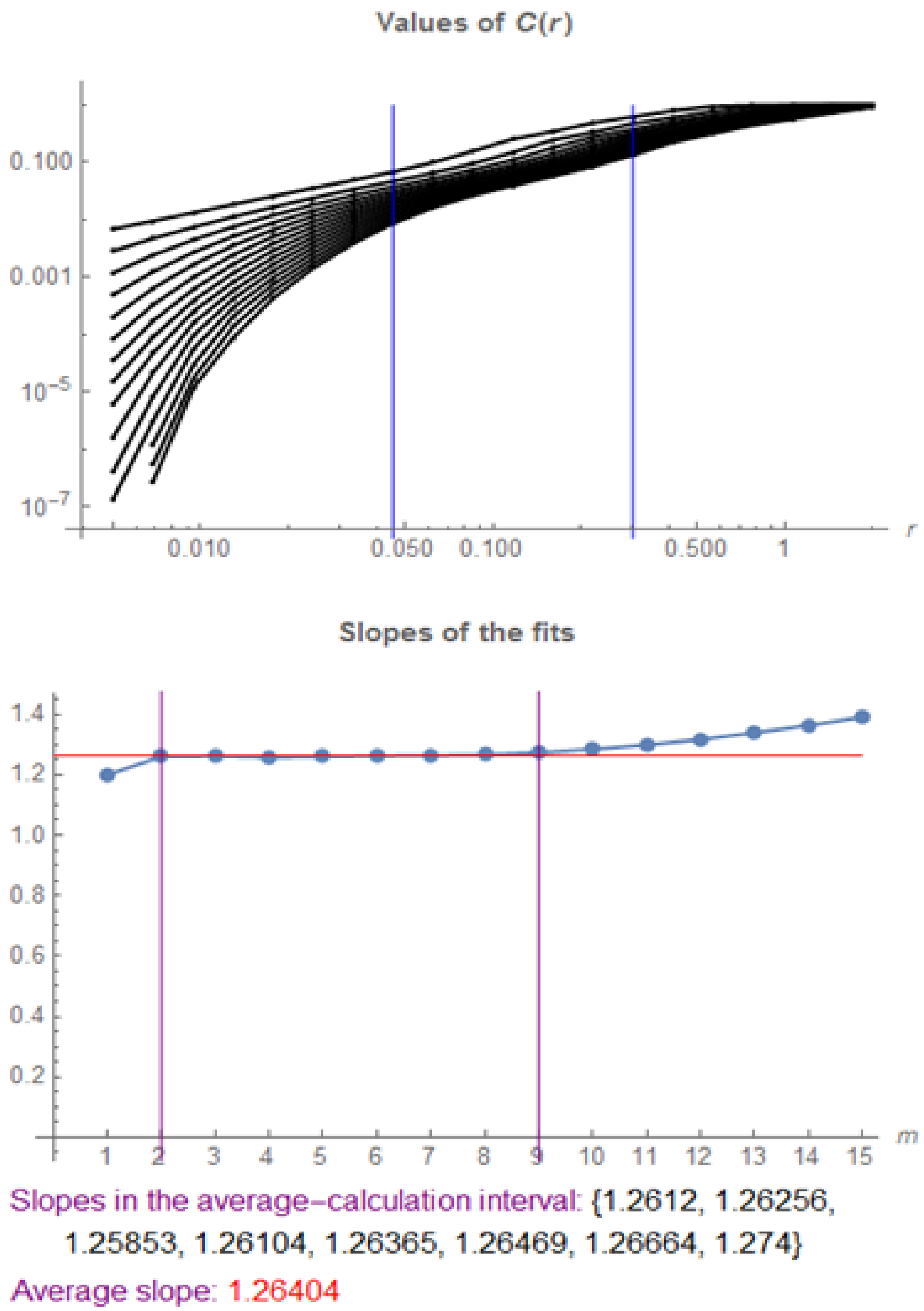

Figure 8. Correlation integral plot for observed daily TRY/USD exchange rate. For m = 2, ..., 9, the average slope in the scaling region is 1.26404; this is the estimate of the correlation dimension.

2.9. Maximal Lyapunov exponent

In the present study, the method of local divergence rates is used to estimate the maximal Lyapunov exponent. Fig. 9(a) presents the S (δ) function for δ = 0, 1, ..., 30 and for m = 1, ..., 15. A scaling region is among δ = 13 and δ = 17. The curve's slopes in the scaling are presented in Fig. 9(b). It can be seen that, the slopes are quite constant for m = 3, ..., 8; and the average slope is 0.025 (a calculation of the maximal Lyapunov exponent). The positive value of the exponent is an indication of chaoticity.

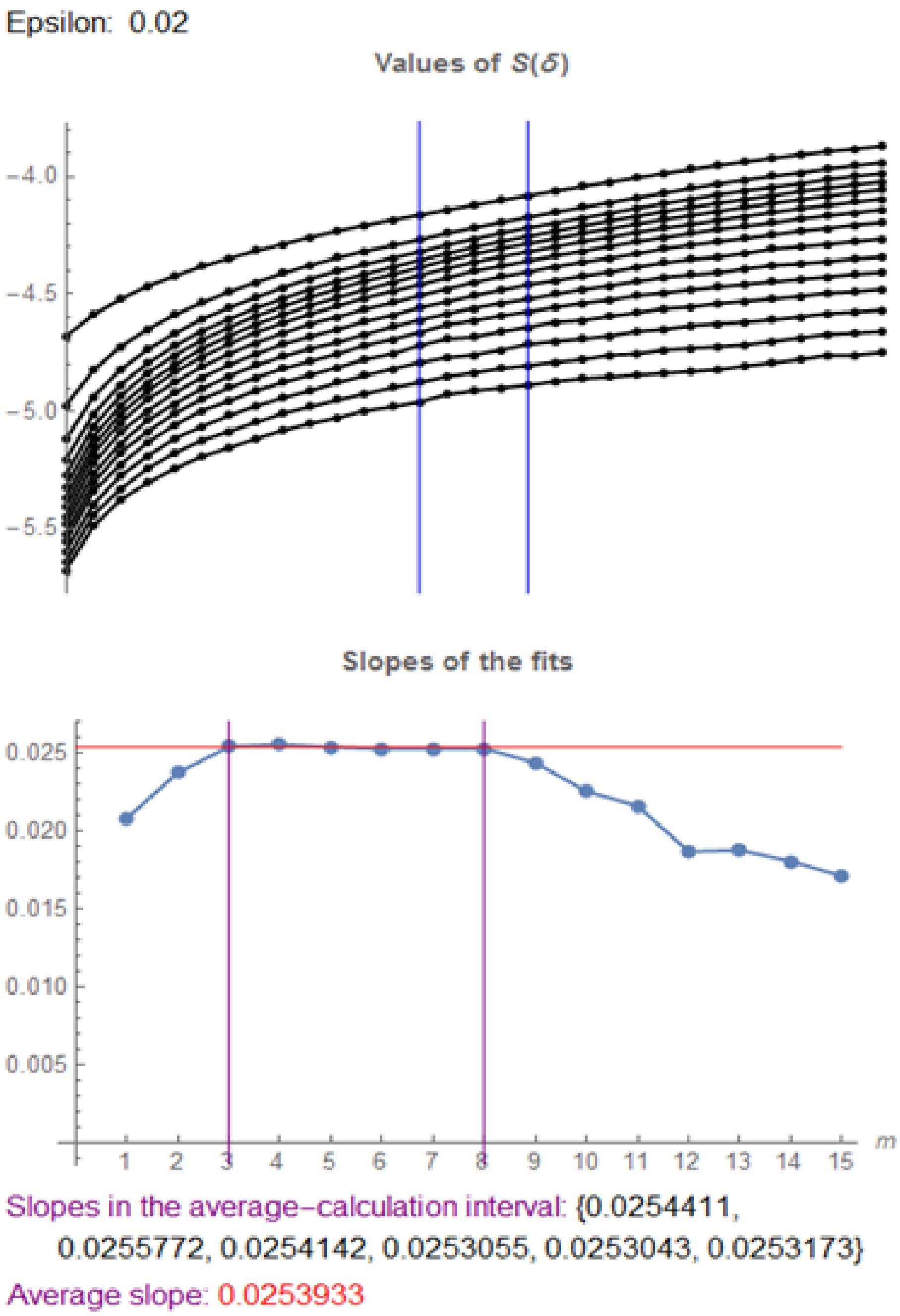

Figure 9. Plot of local divergence rates for observed daily TRY/USD exchange rate. For m = 3, ..., 8, the average slope in the scaling region is 0.025; this is an estimate of the maximal Lyapunov exponent.

2.10. Strength

The strength of nodes for the TRY/USD exchange rate is presented in Figure 10(a). This exchange rate exhibits variations in strength over all the nodes. The strength varies approximately between 40 and 80. There is not a considerable variation in the strength. Some nodes have high variations (about 5) and the other ones have little variations (about 0.05). the variation of strengths does not follow the trend of the TRY/USD time series.

The distribution of strengths for the TRY/USD exchange rate time series (figure 10b) presents different treatment. The strength distribution is left-skewed. The highest count is 78 for the strength value of about 2000 among the strength values between 40 and 80.

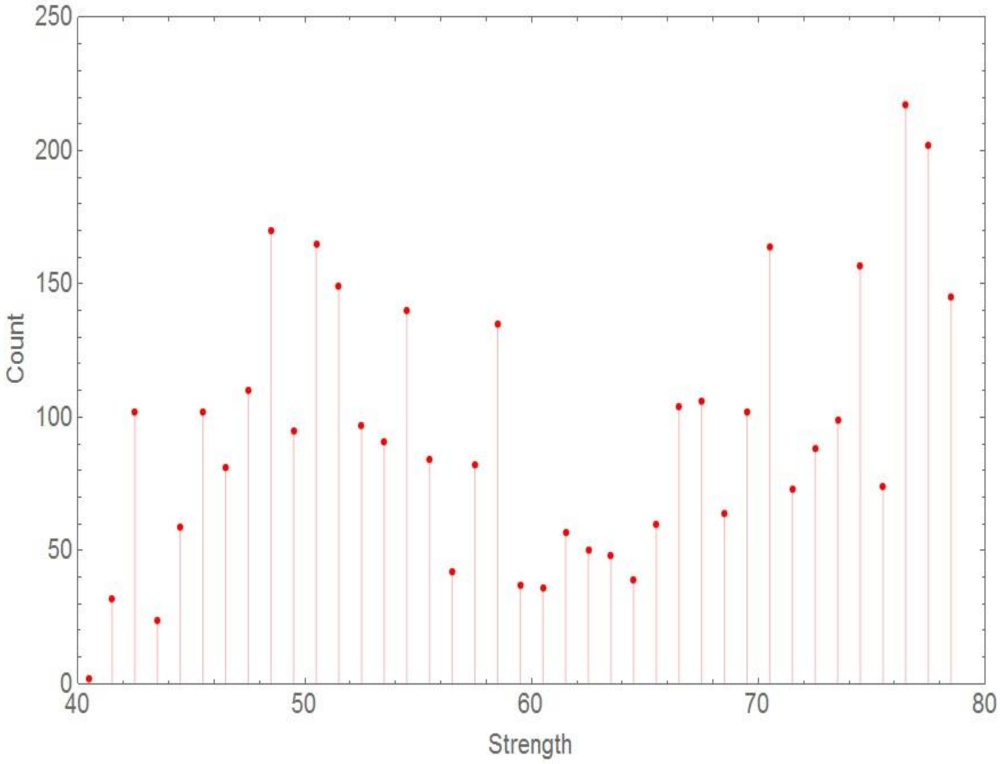

Figure 10. Analysis of the TRY/USD exchange rate: a) the node strength; b) the strength distribution

2.11. Recurrence Plot

Based on the attractor and correlation dimension, the TRY/USD exchange rate is non-linear, and according to the chaos analysis methods, the TRY/USD exchange rate time series chaotic. We use the RP to determine whether the PM $_{2.5}$ time series in Delhi and Beijing is periodic, chaotic or stochastic. Recurrence Plot (RP) is a method to determine the behavior of the non-linear time series. The recurrence plot of the Henon map has irregular line segments in the uniform pattern. This plot indicates the chaotic behavior of the TRY/USD exchange rate (Figure 11).

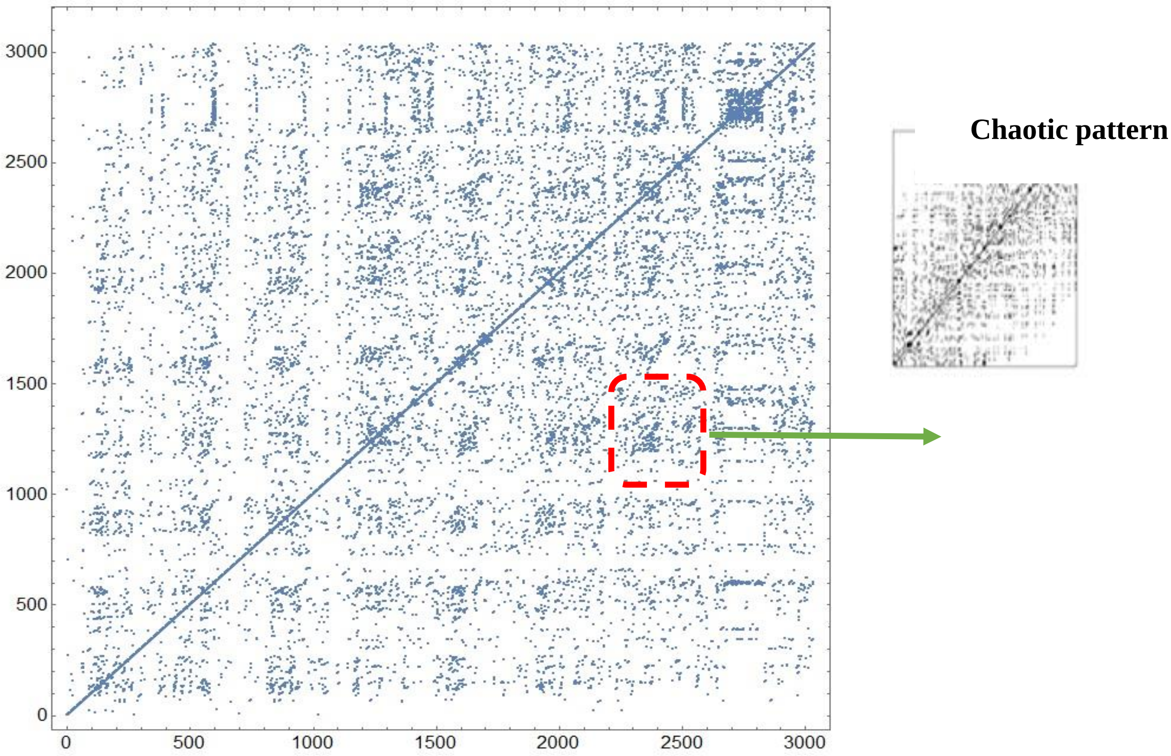

Figure 11. Recurrence plot for the TRY/USD exchange rate

4. Conclusion

In this paper, we analyzed the patterns of the TRY/USD exchange rate during 2005-2021. For this end we utilized the correlation dimension, the embedding dimension, and the maximal Lyapunov exponent. Three methods (method of FNN, method of correlation exponent, and method of local divergence rates) show embedding dimension of this time series should between 3 and 8. The maximal Lyapunov exponent, correlation dimension, and recurrence plot also indicate that the TRY/USD exchange rate is chaotic. Understanding that behavior of daily TRY/USD exchange rate time series root in not irregular randomness and chaotic behavior is crucial in modeling and forecasting to make accurate policies and reducing investment risk.

Availability of data and materials:
Data is available at: https://www.investing.com/currencies/usd-try-historical-data.

Competing interests:
The authors declare no conflicts of interest.

Funding:
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Authors' contributions:
All authors conceived the framework and structured the whole manuscript. All authors have read and agreed to the published version of the manuscript.

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