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Research Article

Some New Results On Shrikhande Graph

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ARTICLE INFO | **ABSTRACT**

In the International Conference on Discrete Mathematics [ICDM, 2021] held in Department of Mathematics, Manonmaniam Sundaranar University, Thriunelveli during October 11- 13, 2021, Dr. A Vijayakumar suggested to work on γ - graph of Shrikhande graph. Motivated by this, an attempt to determine γ - graph of Shrikhande graph has been made in this paper. In addition to that some graph parameters such as hop domination number and hop chromatic number of Shrikhande graph have also been determined.

Keywords: Domination, Hop Domination, Dominator Coloring, Hop Graph
2000 Mathematics Subject Classification: 05C05, 05C15, 05C69

1. Introduction:

Graph coloring and domination are two major areas in graph theory that have been well studied.A dominating set S is a subset of the vertices in a graph such that every vertex in the graph either belongs to S or has a neighbour in S. The domination number is the order of a minimum dominating set.[1]

A proper coloring of a graph $\mathrm{G} = (\mathrm{V(G)}, \mathrm{E(G)})$is an assignment of colors to the vertices of the graph, such that any two adjacent vertices have different colors. The chromatic number is the minimum number of colors needed for a proper coloring of a graph.[1]

A dominator coloring of a graph G is a proper coloring in which each vertex of the graph dominates every vertex of some color class. The dominator chromatic number $\chi_d(G)$ is the minimum number of color classes in a dominator coloring of a graph. [9]

Let $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ be a graph. A set $\mathrm{S} \subset \mathrm{V(G)}$ is a hop dominating set of G if for every $\mathrm{v} \in \mathrm{V} - \mathrm{S}$, there exists $\mathrm{u} \in \mathrm{S}$ such that $d(u, v) = 2$. The minimum cardinality of a hop dominating set of G is called a hop dominator number of G and it is denoted by $\gamma_h(G)$. [2]

This paper deals with γ - graph of Shrikhande graph and some graph parameters such as hop domination number and hop chromatic number of Shrikhande graph have also been determined.

1.1 Preliminaries

Definition 1.1. [3]
A vertex $v \in V$ is a hop dominator of a set $S \subseteq V$ if v hop dominates every vertex in S (ie) $d(u, v) = 2$ for every $u \in S$.

Definition 1.2. [3]
A partition $\pi = \{V_1, V_2, \dots, V_k\}$ is called a hop dominator partition if every vertex is a hop dominator of at least one V_i.

Definition 1.3. [3]
Hop dominator partition number equals the minimum K such that G has a hop dominator partition of order K.

Definition 1.4. [3]
We say that a graph G has a hop dominator coloring if G has a proper coloring
$\pi = \{V_1, V_2, \dots V_k\}$ which is a hop dominator partition of G.

Definition 1.5. [3]
The hop dominator chromatic number $\chi_{hd}(G)$ is the minimum number of colors required for a hop dominator coloring of G.

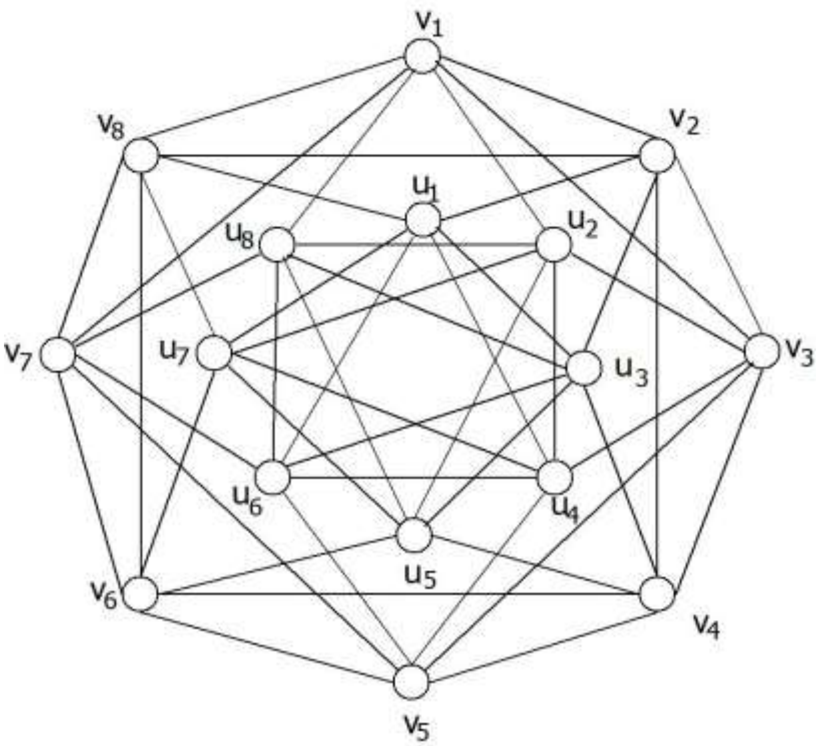
Note 1.6.
Since every vertex is a hop dominator of itself conventionally the partition $\{\{V_1\},\{V_2\},\ldots\{V_n\}\}$ into singleton sets is a hop dominator coloring. Thus every graph n has a hop dominator coloring of order n and therefore hop dominator chromatic number $\chi_{hd}(G)$ is well defined.

2 Shrikhande Graph

Shrikhande Graph discovered by S. S. Shrikhande in 1959 [12] is one of the named graphs playing vital role in Graph Theory. It is a strongly regular graph with 16 vertices and 48 edges, with each vertex having degree 6 .

Observation 2.1.
Shrikhande graph G is a 6 - regular graph. Hence any vertex can dominate atmost seven vertices .
Hence $\gamma(G) \geq \frac{16}{7}$.
(i.e) $\gamma(G) \geq 3$.

For Further discussion let us take G in the following form and label its vertices as shown in the figure.



Shikhande Graph

Observation 2.2.
$\{v_1, v_6, u_1\}$ is a dominating set of G.
$\therefore \gamma(G) \leq 3$ and hence $\gamma(G) = 3$.

Observation 2.3.
Atleast one u_i should be in any γ - set of G.

Proof.
Any v_i covers atmost two u_j 's.
Any γ - set has three elements. Any set $\{v_i, v_j, v_k$ can cover atmost six inner vertices and hence it can not be a γ - set.
So any γ - set of G should have atleast one u_i.

Observation 2.4.

Any γ - set of G should have ateast one v_i.
Reason is similar as that of above observation.

3. γ-sets of Shrikhande Graph

From the above observations it follows that γ - sets are of two types.

$$1.\ \{v_i, v_j, u_k\} \quad 2.\ \{v_i, u_j, u_k\}$$

Shrikhande graph is highly symmetric and so identification of γ - sets becomes simpler.
We got 32 γ - sets for G.

They are listed below:

$$\{v_1,v_4,u_1\}, \{v_1,v_6,u_1\}, \{v_2,v_5,u_2\}, \{v_2,v_7,u_2\}, \{v_3,v_6,u_3\}, \{v_3,v_8,u_3\}$$
$$\{v_4,v_7,u_4\}, \{v_1,v_4,u_4\}, \{v_5,v_2,u_5\}, \{v_5,v_8,u_5\}, \{v_6,v_1,u_6\}, \{v_6,v_3,u_6\}$$
$$\{v_7,v_2,u_7\}, \{v_7,v_4,u_7\}, \{v_8,v_3,u_8\}, \{v_8,v_5,u_8\}, \{u_1,u_2,v_5\}, \{u_2,u_3,v_6\}$$
$$\{u_3,u_4,v_7\}, \{u_4,u_5,v_8\}, \{u_5,u_6,v_1\}, \{u_6,u_7,v_2\}, \{u_7,u_8,v_3\}, \{u_8,u_1,v_4\}$$
$$\{u_4,u_5,v_1\}, \{u_5,u_6,v_2\}, \{u_6,u_7,v_3\}, \{u_7,u_8,v_4\}, \{u_1,u_8,v_5\}, \{u_1,u_2,v_6\}$$
$$\{u_2,u_3,v_7\}, \{u_3,u_4,v_8\}.$$

4 Two different definitions for γ - graph

Definition 4.1. [11]

K. Subramanian, N. Sridharan defined γ - graph of a graph in 2008.

γ - Graph

Let G be a graph. Associate a new graph $\gamma.G$ with G, as follows: The vertex set of $\gamma.G$ is the set of all γ - sets of G. Two vertices D_1 and D_2 of $\gamma.G$ are adjacent if and only if the γ - sets D_1 and D_2 of G are exchangeable with each other. The graph $\gamma.G$ is called the γ - graph of G.

Definition 4.2.[6]

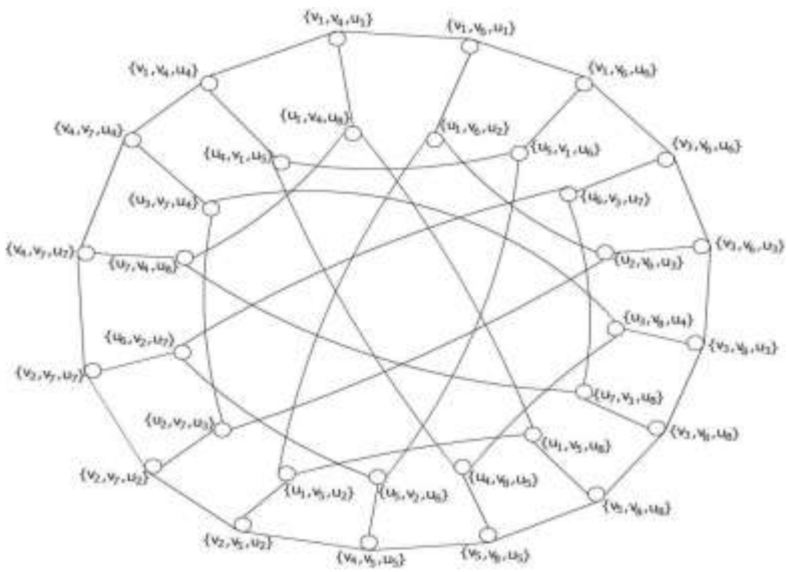
γ - graph introduced by GRED H. FRICKE in 2011.

γ - Graph

Consider the family of all γ - sets of a graph G and define the $\gamma - \boldsymbol{graph}$, $G(\gamma) = (V(\gamma)), E(\gamma))$ of G to be the graph whose vertices $V(\gamma)$ correspond one to one with the γ-sets of G and two γ-sets, say S_1 and S_2, form an edge in $E(\gamma)$, if there exists a vertex $v \in S_1$ and a vertex $w \in S_2$ such that (i) v is adjacent to w and

$$\text{(ii) } S_1 = S_2\ \{w\} \cup \{v\} \text{ and } S_1 = S_2 - \{v\} \cup \{w\}.$$

Here again Shrikhande graph stands as an example for which $\mathrm{G}(\gamma)$ and γ.G coincide. $\mathrm{G}(\gamma) = \gamma.\mathrm{G}$



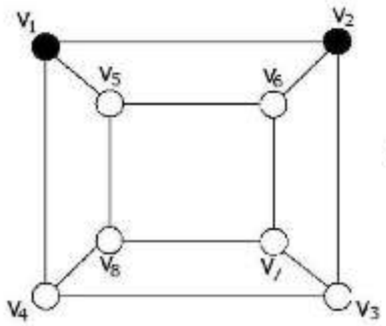
γ - Graph of Shrikhande Graph

5 Hop Graph of Shrikhande Graph

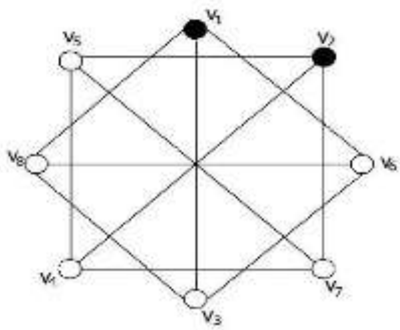
Definition 5.1. [8]

The hop graph $H(G)$ of a graph G is the graph obtained from G by taking $V(H(G)) = V(G)$ and joining two vertices u, v in $H(G)$ if and only if they are at a distance 2 in G.

Example 5.2.



Graph Q_3



Hop graph of Q_3

5.1 Adjacency Matrix of Hop Graph

As the number of vertices of a graph increases, identifying its hop graph becomes much more complicated and time consuming. It is advisable to approach this task using its adjacency matrix. We describe a procedure to obtain the adjacency matrix of the hop graph from that of the given graph.

Let G be a graph with vertex set $\{v_1, v_2, \dots, v_n\}$ and the adjacency matrix be $A = (a_{ij})$.

Take $B = A^2 + A = (b_{ij})$, b_{ij} is equal to the number of $v_i - v_j$ walks.

Construct the matrix $C = (c_{ij})$ from B, by taking

$$c_{ij} = \begin{cases} 0 & \text{if } i = j \text{ or } b_{ij} = 0 \\ 1 & \text{if } i \neq j \text{ and } b_{ij} > 0 \end{cases}$$

Note that $c_{ij} = 1$ if $d_G(v_i, v_j) = 1$ or 2 and $c_{ij} = 0$ otherwise.
Let $D = C - A = (d_{ij})$ (say).

Then

$$d_{ij} = \begin{cases} 1 & \text{if } d_G(v_i, v_j) = 2 \\ 0 & \text{otherwise} \end{cases}$$

Consider the Shrikhande graph.

The adjacency matrix of the Shrikhande graph,

$$A = \begin{array}{cc} & \begin{array}{cccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array} & \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$B = A^2 + A =$

$$\begin{array}{cc} & \begin{array}{cccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array} & \begin{pmatrix} 6 & 3 & 3 & 2 & 2 & 2 & 3 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 3 \\ 3 & 6 & 3 & 3 & 2 & 2 & 2 & 3 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 6 & 3 & 3 & 2 & 2 & 2 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 2 \\ 2 & 3 & 3 & 6 & 3 & 3 & 2 & 2 & 2 & 2 & 3 & 2 & 3 & 2 & 2 & 2 \\ 2 & 2 & 3 & 3 & 6 & 3 & 3 & 2 & 2 & 2 & 2 & 3 & 2 & 3 & 2 & 2 \\ 2 & 2 & 2 & 3 & 3 & 6 & 3 & 3 & 2 & 2 & 2 & 2 & 3 & 2 & 3 & 2 \\ 3 & 2 & 2 & 2 & 3 & 3 & 6 & 3 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 3 \\ 3 & 3 & 2 & 2 & 2 & 3 & 3 & 6 & 3 & 2 & 2 & 2 & 2 & 2 & 3 & 2 \\ 2 & 3 & 2 & 2 & 2 & 2 & 2 & 3 & 6 & 2 & 3 & 3 & 2 & 3 & 3 & 2 \\ 3 & 2 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 6 & 2 & 3 & 3 & 2 & 3 & 3 \\ 2 & 3 & 2 & 3 & 2 & 2 & 2 & 2 & 3 & 2 & 6 & 2 & 3 & 3 & 2 & 3 \\ 2 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 3 & 3 & 2 & 6 & 2 & 3 & 3 & 2 \\ 3 & 2 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 3 & 3 & 2 & 2 & 6 & 3 & 3 \\ 2 & 2 & 2 & 2 & 3 & 2 & 3 & 2 & 3 & 2 & 3 & 3 & 2 & 6 & 2 & 3 \\ 2 & 2 & 2 & 2 & 2 & 3 & 2 & 3 & 3 & 3 & 2 & 3 & 3 & 2 & 6 & 2 \\ 3 & 2 & 2 & 2 & 2 & 2 & 3 & 2 & 2 & 3 & 3 & 2 & 3 & 3 & 2 & 6 \end{pmatrix} \end{array}$$

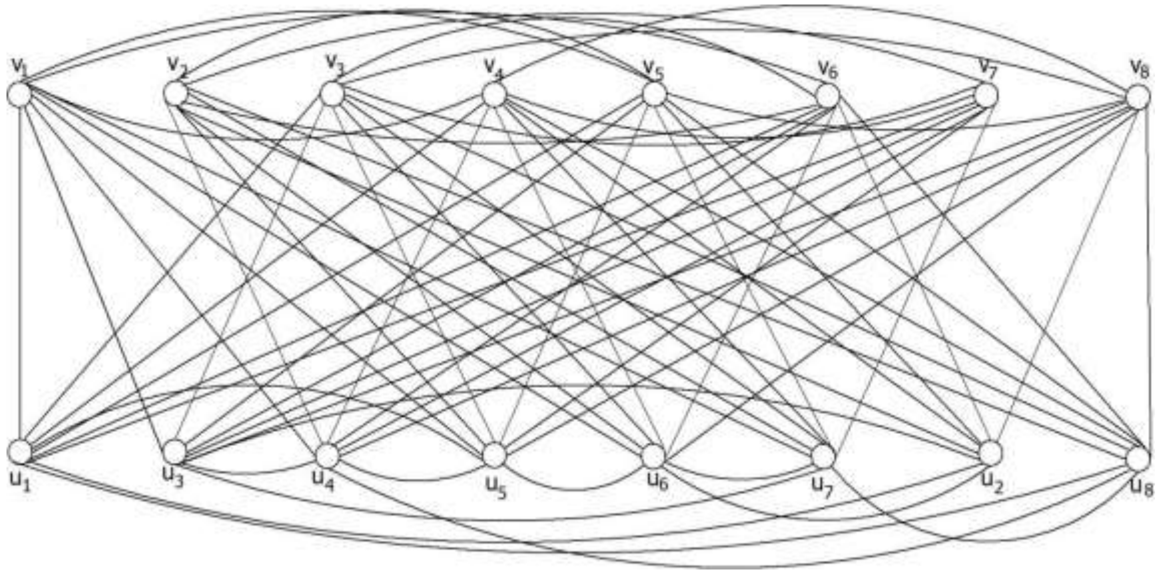
Now in this matrix B, we change all the non zero elements into 1 and all the diagonal elements into zero. Then the matrix C becomes C

$$\begin{array}{cc} & \begin{array}{cccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

$D = C - A =$

$$\begin{array}{cc} & \begin{array}{cccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \end{array} & \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$\Rightarrow$ Adjacency matrix of $H(G) =$



Hop graph of Shrikhande Graph

6 Hop Domination Number of Shrikhande Graph

Definition 6.1. [2]
A set $S \subset V$ of a graph G is a **hop dominating set** of G if for every $v \in V - S$, there exists $u \in S$ such that $d(u, v) = 2$. The minimum cardinality of a hop dominating set of G is called the **hop domination number** and is denoted by $\gamma_h(G)$.

Note 6.2. Any dominating set of $H(G)$ is a hop dominating set of Shrikhande graph.

Observation 6.3. $\{v_1, v_2, v_3\}$ is a hop dominating set of G.
$\therefore \gamma_h(G) \leq 3$.

Observation 6.4. $\gamma_h(G) = 3$.

Proof. Let D be a hop dominating set of G.
With out loss of generality, Let $v_1 \in D$.
The hop neighbourhood of $v_1, N_h(v_1) = \{v_4, v_5, v_6, u_1, u_3, u_4, u_5, u_6, u_7\}$.

So v_1 covers $\{v_1\} \cup N_h(v_1) = D_1$ (Say) Let $D_2 = \text{V} \setminus D_1 = \{v_2, v_3, v_7, v_8, u_2, u_8\}$. The vertices in D_2 are yet to be covered and no single vertex in the neighbour set of D_2 can play this role.

Neighbour set of $D_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} = \text{V}$.

So we need atleast two vertices apart from v_1 to cover all the vertices.
$\therefore \gamma_h(G) \geqslant 3$.
Hence $\gamma_h(G) = 3$.

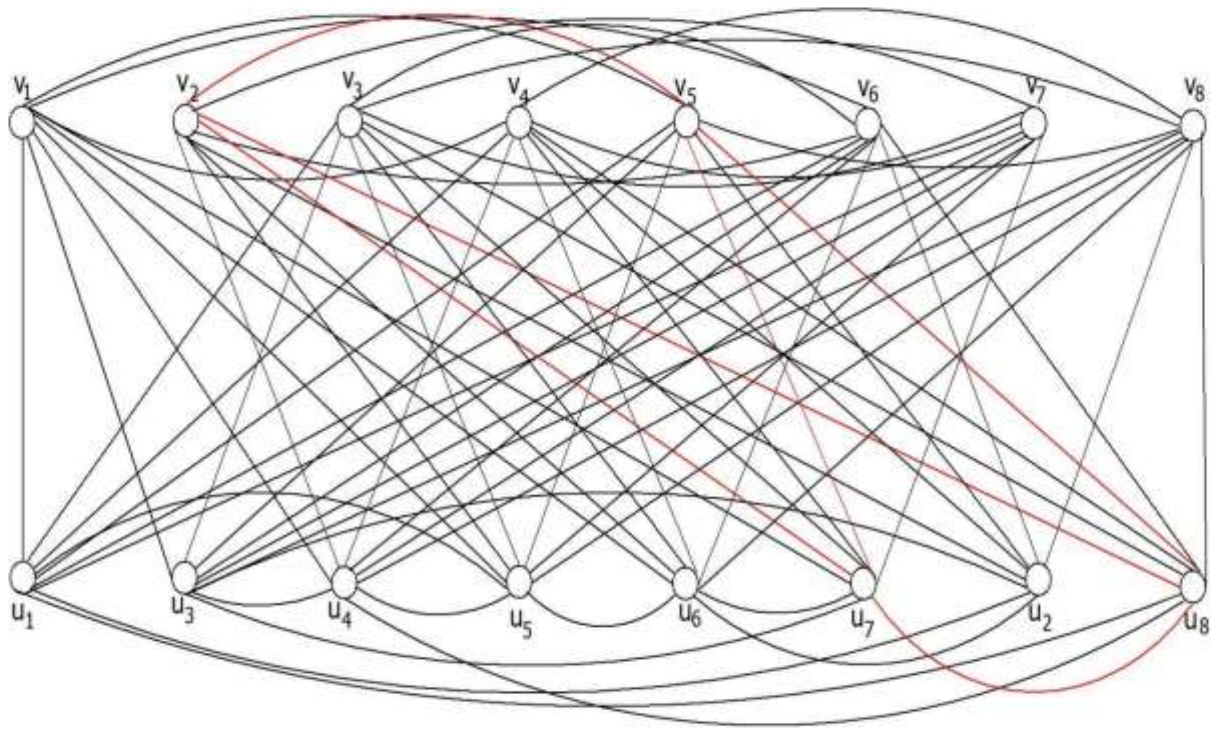
7 Hop Chromatic Number of Shrikhande Graph

Definition 7.1. [13]
A hop coloring of a graph G is a coloring in which no two vertices at a distance two receive the same color. The minimum number of colors required for a hop coloring of G is called **hop chromatic number** of G and is denoted by $\chi_h(G)$.

Definition 7.2. [13]
Let $G = (V, E)$ be a graph. Let $v \in V$ be a vertex of G. The **hop neighbourhood** of v, denoted by $N_h(v)$ is defined as $N_h(v) = \{u \in V / d(u, v) = 2\}$. ie) the set of all vertices at a distance two from v.

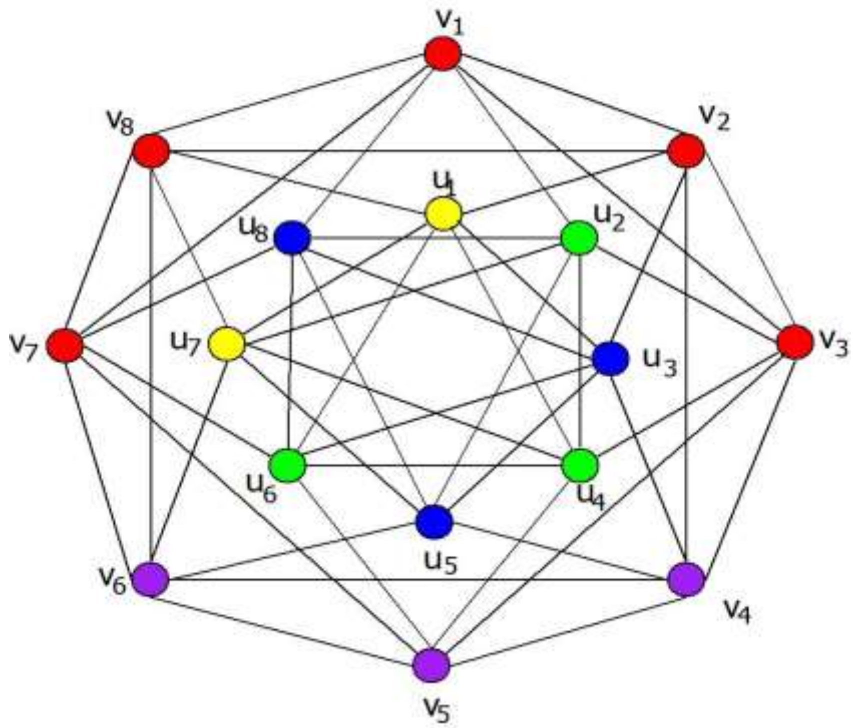
Definition 7.3. [13]
Each n-coloring of G partitions $V(G)$ into n independent sets called color classes. Such a partition induced by a $\chi_h(G)$ coloring of G is called a **hop chromatic partition.**

Observation 7.4. $\chi_h(G) \geqslant 4$.



Hop graph of Shrikhande Graph containing K_4
Hop graph of Shrikhande Graph contains a K_4.
Hence $\chi_h(G) \geqslant 4$.

Hop Coloring of Shrikhande Graph



Hop Coloring of Shrikhande Graph
Hop Chromatic Partition of Shrikhande Graph are, $V_1 = \{v_1, v_2, v_3, v_8, v_7\}$, $V_2 = \{u_1, u_7\}$, $V_3 = \{u_2, u_4, u_6\}$, $V_4 = \{u_3, u_8, u_5\}$, $V_5 = \{v_4, v_5, v_6\}$.
Hop Chromatic Number, $\chi_h(G) \leq 5$.
Hence $4 \leq \chi_h(G) \leq 5$.

References:

1. Arumugam, S., Jay Bagga., Raja Chandrasekar., 2012. On dominator coloring in graphs, Proc. Indian Acad.Sci.(Math. Sci.), **122(4)**, November 2012, PP. 561 - 571.
2. Ayyaswamy, S. K., Natarajan, C., Venkatakrishnan, Y. B., Hop domination in graphs, Discussiones Math. Graph Theory (Submitted).
3. Balamurugan, S., Gnana Jothi R.B., Getchial Pon Packiavathi P., 2018, Hop Dominator Coloring of Graphs, Mathematical Sciences International Research Jour- nal, Vol. 7, Issue 2, ISSN-2278-8697.
4. Gera, R., Horton, S, Rasmussen, C., 2006. Dominator Colorings and Safe Clique partitions, Congress Numerantium.
5. Gera, R., Ralucc., 2007. On dominator coloring in graphs, Graph Theory Notes N.Y.LIT, 25-30.
6. Gerd H. Fricke., 2011. γ -Graphs of Graphs, Discussione Mathematical Graph Theory, **31**, 517-531.
7. Harary, F., Graph Theory, Addisson-Wesley Publishing Company, Inc. USA, 1969. https://doi.org/10.21236/ad0705364
8. Meena Rani, S.M., Hemalatha, T., Hop Graph of a Graph, International Journal of Informative & Futuristic Research (IJIFR), **3(9)**, 3375-3384, ISSN: 2347-1697.
9. Mustapha Chellali., Frederic Maffreay., 2012. Dominator Colorings in Some Graphs of Graphs, Graphs and Combinatorics, **28**, 97-107.
10. Sridharan, N., Subramanian, V.S.A., Elias, M.D., 2002. Bounds on the distance two-domination number of a graph, Graphs and Combinatorics, **18**, 667-675.
11. K. Subramanian and N. Sridharan,2008 γ -graph of a graph, Bull. Kerala Math. Assoc., 5(1), 17-34.
12. Weisstein, Eric W." Shrikhande Graph". Math World.
13. National Level Workshop on Semigraphs and Generalized Graph Structures and Dr.V. Swaminathan Endowment Deliberations, 'Hop Coloring of Graphs' at Depart- ment of Mathematics, V. V. Vanniaperumal College for Women, Virudhunagar, from November 30 to December 02, 2018.