



Statistical Analysis Of Multi-Phase Single Server And Multi-Phase Multi Server: Comparison Study

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ABSTRACT

Queues, or waiting lines, are ubiquitous in various facets of daily life. The study of queuing models proves invaluable for optimizing the operation of systems where waiting times for customers are a critical consideration. This research paper delves into the comparative analysis of two prominent queuing models—multi-phase single-server and multi-phase multi-server—aiming to establish the superiority of the latter in terms of operational efficiency and customer satisfaction. The investigation begins by acknowledging the omnipresence of queues and their impact on service-oriented environments. Recognizing the significance of minimizing customer waiting times, the research focuses on queuing models as effective tools for system optimization. The primary objective is to validate the assertion that the multi-phase multi-server queuing model outperforms its single-server counterpart in delivering enhanced service quality. Through rigorous statistical analysis, the research examines key performance metrics, including throughput, response time, and resource utilization, to provide a comprehensive understanding of the operational dynamics of both queuing models. By comparing the outcomes under varying workloads and conditions, the study aims to delineate the strengths and weaknesses inherent in each system architecture. Also this paper aims to construct code of the Chi-Square Test using MATLAB Software. The findings of this research contribute valuable insights to decision-makers across diverse industries, offering guidance on optimal queuing system design. The multi-phase multi-server queuing model's superiority is substantiated by empirical evidence, paving the way for informed decision-making in sectors ranging from telecommunications and manufacturing to service-oriented industries. Furthermore, the research explores the implications of adopting the multi-phase multi-server approach, emphasizing its potential to address challenges related to scalability and resource allocation. As queues continue to play a pivotal role in shaping customer experiences, this study endeavors to offer practical recommendations for enhancing operational efficiency and customer satisfaction through the implementation of advanced queuing models.

Keywords: $M/E_k/1$ model $M E_k/s$ model ,Multi-phase single server and multi-phase multi server models

I. INTRODUCTION

Delays and queuing issues are pervasive, occurring not only in our daily experiences at places like banks, supermarkets, hospitals, public transportation, and traffic jams but also in technical domains such as manufacturing, computer networking, and telecommunications. Queuing theory emerges as a valuable discipline, offering a mathematical foundation for the study of waiting lines or queues. Constructing queuing models enables the prediction of queue lengths and waiting times, making it an essential tool in understanding and managing these phenomena. Widely regarded as a branch of operations research, queuing theory plays a crucial role in aiding business decisions related to resource allocation for service provision. The comprehensive examination of waiting lines includes factors like the arrival process, service process, the number of servers, system capacity, and the quantity of entities to be served (which could be people, data packets, cars, etc.).

Queuing theory strives to address challenges systematically, applying a scientific understanding to optimize processes and minimize waiting times. By analyzing every component involved in waiting in line, including arrival patterns and service characteristics, queuing theory provides insights that can lead to fully utilized facilities and reduced waiting times. Its applications extend to various fields, guiding decisions related to customer arrival patterns, workstation setups, and workforce requirements based on probability theory. In essence, waiting time (or queuing) theory models offer practical recommendations for managing systems efficiently. By leveraging a probabilistic approach, these models contribute to the optimal utilization of resources, ensuring that facilities operate at their maximum potential while minimizing the waiting times experienced by customers, whether they are individuals, data packets, vehicles, or other entities. The overarching goal is to enhance operational efficiency and improve the overall service experience in diverse settings.

II. RELATED WORK

Ekpenyong and Udoh [1] explores the analysis of a Multi-Server Single Queue System with Multiple Phases, extending the existing Single-Server Single Queue System with Multiple Phases. The study introduces a novel queuing system, denoted as $M/E_k/s: (\infty/FCFS)$, under conditions of First Come First Served, infinite population source, Poisson arrivals, and Erlang service time. The authors derive key performance measures for this multi-server, multi-phase model, including expected total service time, variance, and various queuing properties. Comparative analysis with the Single-Server with Multiple Phases ($M/E_k/1: (\infty/FCFS)$) model demonstrates the enhanced efficiency and effectiveness of the extended model, particularly in handling congestion during peak periods. The literature review contextualizes this contribution within the broader field of queuing theory, emphasizing the significance of extending queuing models to accommodate multiple servers and phases. It underscores the practical implications for managing congestion, improving customer service, and sustaining goodwill. The numerical illustrations provided highlight the tangible benefits of the $M/E_k/s: (\infty/FCFS)$ model over its single-server counterpart, showcasing its superior performance in reducing waiting times and queue lengths. This research extends the theoretical understanding of queuing systems, offering valuable insights for practitioners seeking effective strategies for congestion management in real-world scenarios.

The research paper, "Mathematical Analysis of Single Queue Multi-Server and Multi-Queue Multi-Server Queuing Models: A Comparison Study," by Prasad and Badshah [2], establishes the superiority of the single queue multi-server model over its multi-queue counterpart. Building upon their findings, our study delves into the realm of total cost analysis, introducing waiting cost assumptions to both queuing models. Through meticulous derivation and proof, we demonstrate that the expected total cost is significantly lower for the single queue multi-server model when contrasted with the multi-queue multi-server model. This conclusion, supported by mathematical examples, underscores the cost efficiency of the single queue approach. Our contribution adds a practical dimension to the theoretical discourse, offering valuable insights for decision-makers and system designers aiming to optimize queuing systems for enhanced performance and resource utilization. This study contributes to the ongoing discourse in queuing theory, providing a nuanced perspective on the economic implications of different queuing models.

This study, conducted by Priyangika J.S.K.Cand Cooray T.M.J.A, [3] explores the analysis of sales checkout operations in supermarkets using queuing theory. Acknowledging queues as a pervasive aspect in profit-driven organizations like supermarkets, the research focuses on the checkout service unit, aiming to evaluate key performance metrics such as average service rate, system utilization, and associated costs at various capacity levels. The primary goal is to enhance system efficiency by investigating waiting times and queue lengths, proposing the introduction of additional queues to mitigate customer wait times during peak demand periods. Employing queuing simulation, the research employs a multiple-queue multiple-server model with five checkout sales counters, each accompanied by its respective queue. The empirical data, including arrival and departure times, service duration, and customer feedback, is collected through a developed questionnaire, adding a practical dimension to the analysis.

The study contributes valuable insights for optimizing supermarket checkout operations, providing recommendations for improved customer satisfaction and operational efficacy.

In this insightful study, "A Comparative Analysis of $M/M/1$ and $M/D/1$ Queuing Models in Mitigating Vehicular Traffic Congestion in Kanyakumari District," Dr. K. L. Muruganatha Prasad and B. Usha [4] delve into the application of queuing theory to address traffic issues. Utilizing data from diverse sources, the research assesses the effectiveness of $M/M/1$ and $M/D/1$ queuing models in minimizing congestion at various locations within Kanyakumari district. The findings reveal a traffic intensity parameter (ρ) consistently below 1, indicating successful traffic flow management. The paper adeptly compares results from both queuing models, elucidating their respective strengths and limitations. Dr. K. L. Muruganatha Prasad and B. Usha significantly advance our understanding of queuing theory's practical implications in traffic management, providing valuable insights for decision-makers. Their contribution offers a nuanced perspective on optimizing vehicular flow in diverse locations, enhancing our ability to address real-world traffic dynamics.

Lakshmi C and Sivakumar Appa Iyer's paper [5] critically examines the application of queueing theory in healthcare, specifically focusing on modeling hospital processes. Given the paramount importance of healthcare facilities, where human lives are directly impacted, the central goal is to enhance system performance. The authors emphasize the potential of queueing theory to achieve this objective by categorizing and reviewing its applications. The paper proposes a systematic classification of healthcare areas, expanding on existing literature categories and formulating a detailed taxonomy for subgroups. Through this comprehensive approach, the authors aim to provide analysts with a valuable resource for leveraging queueing theory in healthcare process modeling. By delving into the nuances of different healthcare scenarios, the review equips researchers and practitioners with the necessary insights to locate and apply relevant models effectively. In essence, the paper not only contributes to the theoretical understanding of queueing theory but also serves as a practical guide for healthcare professionals seeking to optimize processes and improve patient outcomes.

III. PROPOSED METHODOLOGY AND DISCUSSION

We will make the following assumptions for queueing system in accordance with queueing theory.

1. Arrivals follow a Poisson probability distribution at an average rate of λ customers per unit of time.
2. The queue discipline is First-Come, First-Served (FCFS) basis by any of the servers. There is no
3. priority classification for any arrival.
4. Service times are distributed exponentially, with an average of μ customers per unit of time.
5. There is no limit to the number of the queue (infinite).
6. The service providers are working at their full capacity.
7. The average arrival rate is greater than average service rate.
8. Service rate is independent of line length; service providers do not go faster because the line is longer.

M/E_k/1 queueing model:(multi-phase single-server)

λ : The mean customers arrival rate

μ : The mean service rate

The average number of customers in the queue:

$$L_{q1} = \left(\frac{k+1}{2k}\right) \left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right)$$

The average number of customers in the system:

$$L_{s1} = \left(\frac{k+1}{2k}\right) \left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right) + \frac{\lambda}{\mu}$$

The average waiting time in the queue:

$$W_{q1} = \left(\frac{k+1}{2k}\right) \left(\frac{\lambda}{\mu(\mu-\lambda)}\right)$$

The average time spent in the system, including the waiting time in the queue:

$$W_{s1} = \left(\frac{k+1}{2k}\right) \left(\frac{\lambda}{\mu(\mu-\lambda)}\right) + \frac{1}{\mu}$$

M/E_k/s queueing model :(multi-phase multi-server)

λ : The mean customers arrival rate

μ : The mean service rate

The average number of customers in the queue:

$$L_{qs} = \left(\frac{k+1}{2k}\right) \left(\frac{\lambda^2}{s\mu(s\mu-\lambda)}\right)$$

The average number of customers in the system:

$$L_{ss} = \left(\frac{k+1}{2k}\right) \left(\frac{\lambda^2}{s\mu(s\mu-\lambda)}\right) + \frac{\lambda}{s\mu}$$

The average waiting time in the queue:

$$W_{qs} = \left(\frac{k+1}{2k}\right) \left(\frac{\lambda}{s\mu(s\mu - \lambda)}\right)$$

The average time spent in the system, including the waiting time in the queue:

$$W_{ss} = \left(\frac{k+1}{2k}\right) \left(\frac{\lambda}{s\mu(s\mu - \lambda)}\right) + \frac{1}{s\mu}$$

We have taken the data from scenario of a factory cafeteria with a four-counter system, the queuing model exhibits characteristics of a Multiple Phases Queue model with a single server in each phase, specifically denoted as M/E_k/1. The process involves customers navigating through four distinct counters for purchasing coupons, selecting snacks, collecting tea, and obtaining dessert. Key performance measurements are determined based on the specified parameters. Client arrivals are modeled as a Poisson process with a mean arrival rate of 9 per hour. Service times follow an Erlang distribution with a mean of 1.5 minutes per customer. The queue discipline is first-come, first-served, and the population is considered infinite. This M/E_k/1 model allows for the analysis of customer wait times, queue lengths, and overall system efficiency, offering valuable insights into the cafeteria's operational performance. The combination of Poisson arrival, Erlang service time distribution, and the specified queue discipline provides a comprehensive framework for understanding and optimizing the cafeteria's queuing dynamics. Customers (workers) in a factory cafeteria must go through four counters. Coupons are purchased at the first counter, snacks are chosen and collected at the second counter, tea is collected at the third counter, and dessert is collected at the fourth counter. The following performance measurements can be achieved if client arrivals follow a Poisson process with a mean arrival rate of 9 per hour, service times follow an Erlang distribution with a mean of 1.5 minutes per customer, and the queue discipline is first come first served with infinite population:

We note that this is a case of Multiple Phases Queue model with a single server in each phase that is M/E_k/1
 $\lambda = 9/\text{hr}$, $\mu = 1.5\text{mins}/\text{person} = 10 \text{ person}/\text{hr}$,
 $k = 4$

The following results are obtained:

$$Lq = \frac{k+1}{2k} \left[\frac{\lambda^2}{u(\lambda - \mu)} \right] = \frac{5}{8} \left[\frac{9^2}{10(10-9)} \right] = 5.0625 \approx 5 \text{ person}$$

$$w_q = \frac{k+1}{2k} \left[\frac{\lambda}{u(\lambda - \mu)} \right] = \frac{5}{8} \left(\frac{9}{10(10-9)} \right) = 0.5625 \text{ hr} = 33.75 \text{ mins}$$

$$w_s = w_q + \frac{1}{\mu} = 0.5625 + \frac{1}{10} = 0.6625 \text{ Hrs} = 39.75 \text{ min}$$

$$Ls = \lambda W_s = 9 \times 0.6625 = 5.9625 \approx 6 \text{ person}$$

$$E(T) = \frac{1}{10} = 0.1 \text{ Hr or } 6 \text{ mins}$$

Case 2: Assuming that each counter has two service points, we can calculate the performance measure using the formula

M/E_k/2.

$$Lq = \frac{5}{8} \left[\frac{9^2}{20(20-9)} \right] = \frac{405}{17600} = 0.023$$

$$W_q = \frac{5 \times 9}{17600} = 0.00256 \text{ hrs} = 0.153 \text{ minutes} = 0.9 \text{ sec}$$

$$w_s = w_q + \frac{1}{s\mu} = 0.00256 + \frac{1}{20} = 0.05256 \\ = 3.15 \text{ mins}$$

$$Ls = \lambda w_s = 9 \times 0.05256 = 0.473$$

$$E(T) = \frac{1}{s\mu} = \frac{1}{20} = 0.05 \text{ hr or } 3 \text{ minutes}$$

Chi square test: The significance of χ^2 lies in its role in determining the statistical significance of the difference between observed and expected frequencies. There are two primary Chi-square tests:

There are two types of Chi square tests.

- (i) The Chi-square goodness of fit test, which assesses whether the distribution of frequencies for a categorical variable deviates significantly from the expected distribution.
- (ii) The Chi-square test of independence, which evaluates the relationship between two categorical variables.

When testing a hypothesis regarding the distribution of a categorical variable, a Chi-square test or another nonparametric test is typically necessary. The "goodness of fit" of a statistical model refers to how effectively it aligns with a given set of observations.

TEST OF SIGNIFICANCE:

$$\chi^2 = \sum \frac{(O-E)^2}{E} \quad ; O = \text{observed data}$$

E= expected data

WORKING RULE:

1. Set a null hypothesis
2. Set up an alternative hypothesis
3. Set a level of significance α
4. Calculate χ^2
5. Find the degree of freedom and corresponding value of χ^2 at given level of significance α
6. If the calculated value of χ^2 is less than tabulated value of χ^2 then null hypothesis is accepted.

Chi square test calculation:

Null hypothesis H_0 : The waiting time in $M/E_k/s$ lesser than $M/E_k/1$.

Alternative hypothesis: In $M/E_k/s$, the wait time exceeds $M/E_k/1$.

Level of significance: $\alpha = 0.05$

Here, $n=7$

Critical value: $v = n-1 = 7-1 = 6$

$$\chi^2_{(0.05)}(V = 6) = 12.59$$

WAITING TIME IN QUEUE FOR $M/ E_k/1(w_q)$

	observed	expected	$(O - E)^2$	$\frac{(O - E)^2}{E}$
1	36	33.75	5.0625	0.15
2	37	33.75	10.5625	0.312962963
3	35	33.75	1.5625	0.046296296
4	33	33.75	0.5625	0.016666667
5	32	33.75	3.0625	0.090740741
6	30	33.75	14.0625	0.416666667
7	34	33.75	0.0625	0.001851852
Total	237	236.25	34.9375	1.035185185 = χ^2

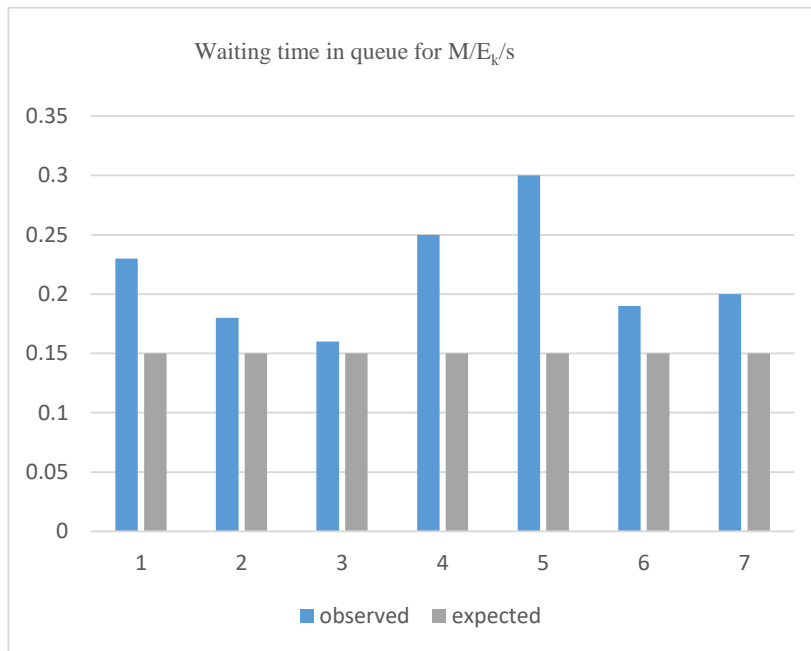
Since, $\chi^2 = 1.0351 < 12.59$ the null hypothesis is accepted at 5% level of significance. i.e, the waiting time in $M/E_k/s$ is lesser than $M/E_k/1$.



	observed	expected	$(O - E)^2$	$\frac{(O - E)^2}{E}$
1	0.23	0.15	0.0064	0.042666667
2	0.18	0.15	0.0009	0.006
3	0.16	0.15	0.0001	0.000666667
4	0.25	0.15	0.01	0.066666667
5	0.3	0.15	0.0225	0.15
6	0.19	0.15	0.0016	0.010666667
7	0.2	0.15	0.0025	0.016666667
Total	1.51	1.05	0.044	0.293333333= χ^2

WAITING TIME IN QUEUE FOR M/ E_k/s(w_q)

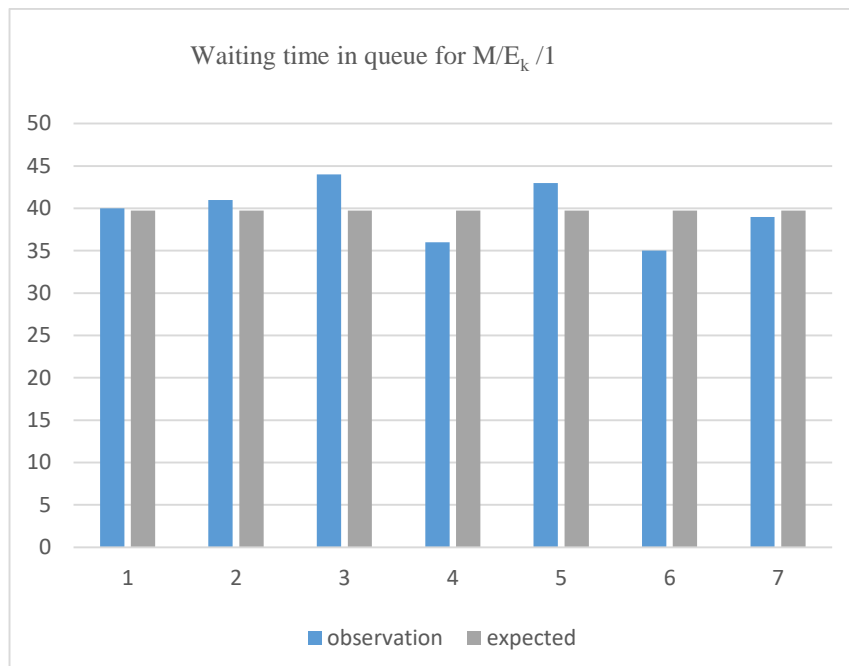
Since, $\chi^2=0.2933 < 12.59$ the null hypothesis is accepted at 5% level of significance. i.e, the waiting time in M/ E_k /s is lesser than M/ E_k /1.



WAITING TIME IN SYSTEM FOR M/ E_k /1 (w_s)

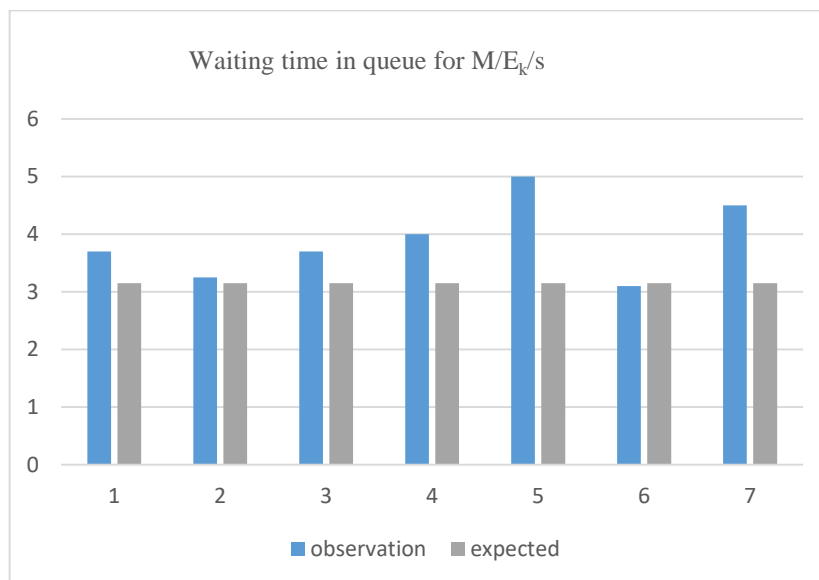
	observed	expected	$(O - E)^2$	$\frac{(O - E)^2}{E}$
1	40	39.75	0.0625	0.00157233
2	41	39.75	1.5625	0.03930818
3	44	39.75	18.0625	0.45440252
4	36	39.75	14.0625	0.35377358
5	43	39.75	10.5625	0.26572327
6	35	39.75	22.5625	0.56761006
7	39	39.75	0.5625	0.01415094
Total	278	278.25	67.4375	1.69654088= χ^2

Since, $\chi^2=1.6965 < 12.59$ the null hypothesis is accepted at 5% level of significance. i.e, the waiting time in M/ E_k /s is lesser than M/ E_k /1.



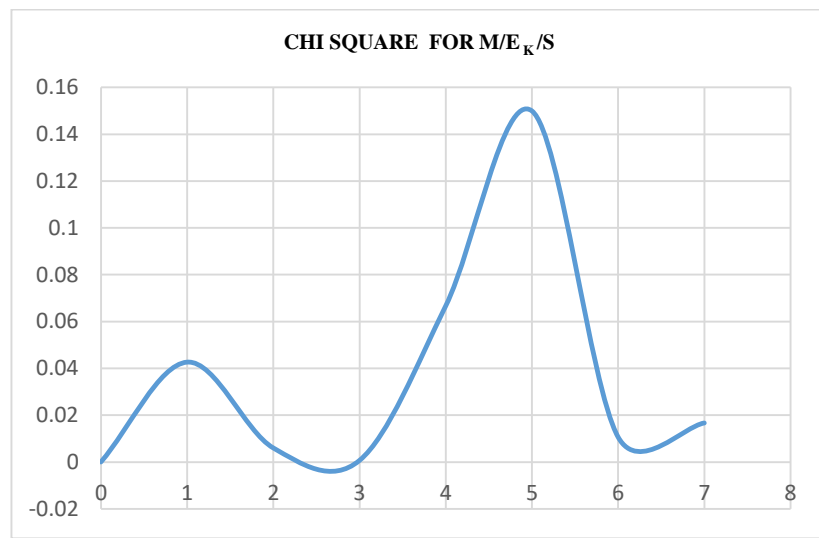
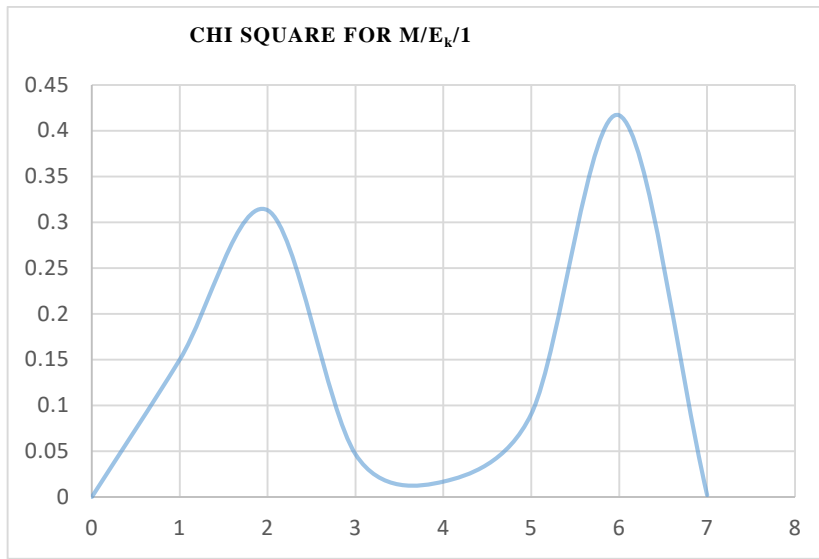
WAITING TIME IN SYSTEM FOR M/ E_k/s(w_s)

	observed	expected	$(O - E)^2$	$\frac{(O - E)^2}{E}$
1	3.7	3.15	0.3025	0.09603175
2	3.25	3.15	0.01	0.0031746
3	3.7	3.15	0.3025	0.09603175
4	4	3.15	0.7225	0.22936508
5	5	3.15	3.4225	1.08650794
6	3.1	3.15	0.0025	0.00079365
7	4.5	3.15	1.8225	0.57857143
total	27.25	22.05	6.585	2.09047619= χ^2

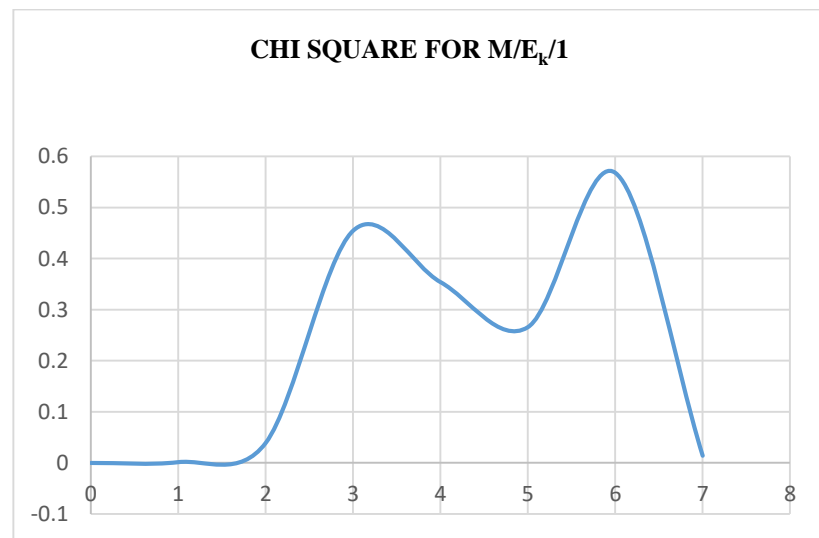


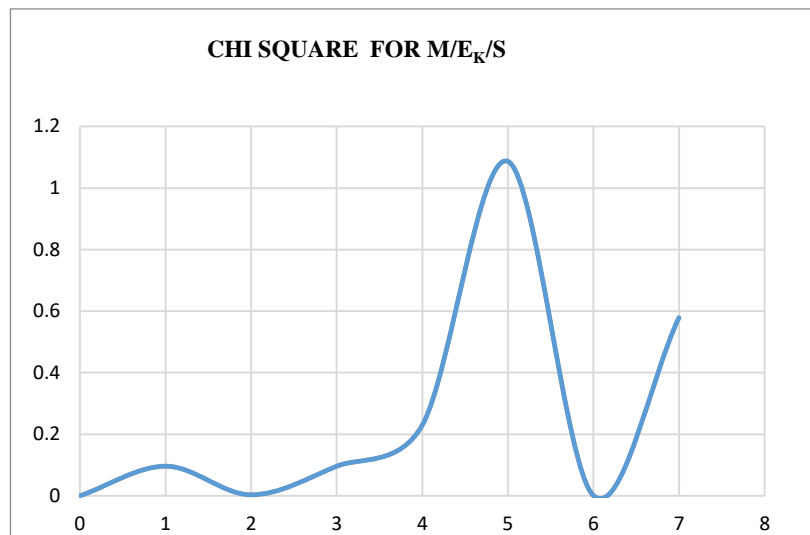
Since, $\chi^2 = 2.090 < 12.59$ the null hypothesis is accepted at 5% level of significance. i.e, the waiting time in M/ E_k/s is lesser than M/ E_k/1.

CHI SQUARE TEST: $M/ E_k /1$ vs $M/ E_k /s (w_q)$



CHI SQUARE TEST: $M/ E_k /1$ vs $M/ E_k /s (w_s)$



**MATLAB:**

MATLAB is a versatile programming language widely used in scientific and engineering domains. Let's delve into its advantages:

Ease of Use:

MATLAB provides an interactive environment where you can evaluate expressions at the command line or execute prewritten programs.

Its integrated development environment (IDE) includes features like an editor, debugger, and extensive demos, making it optimal for fast prototyping.

Platform Independence:

MATLAB runs on various platforms, including Windows, Linux, macOS, and UNIX.

Applications written on one platform can seamlessly run on others, ensuring flexibility.

Predefined Functions:

MATLAB comes with a vast library of predefined functions for common tasks.

These built-in functions handle calculations like mean, standard deviation, and more, saving you time and effort.

Special-purpose toolboxes cater to specific areas like signal processing, control systems, and image processing.

MATLAB CODE for Chi-Square Distribution:

We have constructed the MATLAB code for chi-square test. Observed values, expected values and appropriate chi square tabulated values are to be taken from the user.

```
Observed_values=input('Enter the observed values:\n')
```

```
Expected_values=input('Enter the expected values:\n')
```

```
Chi_tab=input('Enter the Tabulated value =')
```

```
n=length(Expected_values);
```

```
for i=1:n
```

```
  y(i)=(Observed_values(i)-Expected_values(i))^2;
```

```
  Ans(i)=y(i)/Expected_values(i);
```

```
  A=sum(Ans);
```

```
end
```

```
A
```

```
if A < Chi_tab
```

```
  fprintf('the null hypothesis is accepted')
```

```
  else
```

```
    fprintf('the null hypothesis is rejected')
```

```
  end
```

Here are the outputs of the chi square test of the above considered data performed in MATLAB Software.

Output

1.

Enter the observed values:

```
[36 37 35 33 32 30 34]
```

Observed_values =

36 37 35 33 32 30 34

Enter the expected values:

[33.75 33.75 33.75 33.75 33.75 33.75 33.75]

Expected_values =

33.7500 33.7500 33.7500 33.7500 33.7500 33.7500 33.7500

Enter the Tabulated value =

12.59

Chi_tab =

12.5900

A =

1.0352

the null hypothesis is accepted

2.

Enter the observed values:

[0.23 0.18 0.16 0.25 0.3 0.19 0.2]

Observed_values =

0.2300 0.1800 0.1600 0.2500 0.3000 0.1900 0.2000

Enter the expected values:

[0.15 0.15 0.15 0.15 0.15 0.15 0.15]

Expected_values =

0.1500 0.1500 0.1500 0.1500 0.1500 0.1500 0.1500

Enter the Tabulated value =

12.59

Chi_tab =

12.5900

A =

0.2933

the null hypothesis is accepted

3.

Enter the observed values:

[40 41 44 36 43 35 39]

Observed_values =

40 41 44 36 43 35 39

Enter the expected values:

[39.75 39.75 39.75 39.75 39.75 39.75 39.75]

Expected_values =

39.7500 39.7500 39.7500 39.7500 39.7500 39.7500 39.7500

Enter the Tabulated value = 12.59

Chi_tab = 12.5900

A = 1.6965

the null hypothesis is accepted

4.

Enter the observed values:

[3.7 3.25 3.7 4 5 3.1 4.5]

Observed_values =
3.7000 3.2500 3.7000 4.0000 5.0000 3.1000 4.5000

Enter the expected values:
[3.15 3.15 3.15 3.15 3.15 3.15 3.15]

Expected_values =
3.1500 3.1500 3.1500 3.1500 3.1500 3.1500 3.1500

Enter the Tabulated value =
12.59
Chi_tab = 12.5900
A = 2.0905
the null hypothesis is accepted.

Result and declaration

We have made the statistical comparative study between $M/E_k/1$ and $M/E_k/s$ queuing model. The Chi-Square test is a statistical procedure for determining the difference between observed and expected data.

From goodness of fit we can conclude that waiting time in $M/E_k/s$ queuing model is lesser than waiting time in $M/E_k/1$ queuing model.

By using MATLAB Software we have formed the code for Chi square test and verified our results derived from the calculations.

Future Scope

1. The study of $M/E_k/1$ and $M/E_k/s$ models can be done for some other real-life problem.
2. The study of $M/E_k/1$ and $M/E_k/s$ can be conducted by Combine queuing models with machine learning algorithms. Develop hybrid approaches that leverage historical data and predictive models.
3. Investigation of more complex multi-phase multi-server models that incorporate additional features, such as server vacations, priority classes, and heterogeneous servers.
4. Also, One can explore scenarios where servers have different service rates or varying capabilities during different phases.
5. One can incorporate customer behavior dynamics into multi-phase multi-server models. Understand how customer impatience, balking and renege impact system performance.
6. In addition, we can explore advanced statistical methods for parameter estimation, model validation, and hypothesis testing in multi-phase multi-server systems.

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