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Research Article

On The Study Of AVD- Total Coloring Of Triangular Snake Graph Families

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ARTICLE INFO | **ABSTRACT**

Assume that a simple undirected graph is represented by *G = (V, E)*. It is assumed that the correct total coloring has been done when two adjacent vertices have different sets of colors for the incidence edges on them and the vertex itself. This study investigates the total coloring graphs of adjacent vertex distinguishing (AVD) systems. Moreover, we determine the triangle families of the AVD-total color number of Snake Graph.

Keywords: Triangular snake graph, AVD total coloring.

Introduction

Coloring of graphs is one of the most important, well-known, and actively studied subfields of graph theory. The graph coloring problem is one of the most researched because of its theoretical and practical significance. As a result, experts and scholars from all over the world have studied this topic in great detail. Certain network issues can be addressed with adjacent-vertex differentiating edge coloring and adjacent-vertex distinguishing total coloring.
Colors can be assigned to the edges, vertices, or both of a graph G. The vertex coloring is considered accurate if no two vertices acquire the same color. The literature contains a wide range of suitable colorings, including vertex coloring. a-coloring, b-coloring, edge coloring, list coloring, and so on are some examples of coloring techniques. The entire coloring of graphs is the main focus of the current effort. A total coloring of G is a function $f: S \rightarrow C$ where $S = V(G) \cup E)(G)$ and C is a set of colors to satisfies the given Conditions.

No vertex next to it receives the same color more than once.
The color of no two adjacent borders is the same.
None of its edges have the same color applied to its end vertices.
The least cardinality k that allows G to have a total coloring by k-colors is known as the total chromatic number, or $\chi''(G)$ of a graph.

Definition

Chromatic number:
If G has a valid vertex coloring, then the chromatic number of G is the minimal number of colors needed to color G. The chromatic number of G is represented by the symbol
$\chi(G)$.

Definition

Total coloring:
A graph G is said to be fully colored when all of its edges and vertices have the same color applied to them.

Definition
Total chromatic number:
The symbol $\chi''(G)$ represents total- chromatic number, which is the bare minimum number of colors required to get color G.

Definition
AVD-total coloring:
G is a basic graph, and ϕ represents G's overall coloring. ϕ represents an AVD-total color. In the event when ∀ u,v∈V(G) uv adjacent, C(u)≠C(v). In this case, C(u): color set that appears in a vertex u.

Definition
Triangular Snake T_m: [6]A Triangular Snake T_m is obtained from a path $u_1, u_2, u_3, \dots, u_m$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le m-1$.

Definition
Double Triangular snake $D(T_m)$: [6] Two triangular snakes with a common path make up the double triangular snake $D(T_m)$

Definition
Alternate Triangular Snake AT_m :[6]An Alternate Triangular Snake AT_m is obtained from a path $u_1, u_2, u_3, \dots, u_m$ by joining u_i and u_{i+1} (alternatively) to a new vertex v_i.
In the present paper we focusing on AVD coloring for triangular snake graph T_m, Double triangular snake graph DT_m and Alternative triangular snake graph AT_m.

MAIN RESULT AND DISCUSSION

Theorem 1

For any triangular snake graph $\chi(T_m) = \begin{cases} n=3 \ \chi(T_m) \text{ is } 3 & \text{otherwise} \\ \quad \forall n > 3\ \chi(T_m) \text{ is } 5 \end{cases}$ **or** $\chi(T_m) = \Delta(G) + 1$

Proof: Let $V(T_m) = \{u_l : 1 \le l \le m-1\} \cup \{v_l : 1 \le l \le m\}$ represents the vertices in graph and graph contains $2m+1$ number of vertices, where $m = 3,7,9,11 \dots$.
$E(T_m) = \{e_l : 1 \le l \le m-1\} \cup \{s_l : 1 \le l \le m-1\} \cup \{f_l : 1 \le l \le m-1\}$ represents the edges in graph and graph contains $3m$ number of edges, where $m = 3,6,9,12..$
where the edge $\{e_l : 1 \le l \le m-1\}$ represents the edge $\{v_l v_{l+1} : 1 \le l \le m-1\}$, the edge $\{s_l : 1 \le l \le m-1\}$ represents the edge $\{u_l v_l : 1 \le l \le m-1\}$ and
$\{u_l v_{l+1} : 1 \le l \le m-1\}$. From the definition of AVD coloring,
We prove the theorem case by case:

Case 1: for $n = 1$, isolated vertex. AVD coloring exist by having one colorable. But cycle C_3 triangular snake graph does not exist.

Case 2: for $n = 2$, an edge. AVD coloring exist by having three colorable $\chi''(G) = 3$. But cycle C_3 triangular snake graph does not exist.
Example : Suppose for an edge the assigned colors blue(b), red(r), green(g)

Figure 1: an edge.

$C(v_1) = \{b, r\}$,$C(v_2) = \{g, r\}$ $C(v_1) \neq C(v_2)$
AVD - coloring with distinguishable vertices.
Hence for $n < 3$ the cycle C_3 does not exist. Hence triangular snake graph also does not exist. But AVD coloring is possible by taking 1 and 3 colorable.
Case 3: for $n \ge 3$ the cycle C_3 exist. Hence triangular snake graph also exists. AVD coloring is possible.
Case a: for = 3 , it forms a cycle C_3. One triangular snake graph exists by having 3 vertices and 3 edges. AVD coloring of triangular snake graph C_3 is $\chi''(G) = 3$.

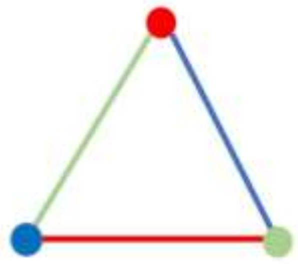

Figure 2: C_3

Example: Consider a cycle C_3 having red(r), blue(b), green(g). From the definition of AVD- coloring we get,

$$C(v_1) = \{g, r, b\}$$
$$C(v_2) = \{g, b, r\}$$
$$C(v_1) \neq C(v_2)$$

AVD - coloring with distinguishable vertices.

Case b: for $n > 3$, it forms a cycle C_3 . more number of triangular snake graph exist by having $2m + 1$ number of vertices, where $m = 3,7,9,11$ .... and $3m$ number of edges, where $m = 3,6,9,12$.. The graph T_m is colored properly with 5 colors. Since triangular snake graph T_m has $3m$ edges and $2m + 1$ vertices. Each T_m is of the form $m\ C_3$'s connected with $(n - 1)$ paths. Each C_3 is 3 – colorable and hence T_m is 5 colorable. Each cycle of three vertices can be colored with three colors.

As seen in the graph, the path can be clearly inserted between thebC_3. Here, (u) is the collection of colours that make up a vertex u. There is a difference between two vertices $u, v \in v(G)$ when $C(u) \neq C(v)$. If not, rearrange the designated colours until then in order to obtain vertices that can be distinguished. The correct AVD-total colorings are one with recognizable vertices. Therefore $T_m = \Delta(G) + 1\ \forall\ n > 3$.
Example : let us assign few colors to figure 3,to apply AVD coloring, $red(r), blue(b), green(g), yellow(y), black(bl)$

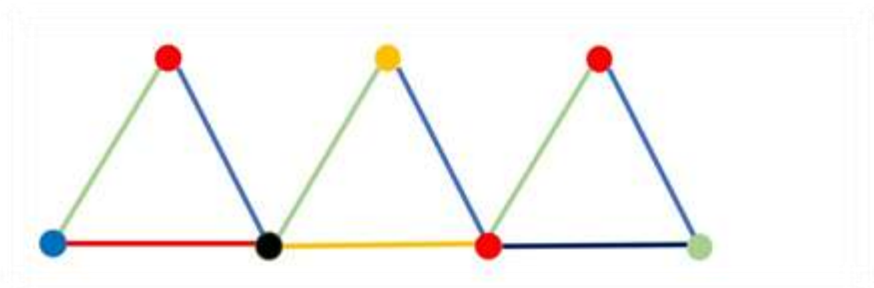

Figure 3 : $m = 3$, $3C_3's$ are attached.

$$C(v_1) = \{g, r, b\}$$
$$C(v_2) = \{g, b, r\},$$
$$C(v_3) = \{r, bl, b, g, y\},$$
$$C(v_4) = \{y, r, b, g, bl\},$$
$$C(v_5) = \{g, y, b\},$$
$$C(v_6) = \{g, r, b\},$$
$$C(v_7) = \{b, g, bl\}$$
$$C(v_3) \neq C(v_4),$$
$$\{r, bl, b, g, y\} \neq \{y, r, b, g, bl\}$$

Here $C(u) \neq C(v)$ AVD - coloring with distinguishable vertices.

Theorem 2
Let DT_m be double triangular snake graph of order $n > 4$, then $\chi''(DT_m) = 7$.
Proof:
Let $V(DT_m) = \{u_l, w_l : 1 \leq l \leq n - 1\} \cup \{v_l : 1 \leq l \leq n\}$ and

$E(DT_m) = \begin{cases} e_l: 1 \le l \le n-1\} \cup \{e_l': 1 \le l \le n-1\} \cup \\ \{e_l'': 1 \le l \le n-1\} \cup \{s_l: 1 \le l \le n-1\} \cup \{s_l': 1 \le l \le n-1\} \end{cases}$, where the edges $\{e_l: 1 \le l \le n-1\}$ represents the edge $\{v_l v_{l+1}: 1 \le l \le n-1\}$, the edges $\{e_l': 1 \le l \le n-1\}$, the edges $\{u_l v_l: 1 \le l \le n-1\}$, the edges $\{e_l'': 1 \le l \le n-1\}$ represents the edge $\{u_l v_{l+1}: 1 \le l \le n-1\}$, the edges $\{s_l: 1 \le l \le n-1\}$ represents the edge $\{w_l v_l: 1 \le l \le n-1\}$ and the edges $\{ s_l': 1 \le l \le n-1\}$ represents the edges $\{w_l v_{l+1}: 1 \le l \le n-1\}$. Using the definition of AVD coloring we are proving this theorem by cases:

Case 1: for $n = 1$, isolated vertex. AVD coloring exists by having one colorable. But cycle C_3 double triangular snake graph does not exist.
Case 2: for n = 2 , an edge. AVD coloring exist by taking $\chi''(G) = 2$. But cycle C_3 double triangular snake graph does not exist.
Case 3: for $n = 3$. A cycle C_3. AVD coloring exists by having three colorable $\chi''(G) = 3$. But double triangular snake graph does not exist.
Case 4: for $n \ge 4$. A double triangular snake graph exists. AVD graph coloring is applied case by case:
Case a: for $n = 4$. A double Cycle C_3 exist by having 4 vertices and 5 edges. Also AVD coloring exist by having $\chi''(G) = 4$
Example: the graph takes AVD 4 coloring , $red(r), blue(b), green(g), yellow(y)$

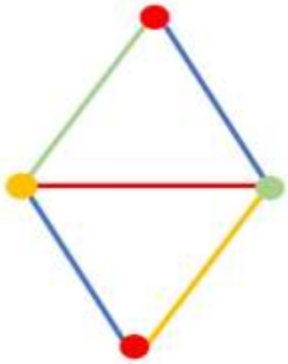

Figure 4: $m = 1$, Double triangle triangular snake.
$C(v_1) = \{g, r, b\}$,
$C(v_2) = \{g, y, r, b\}$,
$C(v_3) = \{b, r, y\}$,
$C(v_4) = \{b, g, r, y\}$
$\{g, y, r, b\} \ne \{b, g, r, y\}$
$C(v_2) \ne C(v_4)$,
Here $C(u) \ne C(v)$ AVD - coloring with distinguishable vertices.

Case b: In this case for $n > 4$, double triangular snake graph takes $3m + 1$ vertices, where $m = 4,7,10,13$....and $5m$ edges where $m = 5,10,15,20,25$.
This double triangular graph has the appropriate coloration with χ"(G)=5. Every DT_m is composed of m double C_3 linked by $(n-1)$ pathways. Since each double C_3 has a color of 5, DT_m has a color of 7. Five colors can be used to color each double cycle with four vertices. As seen in the graph, the path may be clearly inserted in between the double cycle C_3.
The collection of colors present in a vertex u is denoted by (u). There are two identifiable vertices $u, v \in v(G)$ when $C(u) \ne C(v)$. Until then, rearrange the allocated colours to obtain recognisable vertices. An AVD-total colouring that has recognisable vertices is the correct one. Thus If $n > 4$, then $T_m = \Delta(G) + 1$.
Example : Let us consider three double triangular snake graph, having 10 vertices and 15 edges. From the definition of AVD-coloring we get,

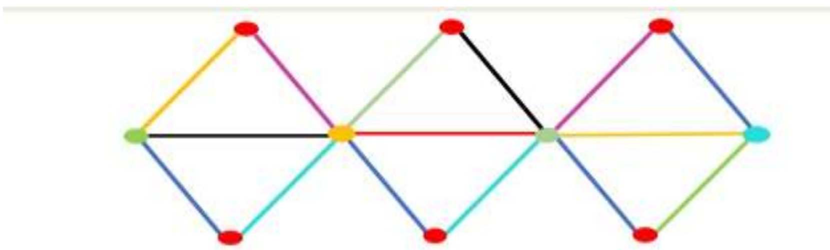

Figure 5: $m = 3$, 3 *Double triangular* $C_3's$ are attached. Here
$red(r), blue(b), black(bl), light\ blue(lb), purple(p), yellow(y), green(g)$.

$C(v_1) = \{y, r, p\}$,
$C(v_2) = \{y, g, bl, b\}$,
$C(v_3) = \{b, r, lb\}$,
$C(v_4) = \{p, y, b, lb, g, r, b\}$
$C(v_5) = \{g, r, b\}$,
$C(v_6) = \{b, r, lb\}$,
$C(v_7) = \{bl, g, r, lb, p, y, b\}$,
$C(v_8) = \{p, r, b\}$,
$C(v_9) = \{b, r, g\}$,
$C(v_{10}) = \{b, lb, y, g\}$

Here $C(u) \neq C(v)$ AVD - coloring with distinguishable vertices.

Theorem 3

Let AT_n be the alternate triangular snake graph, then $\chi''(AT_m) > 4$

Proof: Let $V(AT_m) = \{u_l: l \in \{1,2, \dots, n\}\} \cup \{v_l: l \in \{1,3,5 \dots, n-2\}\}$ represents vertices in alternate triangular graph and graph contain $3m$ number of vertices $m = 3,6,9,12 \dots$ and

Let $E(AT_m) = \{e_l: l \in \{1,2, \dots, n-1\}\} \cup \{e_l': l \in \{1,3, \dots, n-2\}\} \cup \{e_l'': l \in \{1,3, \dots, n-2\}\}$,

Where the edges $\{e_l: l \in \{1,2, \dots, n\}\}$ represents the edges $\{u_l u_{l+1}: l \in \{1,2, \dots, n-1\}\}$ the edges $\{e_l': l \in \{1,3, \dots, n-2\}\}$ represents the edges $\{u_l v_1: l \in \{1,3, \dots, n-2\}\}$, the edges $\{e_l'': l \in \{1,3, \dots, n-2\}\}$ represents the edges $\{v_l u_{l+1}: l \in \{1,3, \dots, n-2\}\}$. Here the alternate triangular graph contains total $4m - 1$ edges, where $m = 3,7,11,15, \dots$

From the definition of AVD coloring we prove the theorem case by case:

Case 1: when $n = 1$. Isolated vertex, an alternative triangular snake graph does not exist. Takes AVD-coloring.

Case 2: When $n = 2$. An edge, takes AVD-coloring but alternative triangular graph does not exist.

Case 3: when $n = 3$. Cycle C_3, consists of 3 vertices and 3 edges but an alternative triangular graph does exist. Graph takes $\chi''(G) = 3$

Case 4: when $n \geq 4$, this case is proved by having subcases:

Case a: when $n = 4$ in this case graph exists with 4 vertices and 4 edges. A cycle C_3 with an extra edge will be attached to a triangle. But alternative triangular graph does not exist, to become alternative triangular graph two cycle of $C_3's$ are placed between one edge.

Example: cycle C_3 is attached with an edge.

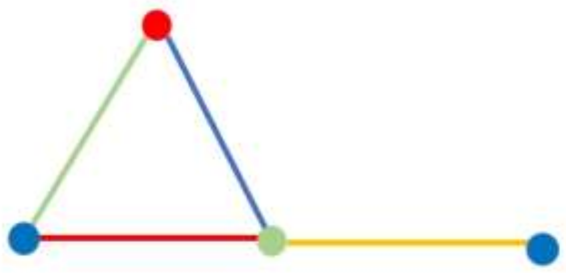

Figure 6: C_3 *with an edge*

$C(v_1) = \{g, r, b\}$,
$C(v_2) = \{g, b, r\}$,
$C(v_3) = \{b, g, r, y\}$,
$C(v_4) = \{b, y\}$
$C(v_1) \neq C(v_2)$,
$C(v_3) \neq C(v_4)$

Here $C(u) \neq C(v)$ AVD - coloring with distinguishable vertices.

Case b: when $n > 4$, the alternative triangular snake graph exists by taking 3m and 4m-1 number of vertices and edges as mentioned above. Since the cycle C_3 is placed between two edges, it takes properly three colorable. Every AT_m is of the type C_3's, which connects the $(n-1)$ pathways. Since each C_3 has three colorations, AT_m has four colorations. You can use three different colours to colour each cycle of three vertices. The path can be clearly inserted as shown in the graph, alternately, between the cycle C_3. The collection of colours present in a vertex u is denoted by (u). There are two identifiable vertices $u, v \in v(G)$ when $C(u) \neq C(v)$. Until then, rearrange the allocated colors to obtain recognizable vertices. An AVD-total colorings that has recognizable vertices is the correct one. Thus If $n > 4$, then $AT_m = \Delta(G)+1$.

Example: consider few colors to show AVD coloring on alternative triangular snake. $red(r), blue(b), green(g), black$(bl)

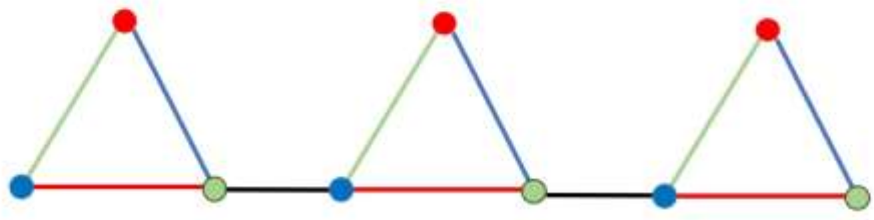

Figure 7 : $m = 3$, Alternative triangular snake graph AT_m.

$C(v_1) = \{g, r, b\}$,
$C(v_2) = \{g, b, r\}$,
$C(v_3) = \{bl, b, r, g\}$,
$C(v_4) = \{bl, b, g, r\}$,
$C(v_5) = \{g, r, b\}$,
$C(v_6) = \{r, g, b, bl\}$,
$C(v_7) = \{bl, b, g, r\}$,
$C(v_8) = \{b, r, g\}$,
$C(v_9) = \{r, g, b\}$

Here $C(u) \neq C(v)$ AVD - coloring with distinguishable vertices.

Conclusion

In this article, we have determined AVD total chromatic number of triangular snake graph T_m, Double triangular snake graph DT_m and Alternative triangular snake graph AT_m. For many other graphs this work can be further extended.

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