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Research Article

Modelling Of Neutrosophic Theory In Bulk Queueing System

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ARTICLE INFO | **ABSTRACT**

This research examines the performance measures of the bulk queue system in a Neutrosophic setting. It creates and improves the performance measures for bulk queue systems with two and three parameters; in notation, this is represented as $NM^{[b]}/NM/1$. We use Neutrosophic values to indicate the arriving rate and serving rate. We calculated the formula for the performance measure in the Neutrosophic environment for both uniform and non-uniform service rates. Additionally, show how to compute the $NM^{[b]}/NM/1$ performance measure for values of two and three parameters. Additionally, a comparison and discussion of the performance metrics between crisp and Neutrosopic values were made.

Key Words: bulk queueing system, neutrosophic values, two parameters, three parameters, performance measure.

1. Introduction:

In the classical queueing theory, the several performance measures were determined by the arrival, service or exit rates. In real life, the arrival rate and service rate are usually erroneous [1,2]. In order to address uncertainty in the queueing system parameters, researchers offered the queueing theory in a fuzzy environment, as in [3,4]. Better models and reality representations are required in order to address the unreliable arrival rate and service rate constraints with queueing theory.
In 1995, Smarandache combined the ideas of intuitionistic fuzzy logic and fuzzy logic to create Neutrosophic logic [5,6,7,8]. Neutronosophic logic has the advantage of handling factual ambiguity in addition to degrees of truth and falsity. Consequently, decision-making is enhanced in a neutrophilic environment [9,10,11,12, 13]. Mohamed Bishe Zeina [14,15,16] provided numerous essential formulas that are an important tool in queueing systems in a Neotrosophic environment in his study of the Erlang service queueing model in a Neutrosophic environment.
An event is considered to be (T) true, (F) false, and (I) indeterminate in Neutrosophic probability, where T, I, and F are real values from the intervals between]-0,1+[, with no restriction on the total T+I+F. However, in many applications, these components may not be presented clearly, and Neutrosophic numbers can be formed by adding the determinant part of the number (D) and the sum of Indeterminacy (I) in the form N=D+I [11,12,13,16].
In queueing theory, a bulk queue—also referred to as a batch queue—is a type of queue discipline. Many researchers have been interested in dimension-related problems. The concept of bulk queues, including bulk arrivals and/or bulk service, is well-established and has garnered significant attention. Recognize that scenarios involving bulk arrivals (also called batch arrivals) and bulk service waits are common in places like utilizing the elevator in government buildings and hospitals, driving traffic, traveling on suburban trains, etc. Consequently, related models find many practical applications. In this research, we compute the performance measure by using the Neutrosophic queueing formula on bulk queues.
The following are the remaining sections of this paper: The preliminary discussion of queueing theory was given in brief in Section 2. In Section 3, the basic principles of Neutrosophic queueing theory are addressed and a bulk queueing system model is presented using Neutrosophic queueing formulas. Numerical examples for the proposed model of NM[b]/NM/1 Neutrosophic formulas with uniform and uneven service rates for two and three parameters are provided in sections 4 and 5. We had discussions and a conclusion in section 6.

2. Bulk Queueing model description:

The model is of notation type $M^{[x]}/M/1/\infty/FCFS$, consider a single service channel
with a Poisson input of batch size X, a fuzzified exponential inter-arrival service model of a queue system with infinite capacity, and FCFS service discipline.
Let c_x to represent assigned probability and let **x** represent the arrival rate of the Poisson Process for batch size **x**. The number of customers in any arrival is a random variable, with $C_x = \frac{\lambda_X}{\lambda}$ and where λ is representing the composite arrival rate of all batches of size x

$$\lambda = \sum_{i=1}^{\infty} \lambda_i.$$

Define Pn (t), the likelihood that n units are present in the system at any time t. By using general Birth Death arguments, it is easy to derive the model's differential difference equations.

$P_n'(t) = -(\lambda + \mu)P_n(t) + \mu P_{n+1}(t) + \lambda \sum_{i=1}^{\infty} . P_{n+1}(t) c_k, \qquad n \geq 1$

$P_0'(t) = -\lambda P_0(t) + \mu P_1(t), \qquad n = 0$

In steady state:
The steady state condition is reached when the behavior of the system becomes independent of thetime. When $t \to \infty$ the steady state equations are

$$0 = -(\lambda + \mu)P_n + \mu P_{n+1} + \lambda \sum_{k=1}^{n} P_{n-k} c_{k,} \qquad n \geq 1$$

$$0 = \lambda P_0 + \mu P_1 \qquad n = 0$$

The various performance measures of this model are derived by Meenu Mittal etal [17].

2.1 Two Parameter value – the batch of two customers arrivals

2.1.1 Expected batch size E(x) for two parameters λ_1 & λ_2 is given by

$E(X) = \frac{(\lambda_1 + 2\lambda_2)}{\lambda}$ where $\lambda = \lambda_1 + \lambda_2$

2.1.2 Expected queue length L_q for two parameters λ_1 & λ_2 is given by

$L_q = \left[\frac{\rho_1 + 3\rho_2}{1 - \rho_1 - 2\rho_2}\right]$ where $\rho = \rho_1 + 2\rho_2;\ \rho_1 = \frac{\lambda_1}{\mu}$, $\rho_2 = \frac{\lambda_2}{\mu}$

2.1.3 Waiting time in queue:

$$W_q = \frac{L_q}{\lambda}$$

2.1.4 Waiting time in the system

$$W_s = W_q + \frac{1}{\mu}$$

2.1.5 Average Utilization $\rho = \frac{\lambda}{\mu}$

2.2 Three Parameters value – the batch of three customers arrival

2.2.1 Expected batch size E(x) for three parameters λ_1, λ_2 & λ_3 is given by

$E(X) = \frac{(\lambda_, + 2\lambda_2 + 3\lambda_3)}{\lambda}$ where $\lambda = \lambda_1 + \lambda_2 + \lambda_3$

2.2.2 Expected queue length for three parameters λ_1, λ_2 & λ_3 is given by

$L_q = \left[\frac{\rho_1 + 3\rho_2 + 6\rho_3}{1 - \rho_1 - 2\rho_2 - 3\rho_3}\right]$ where $\rho = \rho_1 + 2\rho_2 + 3\rho_3;\ \rho_1 = \frac{\lambda_1}{\mu}$, $\rho_2 = \frac{\lambda_2}{\mu}$, $\rho_3 = \frac{\lambda_3}{\mu}$

3. Preliminaries of Neutrosophic theory

In this section, we present the idea of the neutrosophic and construct the neutrosophic formula for performance measures with two and three parameters.

3.1 Neutrosophic Queue

A queueing system known as a neutrosophic queue uses neutrosophic numbers to represent queueing parameters like the average rate of consumers entering the queueing system (λ), and the average rate of customers being served (μ) are neutrosophic numbers [15,18].
In neutrosophic queueing λ is denoted by $\lambda_N = [\lambda_L, \lambda_U]$ and μ is denoted by $\mu_N = [\mu_L, \mu_U]$. Then, theneutrosophic traffic intensity if we have 's' servers is denoted by

$$\rho_N = \frac{\lambda_N}{\mu_N} = \frac{[\lambda_L, \lambda_U]}{[\mu_L, \mu_U]} = \left[\frac{\lambda_L}{\mu_U}, \frac{\lambda_U}{\mu_L}\right]$$

3.2 Arithmetic Operations of Interval Values

Let $[x_1, y_1], [x_2, y_2]$ be two Intervals where $x_1, x_2, y_1, y_2 \in \mathbb{R}$ and for practical cases set $x_1 > 0, x_2 > 0, y_1 > 0, y_2 > 0$ then:

$[x_1, y_1] + [x_2, y_2] = [x_1 + x_2, y_1 + y_2]$

$[x_1, y_1] - [x_2, y_2] = [x_1 - x_2, y_1 - y_2]$

$[x_1, y_1] * [x_2, y_2] = [x_1 x_2, y_1 y_2]$

$[x_1, y_1] \div [x_2, y_2] = [x_1, x_2]*\left[\frac{1}{y2}, \frac{1}{y1}\right] = \left[\frac{x1}{y2}, \frac{x2}{y1}\right]$

3.3 $(NM^{[b]}/NM/1)$: $(FCFS/\infty/\infty)$ Neutrosophic arrival of batch size 'b'

The neutrosophic chance that an arriving customer will locate a batch of two customers in a batch queue or a batch of three customers in bulk arrival is calculated after substituting crisp parameters with neutrosophic parameters.

3.3.1 $(NM^{[2]}/NM/1)$: $(FCFS/\infty/\infty)$

1) Neutrosophic expected number of customers in queue with two parameters:

$$NL_q = \left[\frac{\rho_1 + 3\rho_2}{1 - \rho_1 - 2\rho_2}\right]$$

$$NL_q = \left[\frac{\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}, \frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right] + 3\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}, \frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]}{1 - \left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}, \frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right] - 2\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}, \frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]}\right]$$

$$NL_q = \left[\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}, \quad \frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right]$$

2) Neutrosophic expected waiting time in queue with two parameters:

$$NW_q = \left[\frac{1}{\lambda_N}\left(\frac{\rho_1 + 3\rho_2}{1 - \rho_1 - 2\rho_2}\right)\right]$$

$$NW_q = \left[\frac{1}{\lambda_N}\left(\frac{\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}, \frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right] + 3\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}, \frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]}{1 - \left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}, \frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right] - 2\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}, \frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]}\right)\right]$$

$$NW_q = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}\right), \quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right)\right]$$

3) Neutrosophic expected waiting time in system with two parameters:

$$NW_s = \left[\frac{1}{\lambda_N}\left(\frac{\rho_1 + 3\rho_2}{1 - \rho_1 - 2\rho_2}\right) + \frac{1}{\mu_N}\right]$$

$$NW_s = \left[\frac{1}{\lambda_N}\left(\frac{\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}, \frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right] + 3\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}, \frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]}{1 - \left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}, \frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right] - 2\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}, \frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]}\right) + \frac{1}{\mu_N}\right]$$

$$NW_s = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}\right) + \frac{1}{\mu_U}, \quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right) + \frac{1}{\mu_L}\right]$$

3.3.2 $(NM^{[3]}/NM/1)$: $(FCFS/\infty/\infty)$

4) Neutrosophic expected number of customers in queue with three parameters:

$$NL_q = \left[\frac{\rho_1 + 3\rho_2 + 6\rho_3}{1 - \rho_1 - 2\rho_2 - 3\rho_3}\right]$$

$$NL_q = \left[\frac{\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}},\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right]+3\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}},\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]+6\left[\frac{\lambda_{(3,L)}}{\mu_{(3,U)}},\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}\right]}{1-\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}},\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right]-2\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}},\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]-3\left[\frac{\lambda_{(3,L)}}{\mu_{(3,U)}},\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}\right]}\right]$$

$$NL_q = \left[\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}+3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}+6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1-\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}-2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}-3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}, \quad \frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}+3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}+6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1-\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}-2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}-3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right]$$

5) Neutrosophic expected waiting time in queue with three parameters:

$$NW_q = \left[\frac{1}{\lambda_N}\left(\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right)\right]$$

$$NW_q = \left[\frac{1}{\lambda_N}\left(\frac{\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}},\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right]+3\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}},\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]+6\left[\frac{\lambda_{(3,L)}}{\mu_{(3,U)}},\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}\right]}{1-\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}},\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right]-2\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}},\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]-3\left[\frac{\lambda_{(3,L)}}{\mu_{(3,U)}},\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}\right]}\right)\right]$$

$$NW_q = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}+3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}+6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1-\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}-2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}-3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}\right), \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}+3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}+6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1-\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}-2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}-3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right)\right],$$

6) Neutrosophic expected waiting time in system with three parameters:

$$NW_s = \left[\frac{1}{\lambda_N}\left(\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right)+\frac{1}{\mu_N}\right]$$

$$NW_s = \left[\frac{1}{\lambda_N}\left(\frac{\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}},\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right]+3\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}},\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]+6\left[\frac{\lambda_{(3,L)}}{\mu_{(3,U)}},\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}\right]}{1-\left[\frac{\lambda_{(1,L)}}{\mu_{(1,U)}},\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}\right]-2\left[\frac{\lambda_{(2,L)}}{\mu_{(2,U)}},\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}\right]-3\left[\frac{\lambda_{(3,L)}}{\mu_{(3,U)}},\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}\right]}\right)+\frac{1}{\mu_N}\right]$$

$$NW_s = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}+3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}+6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1-\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}-2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}-3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}\right)+\frac{1}{\mu_U}, \quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}+3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}+6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1-\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}-2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}-3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right)+\frac{1}{\mu_L}\right]$$

7) Average Utilization $\rho_N = \frac{\lambda_N}{\mu_N} = \frac{[\lambda_L,\lambda_U]}{[\mu_L,\mu_U]} = \left[\frac{\lambda_L}{\mu_U},\frac{\lambda_U}{\mu_L}\right]$

4. NUMERICAL EXAMPLE

4.1 (NM[2]/NM/1): (FCFS/∞/∞) batch of 2 arrival neutrosophic values with uniform service rate

Let $\lambda_1 = [9,11], \lambda_2 = [11,13]$ and $\mu = [49,51]$

Then $\lambda = \lambda_1 + \lambda_2 = [20,24]$ and here $\mu_1 = \mu_2 = \mu$

$\mu_{(1,L)} = \mu_{(2,L)} = \mu_L$ and $\mu_{(1,U)} = \mu_{(2,U)} = \mu_U$

By using 3.3.1,

1) Neutrosophic expected number of customers in queue with two parameters service rate is uniform

$$NL_q = \left[\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}+3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1-\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}-2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}, \quad \frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}+3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1-\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}-2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right]$$

$$= \left[\frac{\left(\frac{9}{51}\right)+3\left(\frac{11}{51}\right)}{1-\left(\frac{9}{51}\right)-2\left(\frac{11}{51}\right)}, \quad \frac{\left(\frac{11}{49}\right)+3\left(\frac{13}{49}\right)}{1-\left(\frac{11}{49}\right)-2\left(\frac{13}{49}\right)}\right] = [2.0997, \quad 4.1666]$$

2) Neutrosophic expected waiting time in queue with two parameters service rate is uniform

$$NW_q = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}+3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1-\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}-2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}\right), \quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}+3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1-\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}-2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right)\right]$$

$$= \left[\left(\frac{1}{24}\right)\left(\frac{\left(\frac{9}{51}\right)+3\left(\frac{11}{51}\right)}{1-\left(\frac{9}{51}\right)-2\left(\frac{11}{51}\right)}\right), \quad \left(\frac{1}{20}\right)\left(\frac{\left(\frac{11}{49}\right)+3\left(\frac{13}{49}\right)}{1-\left(\frac{11}{49}\right)-2\left(\frac{13}{49}\right)}\right)\right] = [0.0875, 0.2083]$$

3) Neutrosophic expected waiting time in system with two parameters service rate is uniform

$$NW_s = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}\right) + \frac{1}{\mu_U}, \quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right) + \frac{1}{\mu_L}\right]$$

$$NW_s = \left[\left(\frac{1}{24}\right)\left(\frac{\left(\frac{9}{51}\right) + 3\left(\frac{11}{51}\right)}{1 - \left(\frac{9}{51}\right) - 2\left(\frac{11}{51}\right)}\right) + \left(\frac{1}{51}\right), \ \left(\frac{1}{20}\right)\left(\frac{\left(\frac{11}{49}\right) + 3\left(\frac{13}{49}\right)}{1 - \left(\frac{11}{49}\right) - 2\left(\frac{13}{49}\right)}\right) + \left(\frac{1}{49}\right)\right]$$

$NW_s = [0.1071, 0.2287]$

Average Utilization $\rho_N = \frac{\lambda_N}{\mu_N} = \frac{[20,24]}{[49,50]} = [0.3922, 0.4898]$

4.2 Crisp queue values with two parameters with uniform service rate

Let $\lambda_1 = 10, \lambda_2 = 12$ and μ=50

Then $\lambda = \lambda_1 + \lambda_2 = 22$

By Using 2.1.2, $E(X) = \frac{(\lambda_1 + 2\lambda_2)}{\lambda} = \frac{10+2(12)}{22} = 1.5455$

By using 2.1.3, $L_q = \left[\frac{\rho_1 + 3\rho_2}{1-\rho_1-2\rho_2}\right], \rho_1 = \frac{\lambda_1}{\mu} = \frac{10}{50} = 0.2$, $\rho_2 = \frac{\lambda_2}{\mu} = \frac{12}{50} = 0.24$

$$L_q = \left[\frac{0.2 + 3(0.24)}{1 - 0.2 - 2(0.24)}\right] = 2.875$$

By Using 2.1.4, $W_q = \frac{L_q}{\lambda} = \frac{2.875}{22} = 0.1306$

By Using 2.1.5, $W_s = W_q + \frac{1}{\mu} = 0.1306 + \frac{1}{50} = 0.1506$

Average Utilization ρ= $\frac{\lambda}{\mu} = \frac{22}{50} = [0.44]$

4.3 (NM[3]/NM/1): (FCFS/∞/∞) batch of 3 arrival neutrosophic values with uniform service rate

Let $\lambda_1 = [9,11], \lambda_2 = [11,13], \lambda_3 = [14,16]$ and $\mu = [99,101]$

Then $\lambda = \lambda_1 + \lambda_2 + \lambda_3 = [34, 40], \mu_1 = \mu_2 = \mu_3 = \mu,$

$\mu_{(1,L)} = \mu_{(2,L)} = \mu_{(3,L)} = \mu_L$ and $\mu_{(1,U)} = \mu_{(2,U)} = \mu_{(3,U)} = \mu_U$

By using 3.3.2,

4) Neutrosophic expected number of customers in queue with three parameters service rate is uniform

$$NL_q = \left[\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} + 6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} - 3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}, \quad \frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} + 6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} - 3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right]$$

$$NL_q = \left[\frac{\left(\frac{9}{101}\right) + 3\left(\frac{11}{101}\right) + 6\left(\frac{14}{101}\right)}{1 - \left(\frac{9}{101}\right) - 2\left(\frac{11}{101}\right) - 3\left(\frac{14}{101}\right)}, \quad \frac{\left(\frac{11}{99}\right) + 3\left(\frac{13}{99}\right) + 6\left(\frac{16}{99}\right)}{1 - \left(\frac{11}{99}\right) - 2\left(\frac{13}{99}\right) - 3\left(\frac{16}{99}\right)}\right]$$

$NL_q = [4.5004, 10.4293]$

5) Neutrosophic expected waiting time in queue with three parameters service rate is uniform

$$NW_q = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} + 6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} - 3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}\right), \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} + 6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} - 3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right)\right]$$

$$NW_q = \left[\left(\frac{1}{40}\right)\left(\frac{\left(\frac{9}{101}\right) + 3\left(\frac{11}{101}\right) + 6\left(\frac{14}{101}\right)}{1 - \left(\frac{9}{101}\right) - 2\left(\frac{11}{101}\right) - 3\left(\frac{14}{101}\right)}\right), \left(\frac{1}{34}\right)\left(\frac{\left(\frac{11}{99}\right) + 3\left(\frac{13}{99}\right) + 6\left(\frac{16}{99}\right)}{1 - \left(\frac{11}{99}\right) - 2\left(\frac{13}{99}\right) - 3\left(\frac{16}{99}\right)}\right)\right]$$

$NW_q = [0.1125, 0.3067]$

6) Neutrosophic expected waiting time in system with three parameters service rate is uniform

$$NW_s=\left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}+3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}+6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1-\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}-2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}-3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}\right)+\frac{1}{\mu_U},\quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}+3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}+6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1-\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}-2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}-3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right)+\frac{1}{\mu_L}\right]$$

$$NW_s=\left[\left(\frac{1}{40}\right)\left(\frac{\left(\frac{9}{101}\right)+3\left(\frac{11}{101}\right)+6\left(\frac{14}{101}\right)}{1-\left(\frac{9}{101}\right)-2\left(\frac{11}{101}\right)-3\left(\frac{14}{101}\right)}\right)+\left(\frac{1}{101}\right),\left(\frac{1}{34}\right)\left(\frac{\left(\frac{11}{99}\right)+3\left(\frac{13}{99}\right)+6\left(\frac{16}{99}\right)}{1-\left(\frac{11}{99}\right)-2\left(\frac{13}{99}\right)-3\left(\frac{16}{99}\right)}\right)+\left(\frac{1}{99}\right)\right]$$

$NW_s=[0.1224, 0.3168]$

Average Utilization $\rho_N=\frac{\lambda_N}{\mu_N}=\frac{[34,40]}{[99,101]}=[0.3366, 0.4040]$

4.4 Crisp queue values with 3 parameter and uniform service rate

Let $\lambda_1=10, \lambda_2=12, , \lambda_3=15$ and μ=100

Then $\lambda=\lambda_1+\lambda_2+\lambda_3=37$, here $\mu_1=\mu_2=\mu_3=\mu$

By Using 2.2.1, $E(X)=\frac{(\lambda_1+2\lambda_2+3\lambda_3)}{\lambda}=\frac{10+2(12)+3(15)}{37}=2.14$

By using 2.1.3,

$L_q=\left[\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right], \rho_1=\frac{\lambda_1}{\mu}=\frac{10}{100}=0.1$, $\rho_2=\frac{\lambda_2}{\mu}=\frac{12}{100}=0.12, \rho_3=\frac{\lambda_3}{\mu}=\frac{15}{100}=0.15$

$$L_q=\left[\frac{0.1+3(0.12)+6(0.15)}{1-0.1-2(0.12)-3(0.15)}\right]=6.4762$$

By Using 2.1.4, $W_q=\frac{L_q}{\lambda}=\frac{6.4762}{37}=0.1750$

By Using 2.1.5, $W_s=W_q+\frac{1}{\mu}=0.1750+\frac{1}{100}=0.1850$

Average Utilization ρ= $\frac{\lambda}{\mu}=\frac{37}{100}=[0.37]$

5. In neutrosophic environment when service rate is not uniform

In this section, we take the (NM[2]/NM/1): (FCFS/∞/∞) and (NM[3]/NM/1): (FCFS/∞/∞) type case with arrival rate, service rate is in neutrosophic environment of single server with service rate is not uniform.

5.1(NM[2]/NM/1): (FCFS/∞/∞) two parameter value when service rate is not uniform

Let $\lambda_1=[9,11], \lambda_2=[11,13]$ and $\mu_1=[23,25], \mu_2=[53,55]$

Then $\lambda=\lambda_1+\lambda_2=[20,24]$ and here $\mu_1\neq\mu_2\neq\mu$ also $\mu=\mu_1+\mu_2=[76,80]$

By using 3.3.1,

7) Neutrosophic expected number of customers in queue with two parameters service rate is not uniform

$$NL_q=\left[\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}+3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1-\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}-2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}},\quad \frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}+3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1-\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}-2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right]$$

$$=\left[\frac{\left(\frac{9}{25}\right)+3\left(\frac{11}{55}\right)}{1-\left(\frac{9}{25}\right)-2\left(\frac{11}{55}\right)},\quad \frac{\left(\frac{11}{23}\right)+3\left(\frac{13}{53}\right)}{1-\left(\frac{11}{23}\right)-2\left(\frac{13}{53}\right)}\right]=[4, \quad 38.9135]$$

8) Neutrosophic expected waiting time in queue with two parameters service rate is not uniform

$$NW_q=\left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}+3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1-\frac{\lambda_{(1,L)}}{\mu_{(1,U)}}-2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}\right),\quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}+3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1-\frac{\lambda_{(1,U)}}{\mu_{(1,L)}}-2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right)\right]$$

$$=\left[\left(\frac{1}{24}\right)\left(\frac{\left(\frac{9}{25}\right)+3\left(\frac{11}{55}\right)}{1-\left(\frac{9}{25}\right)-2\left(\frac{11}{55}\right)}\right),\quad \left(\frac{1}{20}\right)\left(\frac{\left(\frac{11}{23}\right)+3\left(\frac{13}{53}\right)}{1-\left(\frac{11}{23}\right)-2\left(\frac{13}{53}\right)}\right)\right]=[0.1667, 1.9457]$$

9) Neutrosophic expected waiting time in system with two parameters service rate is not uniform

$$NW_s = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}}}\right) + \frac{1}{\mu_U}, \quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}}}\right) + \frac{1}{\mu_L}\right]$$

$$NW_s = \left[\left(\frac{1}{24}\right)\left(\frac{\left(\frac{9}{25}\right) + 3\left(\frac{11}{55}\right)}{1 - \left(\frac{9}{25}\right) - 2\left(\frac{11}{55}\right)}\right) + \left(\frac{1}{80}\right), \ \left(\frac{1}{20}\right)\left(\frac{\left(\frac{11}{23}\right) + 3\left(\frac{13}{53}\right)}{1 - \left(\frac{11}{23}\right) - 2\left(\frac{13}{53}\right)}\right) + \left(\frac{1}{76}\right)\right]$$

$$NW_s = [0.1792, 1.9589]$$

Average Utilization $\rho_N = \frac{\lambda_N}{\mu_N} = \frac{[20,24]}{[76,80]} = [0.25, 0.3158]$

5.2 Crisp queue values two parameter value when service rate is not uniform

Let $\lambda_1 = 10, \lambda_2 = 12$ and $\mu_1 = 24, \mu_2 = 54$

Then $\lambda = \lambda_1 + \lambda_2 = 22, \mu = \mu_1 + \mu_2 = 78$

By Using 2.1.2, $E(X) = \frac{(\lambda_1 + 2\lambda_2)}{\lambda} = \frac{10+2(12)}{22} = 1.5455$

By using 2.1.3, $L_q = \left[\frac{\rho_1 + 3\rho_2}{1 - \rho_1 - 2\rho_2}\right], \rho_1 = \frac{\lambda_1}{\mu_1} = \frac{10}{24} = 0.4166$, $\rho_2 = \frac{\lambda_2}{\mu_2} = \frac{12}{54} = 0.2222$

$$L_q = \left[\frac{0.4166 + 3(0.2222)}{1 - 0.4166 - 2(0.2222)}\right] = 7.7999$$

By Using 2.1.4, $W_q = \frac{L_q}{\lambda} = \frac{7.7999}{22} = 0.3545$

By Using 2.1.5, $W_s = W_q + \frac{1}{\mu} = 0.3545 + \frac{1}{78} = 0.3673$

Average Utilization $\rho = \frac{\lambda}{\mu} = \frac{22}{78} = [0.282]$

5.3 (NM[3]/NM/1): (FCFS/∞/∞) three parameters value when service rate is not uniform

Let $\lambda_1 = [9,11], \lambda_2 = [11,13], \lambda_3 = [14,16]$ and

$$\mu_1 = [23,25], \mu_2 = [105,107], \mu_3 = [257,259],$$

Then $\lambda = \lambda_1 + \lambda_2 + \lambda_3 = [34, 40]$ and $\mu = \mu_1 + \mu_2 + \mu_3 = [385,391]$

By using 3.3.2

10) Neutrosophic expected number of customers in queue with two parameters service rate is not uniform

$$NL_q = \left[\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} + 6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} - 3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}, \ \frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} + 6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} - 3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right]$$

$$NL_q = \left[\frac{\left(\frac{9}{25}\right) + 3\left(\frac{11}{107}\right) + 6\left(\frac{14}{259}\right)}{1 - \left(\frac{9}{25}\right) - 2\left(\frac{11}{107}\right) - 3\left(\frac{14}{259}\right)}, \ \frac{\left(\frac{11}{23}\right) + 3\left(\frac{13}{105}\right) + 6\left(\frac{16}{257}\right)}{1 - \left(\frac{11}{23}\right) - 2\left(\frac{13}{105}\right) - 3\left(\frac{16}{257}\right)}\right]$$

$$NL_q = [3.6469, 14.0115]$$

11) Neutrosophic expected waiting time in queue with two parameters service rate is not uniform

$$NW_q = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} + 6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} - 3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}\right), \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} + 6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} - 3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right)\right]$$

$$NW_q = \left[\left(\frac{1}{40}\right)\left(\frac{\left(\frac{9}{25}\right) + 3\left(\frac{11}{107}\right) + 6\left(\frac{14}{259}\right)}{1 - \left(\frac{9}{25}\right) - 2\left(\frac{11}{107}\right) - 3\left(\frac{14}{259}\right)}\right), \left(\frac{1}{34}\right)\left(\frac{\left(\frac{11}{23}\right) + 3\left(\frac{13}{105}\right) + 6\left(\frac{16}{257}\right)}{1 - \left(\frac{11}{23}\right) - 2\left(\frac{13}{105}\right) - 3\left(\frac{16}{257}\right)}\right)\right]$$

$$NW_q = [0.0912, 0.4121]$$

12) Neutrosophic expected waiting time in system with two parameters service rate is not uniform

$$NW_s = \left[\frac{1}{\lambda_U}\left(\frac{\frac{\lambda_{(1,L)}}{\mu_{(1,U)}} + 3\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} + 6\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}{1 - \frac{\lambda_{(1,L)}}{\mu_{(1,U)}} - 2\frac{\lambda_{(2,L)}}{\mu_{(2,U)}} - 3\frac{\lambda_{(3,L)}}{\mu_{(3,U)}}}\right) + \frac{1}{\mu_U}, \quad \frac{1}{\lambda_L}\left(\frac{\frac{\lambda_{(1,U)}}{\mu_{(1,L)}} + 3\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} + 6\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}{1 - \frac{\lambda_{(1,U)}}{\mu_{(1,L)}} - 2\frac{\lambda_{(2,U)}}{\mu_{(2,L)}} - 3\frac{\lambda_{(3,U)}}{\mu_{(3,L)}}}\right) + \frac{1}{\mu_L}\right]$$

$$NW_s = \left[\left(\frac{1}{40}\right)\left(\frac{\left(\frac{9}{25}\right)+3\left(\frac{11}{107}\right)+6\left(\frac{14}{259}\right)}{1-\left(\frac{9}{25}\right)-2\left(\frac{11}{107}\right)-3\left(\frac{14}{259}\right)}\right)+\left(\frac{1}{391}\right),\left(\frac{1}{34}\right)\left(\frac{\left(\frac{11}{23}\right)+3\left(\frac{13}{105}\right)+6\left(\frac{16}{257}\right)}{1-\left(\frac{11}{23}\right)-2\left(\frac{13}{105}\right)-3\left(\frac{16}{257}\right)}\right)+\left(\frac{1}{385}\right)\right]$$

$NW_s = [0.0938, 0.4147]$

Average Utilization $\rho_N = \frac{\lambda_N}{\mu_N} = \frac{[34,40]}{[385,391]} = [0.0870, 0.1039]$

5.4 Crisp queue values three parameter value when service rate is not uniform

Let $\lambda_1 = 10, \lambda_2 = 12,\ , \lambda_3 = 15$ and $\mu_1 = 24, \mu_2 = 106, \mu_3 = 258$

Then $\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 37$, here $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu$. Also $\mu = \mu_1 + \mu_2 + \mu_3 = 388$

By Using 2.2.1, $E(X) = \frac{(\lambda_1+2\lambda_2+3\lambda_3)}{\lambda} = \frac{10+2(12)+3(15)}{37} = 2.14$

By using 2.1.3,

$$L_q = \left[\frac{\rho_1 + 3\rho_2 + 6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right],$$

$$\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{10}{24} = 0.4167 \quad , \qquad \rho_2 = \frac{\lambda_2}{\mu_2} = \frac{12}{106} = 0.1132, \rho_3 = \frac{\lambda_3}{\mu_3} = \frac{15}{258} = 0.0581$$

$$L_q = \left[\frac{0.4167 + 3(0.1132) + 6(0.0581)}{1 - 0.4167 - 2(0.1132) - 3(0.0581)}\right] = 6.0509$$

By Using 2.1.4, $W_q = \frac{L_q}{\lambda} = \frac{6.0509}{37} = 0.1635$

By Using 2.1.5, $W_s = W_q + \frac{1}{\mu} = 0.1635 + \frac{1}{388} = 0.1660$

Average Utilization $\rho = \frac{\lambda}{\mu} = \frac{37}{388} = [0.0954]$

6 Discussions and Conclusions

The efficiency of the system was found to be between [0.3922,0.4898] and [0.3366,0.4040] for the two parameters and three parameters bulk queues, respectively. Additionally, the efficiency of the crisp values [0.44] ∈[0.3922,0.4898] in the 2 parameters queue model and [0.37]∈ [0.3366,0.4040] in the three parameters queue model. The property is also satisfied by the average number of people in the queue, the average length of time in the queue, and the average waiting time throughout the system. Thus, when dealing with uncertain data, neutrosophic models produce better results than crisp values.

Table:1. Uniform Service Rate for Two parameters and Three parameters

Two parameters with uniform service rate		3 parameters with uniform service rate	
Crisp value	Neutrosophic	Crisp value	Neutrosophic
E(X)=1.5455		E(X)=2.14	
L_q=2.875	NL_q = [2.0997,4.166]	L_q=6.4762	NL_q = [4.504, 10.4297]
W_q=0.1306	NW_q = [0.0875,0.2083]	W_q=0.1750	NW_q = [0.1125,0.3067]
w_s=0.1506	NW_s = [0.1071,0.2287]	w_s=0.1850	NW_s = [0.1224,0.3168]
ρ=0.44	ρ_N=[0.3922, 0.4898]	ρ=0.37	ρ_N=[0.3366,0.404]

Table:2. Service Rate is not uniform for Two parameters and Three parameters

Two parameters with service rate is not uniform		3 parameters with service rate is not uniform	
Crisp value	Neutrosophic	Crisp value	Neutrosophic
E(X)=1.5455		E(X)=2.14	
L_q=7.7909	NL_q = [4,38.9135]	L_q=6.0553	NL_q = [3.6469,14.011]
W_q=0.3545	NW_q = [0.1667,1.9457]	W_q=0.1637	NW_q = [0.0912,0.4121]
w_s=0.3673	NW_s = [0.25,0.3158]	w_s=0.1662	NW_s = [0.0938,0.4147]
ρ=0.282	ρ_N= [0.25,0.3158]	ρ=0.0954	ρ_N=[0.0870,0.103]

In both the two parameter and three parameter models in NM[b]/NM/1 proposed model, we found that the expected value E(X) is the same for uniform service rate and non-uniform service rate. The crisp value of two parameter model shows a considerable difference in the average length of the queue (L_q) compared in uniform service rate and non-uniform service rate. However, the three parameters' crisp values of $L_q$varied less. The slightly larger interval is provided by the neutrosophic model NL_q. In three parameter proposed model despite the lengthy queue, there is surprisingly little wait time. As a result, the interval for waiting time in length and system is substantially shorter in a neutrosophic environment. The model's utilization factor ρ and ρ_N is quite low, especially when it comes to the three parameter problem ρ_N=[0.0870,0.103].

We concluded that, in a neutrosophic enviroment, we presented the results of the two parameter bulk queues (NM[2]/NM/1): (FCFS/∞/∞) and the three parameter bulk queues (NM[3]/NM/1): (FCFS/∞/∞) while dealing with imprecise data during bulk arrival, the neutrosophic queueing theory provides better interval values compared to crisp values.

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