Educational Administration: Theory and Practice
2024, 30(4), 174-180
ISSN:2148-2403
https://kuey.net/
Research Article

Bi-Di Fuzzy Lattice KS-Operator Group

M.U. Makandar[1*], A.B. Jadhav[2]

[1*]Assistant Professor, KIT's IMER. Kolhapur. Email:mujirmakandar33@gmail.com
[2]Associate Professor, D.S.M College, Parbhani, Email:arunbjadhao@gmail.com

Citation: M.U. Makandar, A.B. Jadhav, (2024), Bi-Di Fuzzy Lattice KS-Operator Group, Educational Administration: Theory and Practice, 30(4), 174-180
Doi: 10.53555/kuey.v30i5.2818

ABSTRACT

We know a fuzzy set is a function which associates every member of a set with a grade. With the help of fuzzy set one can measure the uncertainty and the belongingness of an element with the set. Bidirectional fuzzy set in short Bi-di fuzzy set is an extension of fuzzy set which defines grade in two directions, one function in this gives membership and other function gives non membership grade to every element of the set. Here we defined Bi-di fuzzy lattice group with operators.

Keywords- Lattice group, fuzzy lattice group, K operator group, fuzzy lattice k^2 operator group, fuzzy lattice KS operator group. Bi-di fuzzy lattice operator group.

1. Introduction

A revolutionary thing happened in the history of mathematics by introduction of Fuzzy set theory by Zadeh [11]. Rosenfield[7] then utilized this theory in algebra and introduced fuzzy groups..In 1994 Ajmal and Thomas [1] combined this fuzzy algebra with the algebraic structure lattice and extended the concept to next level. Nanda [6] used partial ordering in fuzzy set and defined fuzzy lattice. Satya Saibaba[8] used ordering of lattice relation and defined fuzzy lattice ordered group. Goguen [2] generalized the concept of fuzzy lattice by replacing image set of fuzzy set [0,1] by a complete lattice. Solairau and Nagrajan [9] defined fuzzy Q-modules. Murdai and Rajendran [5] defined a new form of fuzzy lattice. Gu[3] done the fuzzy algebra work on operator groups. Subramaniam , Nagrajan & Chellapa [10] brought the research to the next level by giving m fuzzy group concept.Lokhande and Makandar [4] defined the concept of one operator fuzzy algebraic structures. In this paper we introduced the two operator Bi di fuzzy lattice group.

2. PRELIMINARIES

Definition 2.1 Bi-di Fuzzy group

Let $\alpha: X \to [0, 1]$ and $\beta : X \to [0, 1]$ ($0 \le \alpha(x) + \beta(x) \le 1$) are two fuzzy sets & (G,.) is a group which is a subset of X. A Bi-di fuzzy group is a fuzzy set < x, α(x), ß(x)> which satisfy four conditions

1) $\alpha (x y) \ge \text{mini} \{\alpha (x), \alpha(y)\}$
2) $\alpha (x^{-1}) \ge \alpha (x)$
3) $\beta (x y) \le \text{maxi} \{\beta (x), \beta(y)\}$
4) $\beta (x^{-1}) \le \beta (x)$, where $x, y \in G$

Definition 2.2 K-Operator group

A group G is said to be an K- operator group if $kx \in G$ where $k \in K$ (any non empty set called as Operator set) and for all $x \in G$.

Definition 2.3 Bi-di Fuzzy K- operator group

Let $\alpha: X \to [0, 1]$ and $\beta : X \to [0, 1]$ are two fuzzy sets & G is a subset of X which is also a K- operator group. G is a Bi fuzzy K-operator group if it satisfies following four conditions

1) $\alpha (k x k y) \ge \text{mini} \{\alpha (kx), \alpha(ky)\}$
2) $\alpha (kx)^{-1} \ge \alpha (kx)$
3) $\beta (kx\, ky) \le \text{maxi} \{\beta (kx), \beta(ky)\}$
4) $\beta (k x)^{-1} \le \beta (k x)$, where $x, y \in G$, $k \in K$.

Definition 2. Lattice K-operator group

Lattice K-operator group is an algebraic structure (G, . , R) if it satisfy two conditions 1) G is a K-operator group w.r.t '.' 2) G is a lattice w.r.t R

Definition 2.5 KS- operator group-

Let G be a group, K,S be any two nonempty sets if kx ϵ G , sxϵ G. for every x ϵ G, k ϵ K, sϵ S Then G is called a KS- operator group.

Definition 2.6 Bi-di Fuzzy KS- operator group

If α: X → [0, 1] and ß : X → [0, 1] are two fuzzy sets & G is KS- operator group . G subset of X. Then the fuzzy set < x, α(x), ß(x)> is a Bi-di fuzzy KS operator group if

1) $\alpha(kxsy)) \geq$ mini $\{\alpha(kx), \alpha(sy)\}$ 2) $\alpha(kx)^{-1} \geq \alpha(kx)$ & $\alpha(sx)^{-1} \geq \alpha(sx)$

3) $ß(kxsy)) \leq$ maxi $\{ß(kx), ß(sy)\}$ 2) $ß(kx)^{-1} \leq ß(kx)$ & $ß(sx)^{-1} \leq ß(sx)$

for every x, yϵ G, k ϵ K ,s ϵ S

Definition 2.7 Lattice KS operator group

A lattice KS- operator group is an algebraic structure (G, R , .) if it satisfy two conditions 1) G is a KS- operator group w.r.t '.' 2) G is a lattice w.r.t R .

Definition 2.8. Bi-di Fuzzy lattice KS- operator group (BDFL KS- operator group) –

α: X to [0, 1] and β: X to [0, 1] are two fuzzy sets, Let G be a subset of X which is a lattice KS- operator group , K,S(operator sets). Then the fuzzy set < x, α(x), ß(x)> is a Bi-di fuzzy lattice KS- operator group if it satisfy following conditions

1) $\alpha(kxsy) \geq$ mini $\{\alpha(kx), \alpha(sy)\}$
2) $\alpha(kx)^{-1} \geq \alpha(kx)$ & $\alpha(sx)^{-1} \geq \alpha(sx)$
3) $\alpha(kx \vee sy) \geq$ mini$\{\alpha(kx),\alpha(sy)\}$
4) $\alpha(kx \wedge sy) \geq$ mini$\{\alpha(kx),\alpha(sy)\}$
5) $\beta(kxsy) \leq$ maxi $\{\beta(kx),\beta(sy)\}$
6) $\beta(kx)^{-1} \leq \beta(kx)$ & $\beta(sx)^{-1} \leq \beta(sx)$
7) $\beta(kx \vee sy) \leq$ maxi$\{\beta(kx),\beta(sy)\}$
8) $\beta(kx \wedge sy) \leq$ maxi$\{\beta(kx),\beta(sy)\}$ For every x ϵ G, k ϵ K, s ϵ S

Definition 2.9 Bi-di Fuzzy lattice KK -operator group

α: X to [0, 1] and β: X to [0, 1] are two fuzzy sets, G is a K- lattice operator group. Then the fuzzy set < x, α(x), ß(x)> is said to be a Bi-di fuzzy lattice KK-operator group if it satisfy following conditions

1) $\alpha(k_1x\, k_2y) \geq$ mini$\{\alpha(k_1 x),\alpha(k_2 y)\}$
2) $\alpha(k_1 x)^{-1} \geq \alpha(k_1x)$, $\lambda(k_2 x)^{-1} \geq \alpha(k_2x)$,
3) $\alpha(k_1x \vee k_2y) \geq$ mini$\{\alpha(k_1 x),\alpha(k_2 y)\}$
4) $\alpha(k_1x \wedge k_2y) \geq$ mini$\{\alpha(k_1 x),\alpha(k_2 y)\}$,
5) $ß(k_1x\, k_2y) \leq$ maxi$\{ß(k_1 x),ß(k_2 y)\}$
6) $ß(k_1 x)^{-1} \leq ß(k_1x)$, $ß(k_2 x)^{-1} \leq ß(k_2x)$,
7) $ß(k_1x \vee k_2y) \leq$ maxi$\{ß(k_1 x),ß(k_2 y)\}$
8) $ß(k_1x \wedge k_2y) \leq$ maxi $\{ß(k_1 x),ß(k_2 y)\}$, For all x, yϵ G , k_1, k_2 ϵK

Definition 2.10 Bi-di Fuzzy lattice K^2-operator group

α: X to [0, 1] and β: X to [0, 1] are two fuzzy sets, Let G be a subset of X which is a lattice K^2- operator group , K(operator set). Then the fuzzy set < x, α(x), ß(x)> is a Bi-di fuzzy lattice K^2- operator group if it satisfy following conditions

1) $\alpha(kxky) \geq$ mini $\{\alpha(kx),\alpha(ky)\}$
2) $\alpha(kx)^{-1} \geq \alpha(kx)$
3) $\alpha(kx \vee ky) \geq$ mini$\{\alpha(kx),\alpha(ky)\}$
4) $\alpha(kx \wedge ky) \geq$ mini$\{\alpha(kx),\alpha(ky)\}$
5) $\beta(kxky) \leq$ maxi $\{\beta(kx),\beta(ky)\}$
6) $\beta(kx)^{-1} \leq \beta(kx)$
7) $\beta(kx \vee ky) \leq$ maxi$\{\beta(kx),\beta(ky)\}$
8) $\beta(kx \wedge ky) \leq$ maxi$\{\beta(kx),\beta(ky)\}$ For every x ϵ G, k ϵ K

3 PROPERTIES OF BDFL KS- OPERATOR GROUP

Preposition 3.1: Let T and T' be two Lattice KS-operator groups and α: T→T' and ß: T→T' are two lattice KS homomorphisms. If P'is a BDFL KS operator group of T' then the pre-image P'^{-1} is a BDFL KS operator group of T.

Proof- Assume P' is a BDFL KS- operator group of T'. Let x, y ϵ T

i) $\alpha^{-1}(P') = \mu_{\alpha^{-1}(P')}(kxsy))$
$= \mu_{P'}\alpha(kxsy))$
$= \mu_{P'}(k\alpha(x)s\alpha(y))$
$\geq \text{mini}\{\mu_{P'}(k\alpha(x)),\mu_{P'}(s\alpha(y))\}$
$\geq \text{mini}\{\mu_{P'}(\alpha(kx)),\mu_{P'}(\alpha(sy))\}$
$= \text{mini}\{\mu_{\alpha^{-1}(P')}(kx), \mu_{\alpha^{-1}(P')}(sy)\}$
ii) $\mu_{\alpha^{-1}(P')}(kx)^{-1} = \mu_{P'}\alpha(kx)^{-1}$
$=\mu_{P'}[\alpha(kx)]^{-1}$

$=\mu_{P'}[k\alpha(x)]^{-1}$
$\geq \mu_{P'}k\alpha(x)$
$=\mu_{P'}(\alpha(kx))$
$=\mu_{\alpha^{-1}(P')}(kx)$
$\mu_{\alpha^{-1}(P')}(sx)^{-1} = \mu_{P'}\alpha(sx)^{-1}$
$=\mu_{P'}[\alpha(sx)]^{-1}$
$=\mu_{P'}[s\lambda(x)]^{-1}$
$\geq \mu_{P'}s\alpha(x)$
$=\mu_{P'}(\alpha(sx))$
$=\mu_{\alpha^{-1}(P')}(sx)$
iii) $\mu_{\alpha^{-1}(P')}(kx \vee sy) = \mu_{P'}\alpha(kx \vee sy)$
$=\mu_{P'}\alpha(kx)\vee\alpha(sy)$
$\geq \text{mini}\{\mu_{P'}\alpha(kx),\mu_{P'}\alpha(sy))$
$\geq \text{mini}\{\mu_{\alpha^{-1}(P')}(kx),\mu_{\alpha^{-1}(P')}(sy)\}$
iv) $\mu_{\alpha^{-1}(P')}(kx \wedge sy) = \mu_{P'}\alpha(kx \wedge sy)$
$=\mu_{P'}\alpha(kx)\wedge\alpha(sy)$
$\geq \text{mini}\{\mu_{P'}\alpha(kx),\mu_{P'}\alpha(sy))$
$\geq \text{mini}\{\mu_{\alpha^{-1}(P')}(kx),\mu_{\alpha^{-1}(P')}(sy)\}$
v) $\beta^{-1}(P') = \mu_{\beta^{-1}(P')}(kxsy))$
$= \mu_{P'},\beta(kxsy))$
$= \mu_{P'}(k\beta(x)s\beta(y))$
$\leq \text{maxi}\{\mu_{P'}(k\beta(x)),\mu_{P'}(s\beta(y))\}$
$\leq \text{maxi}\{\mu_{P'}(\beta(kx)),\mu_{P'}(\beta(sy))\}$
$= \text{maxi}\{\mu_{\beta^{-1}(P')}(kx), \mu_{\beta^{-1}(P')}(sy)\}$
vi) $\mu_{\beta^{-1}(P')}(kx)^{-1} = \mu_{P'}\beta(kx)^{-1}$
$=\mu_{P'}[\beta(kx)]^{-1}$
$=\mu_{P'}[k\beta(x)]^{-1}$
$\leq \mu_{P'}k\beta(x)$
$=\mu_{P'}(\beta(kx))$
$=\mu_{\beta^{-1}(P')}(kx)$
$\mu_{\beta^{-1}(P')}(sx)^{-1} = \mu_{P'}\beta(sx)^{-1}$
$=\mu_{P'}[\beta(sx)]^{-1}$
$=\mu_{P'}[s\beta(x)]^{-1}$
$\leq \mu_{P'}s\beta(x)$
$=\mu_{P'}(\beta(sx))$
$=\mu_{\beta^{-1}(P')}(sx)$
vii) $\mu_{\beta^{-1}(P')}(kx \vee sy) = \mu_{P'}\beta(kx \vee sy)$
$=\mu_{P'}\beta(kx)\vee\beta(sy)$
$\leq \text{maxi}\{\mu_{P'}\beta(kx),\mu_{P'}\beta(sy))$
$\leq \text{maxi}\{\mu_{\beta^{-1}(P')}(kx),\mu_{\beta^{-1}(P')}(sy)\}$
viii) $\mu_{\beta^{-1}(P')}(kx \wedge sy) = \mu_{P'}\beta(kx \wedge sy)$
$=\mu_{P'}\beta(kx)\wedge\beta(sy)$
$\leq \text{maxi}\{\mu_{P'}\beta(kx),\mu_{P'}\beta(sy))$
$\leq \text{maxi}\{\mu_{\beta^{-1}(P')}(kx),\mu_{\beta^{-1}(P')}(sy)\}$

Preposition 3.2: If T and T' are two Lattice KS operator groups and α:T→T' ,ß: T→T' are two lattice KS epimorphisms. P' is a fuzzy set in T'. If $(P')^{-1}$ is a BDFL KS operators group of T then P' is a BDFL KS-operator group of T'.

Proof- Consider x, y ϵ T', hence there are elements m, n , r , t ϵT so that $\alpha(m)= x$,
$\alpha(n) = y$. $\beta(r)= x$,$\beta(t) = y$.
i) $\mu_{P'}(kxsy) = \mu_{P'}(k\alpha(m)s\alpha(n))$

$=\mu_{P'}(\alpha(kmsn))$
$=\mu_{\alpha^{-1}(P')}(kmsn)$
$\geq mini\{\mu_{\alpha^{-1}(P')}(km)),\mu_{\alpha^{-1}}(P')(sn)\}$
$\geq mini\{\mu_{P'}\alpha(km),\mu_{P'}\alpha(sn)\}$
$\geq mini\{\mu_{P'}k\alpha(m),\mu_{P'}s\alpha(n)\}$
$\geq mini\{\mu_{P'}(kx),\mu_{P'}(sy)\}$
ii) $\mu_{P'}(kx)^{-1}=\mu_{P'}(k\alpha(m))^{-1}$
$=\mu_{P'}(\alpha(km)^{-1}$
$=\mu_{\alpha^{-1}(P')}(km)^{-1}$
$\geq\mu_{\alpha^{-1}(P')}(km)$
$=\mu_{P'}\alpha(km)$
$=\mu_{P'}k\alpha(m)$
$=\mu_{P'}(kx)$
$\mu_{P'}(sx)^{-1}=\mu_{P'}(s\alpha(m))^{-1}$
$=\mu_{P'}(\alpha(sm)^{-1}$
$=\mu_{\alpha^{-1}(P')}(sm)^{-1}$
$\geq\mu_{\alpha^{-1}(P')}(sm)$
$=\mu_{P'}\alpha(sm)$
$=\mu_{P'}s\ \alpha\ (m)$
$=\mu_{P'}(sx)$
iii) $\mu_{P'}(kx\vee sy)$ $=\mu_{P'}(k\alpha(m)\vee s\alpha(n))$
$=\mu_{P'}(\alpha(km)\vee\alpha(sn))$
$=\mu_{P'}(\alpha(km\vee sn))$
$=\mu_{\alpha^{-1}(P')}(km\vee sn)$
$\geq mini\{\mu_{\alpha^{-1}(P')}(km),\mu_{\alpha^{-1}(P')}(sn)\}$
$=mini\{\mu_{P'}\alpha(km),\mu_{P'}\alpha(sn)\}$
$=mini\{\mu_{P'}k\alpha(m),\mu_{P'}s\alpha(n)\}$
$=mini\{\mu_{P'}(kx),\mu_{P'}(sy)\}$
iv) $\mu_{P'}(kx\wedge sy)$ $=\mu_{P'}(k\alpha(m)\wedge s\alpha(n))$
$=\mu_{P'}(\alpha(km)\wedge\alpha(sn))$
$=\mu_{P'}(\alpha(km\wedge sn))$
$=\mu_{\alpha^{-1}(P')}(km\wedge sn)$
$\leq mini\{\mu_{\alpha^{-1}(P')}(km),\mu_{\alpha^{-1}}(P')(sn)\}$
$=mini\{\mu_{P'}\alpha(km),\mu_{P'}\alpha(sn)\}$
$=mini\{\mu_{P'}k\alpha(m),\mu_{P'}s\alpha(n))$
$=mini\{\mu_{P'}(kx),\mu_{P'}(sy)\}$
v) $\mu_{P'}(kxsy)$ $=\mu_{P'}(k\beta(r)s\beta(t))$
$=\mu_{P'}(\beta(krst))$
$=\mu_{\beta^{-1}(P')}(krst)$
$\leq maxi\{\mu_{\beta^{-1}(P')}(kr)),\mu_{\beta^{-1}}(P')(st)\}$
$\leq maxi\{\mu_{P'}\beta(kr),\mu_{P'}\beta(st)\}$
$\leq maxi\{\mu_{P'}k\beta(r),\mu_{P'}s\beta(t)\}$
$\leq maxi\{\mu_{P'}(kx),\mu_{P'}(sy)\}$
vi) $\mu_{P'}(kx)^{-1}=\mu_{P'}(k\beta(r))^{-1}$
$=\mu_{P'}(\beta(kr)^{-1}$
$=\mu_{\beta^{-1}(P')}(kr)^{-1}$
$\leq\mu_{\beta^{-1}(P')}(kr)$
$=\mu_{P'}\beta(kr)$
$=\mu_{P'}k\beta(r)$
$=\mu_{P'}(kx)$
$\mu_{P'}(sx)^{-1}=\mu_{P'}(s\beta(r))^{-1}$
$=\mu_{P'}(\beta(sr)^{-1}$
$=\mu_{\beta^{-1}(P')}(sr)^{-1}$
$\leq\mu_{\beta^{-1}(P')}(sr)$
$=\mu_{P'}\beta(sr)$
$=\mu_{P'}s\beta(r)$
$=\mu_{P'}(sx)$
vii) $\mu_{P'}(kx\vee sy)$ $=\mu_{P'}(k\beta(r)\vee s\beta(t))$
$=\mu_{P'}(\beta(kr)\vee\beta(st))$
$=\mu_{P'}(\beta(kr\vee st))$
$=\mu_{\beta^{-1}(P')}(kr\vee st)$

$\leq$maxi{$\mu_{ß^{-1}(P')}$(kr),$\mu_{ß^{-1}(P')}$(st)}
=maxi{$\mu_{P'}$ß(kr),$\mu_{P'}$ß(st)}
=maxi{$\mu_{P'}$kß(r),$\mu_{P'}$sß(t)}
=maxi{$\mu_{P'}$(kx),$\mu_{P'}(sy)$}
viii) $\mu_{P'}(kx \wedge sy)$ =$\mu_{P'}$(kß(r)∧sß(t))
=$\mu_{P'}$(ß(kr)∧ß(st))
=$\mu_{P'}$(ß(kr∧st))
=$\mu_{ß^{-1}(P')}$(kr∧st)
$\leq$maxi{$\mu_{ß^{-1}(P')}$(kr),$\mu_{ß^{-1}}$(P')(st)}
=maxi{$\mu_{P'}$ß(kr),$\mu_{P'}$ß(st)}
=maxi{$\mu_{P'}$kß(r),$\mu_{P'}$sß(t))
=maxi{$\mu_{P'}(kx)$,$\mu_{P'}(sy)$}
Therefore P' is a BDFL KS operator group of T'.

Preposition 3.3: If {A_i} is a family of BDFL KS operator group of T then ∩A_i is a BDFL KS operator group of T where ∩A_i = { x , ∧ α_{Ai}(x) , ∨ $ß_{Ai}$(x) / x ϵ T }

Proof- Consider x, y ϵ T
i) $(\cap_{Ai})(kxsy)$ = ∧α_{Ai}(kxsy)
≥∧ mini{$\alpha_{Ai}(kx)$,$\alpha_{Ai}(sy)$}
=mini{$(\cap A_i)(kx)$,$(\cap A_i)(sy)$}
ii) $(\cap_{Ai})(kx)^{-1}$ =∧$\alpha_{Ai}(kx)^{-1}$
≥∧$\alpha_{Ai}(kx)$
=$(\cap Ai)(kx)$
$(\cap_{Ai})(sx)^{-1}$ =∧$\alpha_{Ai}(sx)^{-1}$
≥∧$\alpha_{Ai}(sx)$
= $(\cap Ai)(sx)$
iii) $(\cap_{Ai})(kx \vee sy)$ =∧α_{Ai}(kx∨sy)
≥∧ mini{α_{Ai} (kx),$\alpha_{Ai}(sy)$}
=mini{$(\cap_{Ai})(kx)$,$(\cap_{Ai})(sy)$}
iv) $(\cap_{Ai})(kx \wedge sy)$ =∧$\alpha_{Ai}(kx \wedge sy)$
≥∧ mini{$\alpha_{Ai}(kx)$,$\alpha_{Ai}(sy)$}
≥mini{$(\cap_{Ai})(kx)$,$(\cap_{Ai})(sy)$}
v) $(\cap_{Ai})(kxsy)$ = ∨$ß_{Ai}$(kxsy)
≤∨ maxi{$ß_{Ai}(kx)$,$ß_{Ai}(sy)$}
= maxi{∨$ß_{Ai}(kx)$, ∨$ß_{Ai}(sy)$}

=maxi{$(\cap A_i)(kx)$,$(\cap A_i)(sy)$}
vi) $(\cap_{Ai})(kx)^{-1}$ =∨$ß_{Ai}(kx)^{-1}$
≤∨ $ß_{Ai}(kx)$
=$(\cap Ai)(kx)$
$(\cap_{Ai})(sx)^{-1}$ =∨ $ß_{Ai}(sx)^{-1}$
≤∨ $ß_{Ai}(sx)$
= $(\cap Ai)(sx)$
vii) $(\cap_{Ai})(kx \vee sy)$ =∨$ß_{Ai}$(kx∨sy)
≤∨ maxi{$ß_{Ai}$ (kx),$ß_{Ai}(sy)$}
≤maxi{ ∨$ß_{Ai}$ (kx), ∨$ß_{Ai}(sy)$}
=maxi{$(\cap_{Ai})(kx)$,$(\cap_{Ai})(sy)$}
viii) $(\cap_{Ai})(kx \wedge sy)$ =∨$ß_{Ai}(kx \wedge sy)$
≤∨ maxi{$ß_{Ai}(kx)$,$ß_{Ai}(sy)$}
≤ maxi{∨$ß_{Ai}(kx)$, ∨$ß_{Ai}(sy)$}
≤ maxi{$(\cap_{Ai})(kx)$,$(\cap_{Ai})(sy)$}
Therefore ∩A_i is a BDFL KS operator group of T

Preposition 3.4: P is a BDFL KS operator group of T. If Q is a bi-di fuzzy set in T given by Q(x)=< α(x)−α(e)+1, ß(e)−ß(x)+1> for every x ϵT. Then Q is a BDFL KS operator group of T .

Proof- Consider x, y ϵ T
i) Q$(kxsy)$ =α(kxsy)+1-α(e)
≥mini({α(kx),α(sy)}+1−α(e))
≥mini(α(kx)+1-α(e),α(sy)+1−α(e))

$\geq$mini{Q(kx),Q(sy)}

ii) $Q((kx)^{-1}) = \alpha((kx)^{-1})+1-\alpha(e)$

$\geq \alpha(kx)+1-\alpha(e)$

$\geq Q(kx)$

$Q((sx)^{-1}) = \alpha(sx)^{-1}+1-\alpha(e)$

$\geq \alpha(sx)+1-\alpha(e)$

$\geq Q(sx)$

iii) $Q(kx \vee sy) = \alpha(kx \vee sy)+1-\alpha(e)$

$\geq mini\{\alpha(kx),\alpha(sy)\}+1-\alpha(e)$

$\geq mini\{\alpha(kx)+1-\alpha(e),\alpha(sy)+1-\alpha(e)\}$

$\geq mini\{Q(kx),Q(sy)\}$

iv) $Q(kx \wedge sy) = \alpha(kx \wedge sy)+1-\alpha(e)$

$\geq mini\{\alpha(kx),\alpha(sy)\}+1-\alpha(e)$

$\geq mini\{\alpha(kx)+1-\alpha(e),\alpha(sy)+1-\alpha(e)\}$

$\geq mini\{Q(kx),Q(sy)\}$

v) $Q(kxsy) = \beta(e)-\beta(kxsy)+1$

$\leq \beta(e)- maxi(\{\beta(kx),\beta(sy)\}+1$

$\leq maxi\{\beta(e)-\beta(kx)+1,\beta(e)-\beta(sy)+1\}$

$=maxi\{Q(kx),Q(sy)\}$

vi) $Q(kx)^{-1} = \beta(e)-\beta((kx)^{-1})+1$

$\leq \beta(e)-\beta((kx)+1$

$=Q(kx)$

$Q(sx)^{-1} = \beta(e)-\beta(sx)^{-1}+1$

$\leq \beta(e)-\beta(sx)+1$

$\leq Q(sx)$

vii) $Q(kx \vee sy) = \beta(e)-\beta(kx \vee sy)+1$

$\leq \beta(e)- maxi\{\beta(kx),\beta(sy)\}+1$

$\leq maxi\{\beta(e)-\beta(kx)+1, \beta(e)-\beta(sy)+1\}$

$=maxi\{Q(kx),Q(sy)\}$

viii) $Q(kx \wedge sy) =-\beta(e)-\beta(kx \wedge sy)+1$

$\leq \beta(e)-maxi\{\beta(kx),\beta(sy)\}+1$

$\leq maxi\{\beta(e)-\beta(kx)+1, \beta(e)-\beta(sy)+1\}$

$=maxi\{Q(kx),Q(sy)\}$

Preposition 3.5: Direct product of Bi-di fuzzy lattice KS operator groups is also a Bi-di fuzzy lattice KS operator group.

Proof- Let $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n) \in G_1 \times G_2 \times \ldots \times G_n$

Let $A_1 \times A_2 \times \ldots \times A_n = A$

i) $\alpha_A(k\,x\,s\,y)) = \alpha_A(k(x_1, x_2, \ldots, x_n)\, s(y_1, y_2, \ldots, y_n))$

$= \alpha_A(k x_1 s y_1, k x_2 s y_2, \ldots, k x_n s y_n))$

$= mini\{\alpha A_1(k x_1 s y_1), \alpha A_2(k x_2 s y_2), \ldots, \alpha A_n(k x_n s y_n)\}$

$\geq mini\{mini[\alpha A_1(kx_1), \alpha A_1(sy_1)], min[\alpha A_2(kx_2), \alpha A_2(sy_2)], \ldots$

$mini[\alpha A_n (k x_n), \alpha A_n(s y_n)]\}$

$\geq mini\{mini[\alpha A_1(k x_1), \alpha A_2(k x_2), \ldots \alpha A_n(k x_n)],$

$mini[\alpha A_1(s y_1), \alpha A_2(s y_2), \ldots \alpha A_n(s y_n)]\}$

$\geq mini\{(\alpha A_1 \times \alpha A_2 \times \ldots \times \alpha A_n)\, k(x_1, x_2, \ldots, x_n),$

$(\alpha A_1 \times \alpha A_2 \times \ldots \times \alpha A_n)\, s(y_1, y_2, \ldots, y_n)$

$\geq mini\{\alpha A(kx), \alpha A(sy)\}$

ii) $\alpha A(k\,x)^{-1} = \alpha A((k x_1)^{-1}, (k x_2)^{-1}, \ldots, (k x_n)^{-1})$

$= mini\{\alpha A_1(k x_1)^{-1}, \alpha A_2(k x_2)^{-1}, \ldots, \alpha A_n(k x_n)^{-1})\}$

$\geq mini\{\alpha A_1(k x_1), \alpha A_2(k x_2), \ldots, \alpha A_n(k x_n)\}$

$= \alpha A\, k(x_1, x_2, \ldots, x_n)$

$\geq \alpha A(kx)$

$\alpha A(s\,x)^{-1} = \alpha A((s x_1)^{-1}, (s x_2)^{-1}, \ldots, (s x_n)^{-1})$

$= mini\{\alpha A_1(s x_1)^{-1}, \alpha A_2(s x_2)^{-1}, \ldots, \alpha A_n(s x_n)^{-1})\}$

$\geq mini\{\alpha A_1(s x_1), \alpha A_2(s x_2), \ldots, \alpha A_n(s x_n)\}$

$= \alpha A\, s(x_1, x_2, \ldots, x_n)$

$\geq \alpha A(sx)$

iii) $\alpha A(k\,x \vee s\,y)) = \alpha A(k x_1 \vee s y_1, k x_2 \vee s y_2, \ldots, k x_n \vee s y_n))$

$= mini\{\alpha A_1(k x_1 \vee s y_1), \alpha A_2(k x_2 \vee s y_2), \ldots, \alpha A_n(k x_n \vee s y_n)\}$

$\geq mini\{mini[\alpha A_1(kx_1), \alpha A_1(ky_1)], mini[\alpha A_2(kx_2), \alpha A_2(ky_2)], \ldots mini[\alpha A_n(kx_n), \alpha A_n(ny_n)]\}$

$\geq mini\{mini[\alpha A_1(kx_1), \alpha A_2(kx_2), \ldots \alpha A_n(kx_n)], mini[\alpha A_1(sy_1), \alpha A_2(sy_2), \ldots \alpha A_n(sy_n)]\}$

$\geq$ mini{$(\alpha A_1 x \alpha A_2 x \ldots x \alpha A_n)$ k ($x_1, x_2, \ldots, x_n$) , $(\alpha A_1 x \alpha A_2 x \ldots x \alpha A_n)$ s ($y_1, y_2, \ldots, y_n$)
$\geq$ mini{$\alpha A(k x), \alpha A(s y)$}
iv) $\alpha A(k x \wedge sy)) = \alpha A (k x_1 \wedge sy_1, k x_2 \wedge sy_2, \ldots, k x_n \wedge s y_n))$
= mini{$\alpha A_1 (k x_1 \wedge sy_1), \alpha A_2 (k x_2 \wedge sy_2), \ldots, \alpha A_n (kx_n \wedge s y_n)$ }
$\geq$ mini{mini [$\alpha A_1(k x_1), \alpha A_1 (s y_1)$], mini [$\alpha A_2 (k x_2), \alpha A_2(s y_2)$],...mini[$\alpha A_n (k x_n), \alpha A_n (s y_n)$] }
$\geq$ mini {mini [$\alpha A_1(k x_1), \alpha A_2 (k x_2), \ldots \alpha A_n (k x_n)$], mini [$\alpha A_1(s y_1), \alpha A_2 (s y_2), \ldots \alpha A_n (s y_n)$]}
$\geq$ mini{$(\alpha A_1 x \alpha A_2 x \ldots x \alpha A_n)$ k ($x_1, x_2, \ldots, x_n$) , $(\alpha A_1 x \alpha A_2 x \ldots x \alpha A_n)$ s ($y_1, y_2, \ldots, y_n$)
$\geq$ mini{$\alpha A(k x), \alpha A(s y)$}
v) $\beta_A(k x s y)) = \beta_A (k (x_1, x_2, \ldots, x_n) s(y_1, y_2, \ldots, y_n)$
= $\beta_A (k x_1 s y_1, k x_2 sy_2, \ldots, kx_n s y_n))$
= maxi {$\beta A_1 (k x_1 sy_1), \beta A_2 (k x_2 sy_2), \ldots, \beta A_n (k x_n sy_n)$ }
$\leq$ maxi{maxi [$\beta A_1(kx_1), \beta A_1(sy_1)$], min [$\beta A_2 (kx_2), \beta A_2(sy_2)$],...
maxi[$\beta A_n (k x_n), \beta A_n (s y_n)$] }
$\leq$ maxi {maxi [$\beta A_1(k x_1), \beta A_2 (k x_2), \ldots \beta A_n (k x_n)$],
maxi [$\beta A_1(s y_1), \beta A_2 (s y_2), \ldots \beta A_n (s y_n)$]}
$\leq$ maxi {$(\beta A_1 x \beta A_2 x \ldots x \beta A_n)$ k ($x_1, x_2, \ldots, x_n$) ,$(\beta A_1 x \beta A_2 x \ldots x \beta A_n)$s($y_1, y_2, \ldots, y_n$)
$\leq$ maxi{$\beta A(kx), \beta A(sy)$}
vi) $\beta A (k x)^{-1} = \beta A ((k x_1)^{-1}, (k x_2)^{-1}, \ldots, (k x_n)^{-1})$
= maxi {$\beta A_1 (k x_1)^{-1}, \beta A_2(k x_2)^{-1}, \ldots, \beta A_n (k x_n)^{-1})$}
$\leq$ maxi {$\beta A_1 (k x_1), \beta A_2(k x_2), \ldots, \beta A_n (k x_n)$}
= βA k($x_1, x_2, \ldots, x_n$)
$\leq \beta A (kx)$
$\beta A (s x)^{-1} = \beta A ((s x_1)^{-1}, (s x_2)^{-1}, \ldots, (s x_n)^{-1})$
= maxi {$\beta A_1 (s x_1)^{-1}, \beta A_2(s x_2)^{-1}, \ldots, \beta A_n (s x_n)^{-1})$}
$\leq$ maxi {$\beta A_1 (s x_1), \beta A_2(s x_2), \ldots, \beta A_n (s x_n)$}
= βA s($x_1, x_2, \ldots, x_n$)
$\leq \beta A (sx)$
vii) $\beta A(k x \vee s y)) = \beta A (k x_1 \vee s y_1, k x_2 \vee s y_2, \ldots, k x_n \vee s y_n))$
= maxi {$\beta A_1 (k x_1 \vee sy_1), \beta A_2 (k x_2 \vee sy_2), \ldots, \beta A_n (k x_n \vee s y_n)$ }
$\leq$ maxi{maxi[$\beta A_1(kx_1), \beta A_1 (ky_1)$], maxi[$\beta A_2 (kx_2), \beta A_2(ky_2)$],...maxi[$\beta A_n(kx_n), \beta A_n(ny_n)$] }
$\leq$maxi{maxi[$\beta A_1(kx_1), \beta A_2(kx_2), \ldots \beta A_n(kx_n)$],maxi[$\beta A_1(sy_1), \beta A_2(sy_2), \ldots \beta A_n(sy_n)$]}
$\leq$ maxi{$(\beta A_1 x \beta A_2 x \ldots x \beta A_n)$ k ($x_1, x_2, \ldots, x_n$) , $(\beta A_1 x \beta A_2 x \ldots x \beta A_n)$ s ($y_1, y_2, \ldots, y_n$)
$\leq$ maxi{$\beta A(k x), \beta A(s y)$}
viii) $\beta A(k x \wedge sy)) = \beta A (k x_1 \wedge sy_1, k x_2 \wedge sy_2, \ldots, k x_n \wedge s y_n))$
= maxi{$\beta A_1 (k x_1 \wedge sy_1), \beta A_2 (k x_2 \wedge sy_2), \ldots, \beta A_n (kx_n \wedge s y_n)$ }
$\leq$ maxi{maxi [$\beta A_1(k x_1), \beta A_1 (s y_1)$], maxi [$\beta A_2 (k x_2), \beta A_2(s y_2)$],...maxi[$\beta A_n (k x_n), \beta A_n (s y_n)$] }
$\leq$ maxi {maxi [$\beta A_1(k x_1), \beta A_2 (k x_2), \ldots \beta A_n (k x_n)$], maxi [$\beta A_1(s y_1), \beta A_2 (s y_2), \ldots \beta A_n (s y_n)$]}
$\leq$ maxi{$(\beta A_1 x \beta A_2 x \ldots x \beta A_n)$ k ($x_1, x_2, \ldots, x_n$) , $(\beta A_1 x \beta A_2 x \ldots x \beta A_n)$ s ($y_1, y_2, \ldots, y_n$)
$\leq$ maxi{$\beta A(k x), \beta A(s y)$}

REFERENCES

1. Ajmal and Thomas, The Lattice of Fuzzy subgroups and fuzzy normal subgroups, Inform. sci. 76 (1994), 1 – 11.
2. Goguen: L – Fuzzy Sets, J. Math Anal.Appl. 18, 145-174 (1967).
3. Gu. Li and Chen, fuzzy groups with operators, fuzzy sets andsystem,66 (1994) ,363-371.
4. Lokhande and Makandar , fuzzy lattice ordered m group International Journal of Computer Applications (0975 – 8887) Volume 82 – No.8, November 2013.
5. Marudai & Rajendran: Characterization of Fuzzy Lattices on a Group International Journal of Computer Applications with Respect to T-Norms,8(8),0975 – 8887 (2010)
6. Nanda : Fuzzy Lattices, Bulletin Calcutta Math. Soc. 81 (1989) 1 – 2.
7. Rosenfeld : Fuzzy groups, J. Math. Anal. Appl. 35, 512 – 517 (1971).
8. Satya Saibaba. Fuzzy lattice ordered groups, South east Asian Bulletin of Mathematics 32,749-766 (2008).
9. Solairaju and Nagarajan : Lattice Valued Q-fuzzy left R – Submodules of Neat Rings with respect to T-Norms, Advances in fuzzy mathematics 4(2), 137 – 145 (2009).
10. Subramanian, Nagrajan & Chellappa ,Structure Properties of M-Fuzzy Groups Applied Mathematical Sciences,6(11),545-552(2012)
11. Zadeh : Fuzzy sets, Information and Control, 8, 338-353 (1965).