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Research Article

Typology Of The Most Common Errors In The Application Of The Algotitm Of Addition In Middle School Students.

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ABSTRACT

The purpose of this research is to classify the most common mistakes made by students in the tenth and eleventh grades of secondary education when faced with the need to perform operations such as addition or subtraction, to use them as an input for leveling and enhancing the development of mathematical thinking through error feedback processes in the evaluation.
A mixed approach was adopted, case study type; where the information was collected through different tests on basic operations taking into account the before, during and after, where the purpose is to reach a minimum assertiveness of 60% to be able to advance in the implementation of the different stages of the present study.
For more than five years, students in the tenth and eleventh grades between 15 and 19 years of age have been intervened at the beginning of the school year in order to reach a basic operability to develop mathematical thinking through the feedback of the error in the evaluation according to the grade in which they are. The productions, group constructions and their answers to the interviews were analyzed to conclude that the most common errors are: The poor handling of the algorithm in basic operations, the lack of knowledge of mathematics as a language generating the absence of meaning in various activities and the abandonment of the training process in the area of mathematics linked to feelings of low self-esteem or the inability to address operations that require considerable concentration times.

Keywords: error feedback; development of mathematical thinking, error, addition algorithm, evaluation.

Introduction.

Mathematics is an essential part in the intellectual growth of individuals, so the school plays a role of developer of mathematical thinking of utmost importance throughout each school grade. Each person should develop the ability to perform basic mathematical operations, including addition, which is the basis for learning the others. However, it has been observed that students often have difficulties with the application of the addition algorithm, since they accumulate with difficulty, especially surprising the state in which they reach the eleventh grade of secondary education in the educational institution of the city and Colombia in general; since of 280 people evaluated, 80% reach deficit in this operative part in basic operations, more than 40% do not dominate the addition algorithm with property, and present a deficient handling of the multiplication tables among many other inappropriate mathematical phenomena. This struggle can be attributed to several factors, such as the lack of conceptual understanding, since success in the application of addition consists in understanding the procedure of how to perform or implement its algorithm accompanied by its different processes. In the last PISA tests carried out in the year 2022, Colombia ranked 64th among the 81 countries participating in the evaluation of mathematics, an aspect that evidences the limitation that our students present in relation to the ability to extrapolate what they have learned and apply their knowledge to new circumstances, since their management of basic operations is limited.
Mathematics is fundamental in the Colombian curriculum because it is the basis of many areas of knowledge and is present in people's daily lives. Mathematics is essential for the development of cognitive skills, such as

logical reasoning, analysis and problem solving. In addition, mathematics is an important tool in the working world and in decision making in everyday life, so it is not possible to achieve the development of mathematical thinking properly if they do not master the management of basic operations including addition and subtraction fundamentally as they are the basis for addressing problem solving, in educational institution in recent years we have gone from getting 54 on average to get more 58 in the tests to know through the implementation of this methodology.

In Colombia, mathematics is one of the areas evaluated in the Saber and PISA tests, which reflects the importance given to this subject in the country's education since it opens the door to broader horizons. In addition, mathematical knowledge is essential for the development of other areas, such as science, technology, engineering and economics, which makes the teaching of mathematics in high school and other educational levels crucial for the development of the country, thus resignifying the need to acquire the proper operability in the management of basic operations; since students are completing their high school cycle without possessing this competence or have it rooted with errors in a large percentage.

To address this problem, the error feedback methodology has been proposed in the evaluation focused on problem solving as a way to identify and correct common mistakes made by students in basic mathematical operations and from there to the mastery of the respective algorithms and then teach them to solve problems in specific contexts. The error feedback methodology is based on the premise that feedback is essential for learning, and errors can be used as a means to provide assertive knowledge to students, since they in themselves represent experiences, memories, corrections and conceptual journey. By identifying and addressing errors, students can develop their mathematical thinking. Reaching a better understanding of mathematical concepts, algorithmic processes, of the possibilities of development, which leads to an improvement in performance in these basic operations until they reach mastery. Guerrero (2012) considers it propitious to adapt the education that students receive, contextualizing it and adapting it to their needs, also incorporating problem-solving activities that allow them to get used to and acquire skills for this activity; which lets see the mastery of basic operations as a starting point in each grade of secondary education in mathematics and other areas.

The purpose of this article is to present a typology of common errors made by eleventh grade students in basic mathematical operations that evidence the need for feedback and to explore the effectiveness of error feedback methodology through assessment where these errors are addressed as input for new and improved knowledge constructions. Specifically, this study will examine the use of error feedback methodology in the identification and correction of errors in addition, subtraction, multiplication, and division prior to standardized testing.

The importance of this study lies in its relevance to national and international assessments, such as the Saber and PISA tests. These assessments are designed to evaluate the knowledge and skills of students in various subject areas, including mathematics where the results have not been ideal. The results of this study account for the development of strategies and interventions aimed at improving student performance which is ultimately reflected in effective improvement in these types of assessments.

As Cumming (2016) states, "the ability to perform basic mathematical operations is critical to success in mathematics and other disciplines requiring mathematical competence." Similarly, McLeod and Vasinda (2017) note that "error feedback is an effective tool for promoting learning and improving performance in mathematics." These statements highlight the importance of this study and the potential impact it can have on the educational landscape.

In conclusion, this article aims to provide a comprehensive understanding of common errors made by eleventh grade students in basic mathematical operations by typifying them so that they can be appropriately fed back and highlight the effectiveness of the error feedback methodology in assessment as it addresses these errors by removing them from the negative signifier that traditionally accompanies them. The results of this study have the potential to inform teaching practices and improve student performance on national and international assessments. Starting from the premise that those who do not correct their mistakes are condemned to repeat them.

1. THEORETICAL FOUNDATIONS

This section deals with the development of mathematical thinking, the treatment of error, evaluation and feedback, and finally, pedagogical positions.

1.1 Mathematical thinking

According to Bosch, quoting Cantoral et al. (2005), mathematical thinking is interpreted according to the focus of attention of the protagonist, where this concept is attributed to the way in which people who are professionally engaged in mathematics think. In addition, it starts from a scientific environment in which mathematical concepts and techniques arise in problem solving and includes thoughts on mathematical topics and processes of abstraction, justification, visualization, estimation and reasoning under hypothesis, that is, all forms of construction of mathematical ideas at different levels of knowledge, which applies to all human beings. (Bosch Saldaña, 2012).

According to Reyes and Karg (2009), the mastery of mathematical fields involves traits such as persistence and perseverance, which motivates and are metacognitive abilities, which help to grasp and manipulate mathematical information. In this sense, thinking styles, which influence mathematical thinking styles (MTS), (Borromeo-Ferri, 2004). MWEs are personality attributes, preferences for the use of a mathematical skill

(Reyes Santander et al, 2018. However, the development of mathematical thinking is only viable through adequate oral communication between teacher-student, since it is fundamental as an instrument for the communication of mathematical concepts and procedures (Espinosa & Jiménez, 2019).

In this study, mathematical thinking is understood as a dynamic, analytical process whose degree of complexity and development increases as the student acquires basic skills, i.e., he/she will not only be appropriating the basic concepts, but also knows how to apply them in a given context in favor of problem solving. Thus, the development of mathematical thinking as a process implies the acquisition, mastery, deepening and improvement of the five general processes of mathematical activity, which are 1. formulating and solving problems, 2. modeling processes and phenomena of reality, 3. communicating, 4. reasoning, 5. formulating, comparing and exercising procedures and algorithms (Ministry of National Education [MEN], 2006), which would lead to form a mathematically competent individual and to develop his/her mathematical thinking.

1.2 The importance of error treatment

In relation to errors, the research brought as a reference highlights the importance of an ethical stance in the treatment of errors, as well as in being clear that error is not so much a lack of knowledge as an inadequate cognitive scheme. Popper (2001), in order to introduce the importance of errors for the acquisition of knowledge, takes up the Greeks, specifically Socrates, and states that "wisdom consisted in the knowledge of our limitations and, most important of all, in the knowledge of our own ignorance" (Popper, 2001, p. 1).

According to Popper (2001), approaching knowledge implies an attitude of tolerance and ethics. Thus, learning implies recognizing the other as a potential equal so that knowledge can really take place; all this with the understanding that one should not lose the critical spirit that allows one to be clear that it is not possible to be certain of having reached the truth. Likewise, since it is not possible to avoid making mistakes, one must change one's attitude towards them, assuming an ethical posture, since they should no longer be hidden, but should be corrected and learned from. This is possible if there is a level of awareness and self-criticism that allows one to accept the criticism of others not as something negative but as something necessary, since one needs others to discover and correct one's own mistakes.

In their research - related to the literature review on the characteristics, categorization, classification and evolution of the study of errors - Engler, Gregorini, Müller, Vrancken & Hecklein, establish that errors are systematic in the process of learning mathematical knowledge, so that "this process should include criteria for diagnosis, correction and overcoming through activities that promote the exercise of criticism of one's own productions" (Engler & Others, 2004, p. 23). These authors argue that if a quality mathematics education is to be achieved, it is essential that the teacher knows about errors, since both the importance and the treatment given to these errors influence the

They emphasize that the role of the student is active and open to learning, and that of the teacher is to "facilitate activities that provoke conflict and make them rethink the erroneous cognitive structure, forcing them to actively participate in the solution of their own conflicts by trying to replace false concepts with adequate conceptual understanding" (p. 31). (p. 31).

Rico (1995), states that error should be considered, generally, as incomplete knowledge, although it could be due to lack of attention to the processes; but surely, this is a permanent possibility of acquisition and consolidation of knowledge, which requires some conditions, and a permanent reality in the path of education that is the object of study itself. Thus, Rico takes up Popper's idea in which it is established that knowledge, being a human construction, cannot be considered as absolute truth since it is loaded with both errors and prejudices (Rico, 1995).

De la Torre (2004) states that error should be assumed as a strategy for change, as an opportunity for cognitive orientation or cognitive association, since it is part of the hidden curriculum and cultural values. Thus, error is part of popular wisdom, but with a generally negative connotation, for It is therefore urgent to change this meaning by bringing them to light and adopting constructivist, didactic and creative postures, i.e., we must have a didactic and pedagogical vision of the error, understood not as an end but as a strategy; therefore, to achieve a change in the cognitive facets there must be a change, an innovative posture, since the error must be understood as an inadequate process that must be corrected (De la Torre, 2004).

The research conducted by Ruano, Socas & Palarea (2008) - which was carried out with high school students and whose object was the analysis of errors in the algebraic processes of generalization, formal substitution and modeling - focuses on the errors made by students in the interpretation and use of algebraic procedures, highlighting that it is important not only that the teacher identifies the errors made by his students, but that based on this identification he should establish the appropriate mechanisms so that the students can correct their own errors. The error is considered "as an inadequate cognitive scheme and not only as a consequence of a lack of knowledge or an absent-mindedness" (Ruano, Socas, & Palarea, 2008).

Socas (1997) establishes three different axes in relation to the study of error: [1] Obstacle, [2] Absence of meaning and [3] Affective and emotional attitudes. The obstacle refers to the knowledge acquired, but when used by the student out of context, the responses generated are not adequate. On the other hand, the absence of meaning refers to the student's systems of representation; therefore, since they are not correctly developed, they make mistakes when solving either an exercise or an algebraic problem. The last axis, which is that of affective and emotional attitudes, refers to the fact that errors can arise due to forgetfulness, lack of attention, emotional blocks, among others.

For Popper (1991) the fundamental question about the source of knowledge is not the one that leads to think of knowledge as absolute and supreme truth, so he poses the following question:

How can we detect and eliminate error? The question about the sources of our knowledge, like so many other authoritative questions, is of a genetic character. It inquires about the origin of knowledge in the belief that it can be legitimized by its genealogy. The nobility of racially pure knowledge, of immaculate knowledge, of knowledge that derives from the highest authority, if possible from God: such are the (often unconscious) metaphysical ideas behind this question. It may be said that the question I have proposed as a replacement for the other, how can we detect error, derives from the idea that such pure, immaculate, certain sources do not exist, and that questions of origin or purity should not be confused with questions of validity or truth…our knowledge is conjecture, opinion-doxa,
rather than episteme-. (p. 49).

Therefore, in the light of this author, learning occurs from the conjectures and refutations that are made in the face of a given event; therefore, identifying errors and eliminating them is fundamental for the generation of knowledge. In this order of ideas, if this concept is applied to the teaching of mathematics -specifically to the evaluation of problem solving- it is intended as In addition, the student should learn not only to make conjectures when faced with the solution of a problem, but also to have the ability to defend and/or criticize his position, all this in order to achieve not only the solution of the problem, but also to make his learning process reflective, which would give him the opportunity to develop his mathematical thinking.

From the approaches outlined above, it can be said, first, that in mathematics errors are recurrent, paradigmatic and that they can be typified, in their great majority. Second, that some of them are generated due to paradigms that reflect the lack of well-developed competencies, which leads to an inappropriate development of mathematical thinking. And third, if the student is led to recognize, correct and build on the basis of the recurrent errors in his or her procedure, he or she develops his or her mathematical thinking, taking it to a more advanced level of operation and understanding at a conceptual level.

In addition, from the teaching position, the identification of errors allows the construction of didactic and pedagogical proposals that facilitate a contextualized management of the error in the classroom (Fong-Flores, 2021). Thus, error correction achieves its objective to the extent that it is integrated into the teaching-learning process, and as long as the excess of these, based on relapses, do not frustrate teachers and students (Merino-Mañueco, 2018).

Therefore, for the purposes of this study, it can be stated that identifying the error in itself does not lead to anything if it is not accompanied by reflection, if the error is not used as a constructive didactic tool with which to indicate inadequate strategies used by the student, failures in comprehension, lapses in execution and in any case, the possibility of diagnosing and establishing routes for improvement in the evaluation and teaching processes.

1.3 Evaluation and Feedback

Among the research related to the treatment of evaluation and feedback according to Florez & Gómez (2009), in the educational process, evaluation is conceived as the mechanism to pass a course, so that for the student learning takes a back seat since what is really important is to pass the exams and obtain good grades (Flores & Gómez, 2009). According to this research, the teaching of mathematics is no exception, despite studies that seek to focus the importance of the educational process on the student. The traditional form, teacher-centered teaching, establishes that the teacher is in charge of deciding "who learned and who did not, and the instrument to make this decision is the exam" (Ibid., 2009). Whereas, when the student is the protagonist of the pedagogical process -which is the proposal of these authors- he becomes responsible for acquiring his own knowledge, basic competencies and a positive attitude are promoted (Cfr. Ibid., 2009).

Additionally, they state that the evaluation instruments must be in accordance with the educational reality. That is, if the teaching process is student-centered, "it is necessary to change the concept of evaluation and design the appropriate instruments to make such evaluation effective." (p. 139). Therefore, in order to assess student learning it is not necessary to make a break
in the educational process, since "the same activities designed for the student to learn mathematics will serve for such evaluation" (Ibid. 2009).

Toranzos (2014), states that evaluation still has little pedagogical value since, despite the efforts to change the paradigm, the sanctioning or penalizing character it has still persists, in addition to being considered as a control mechanism. It takes up what Tenbrink (1981) stated, for whom "evaluation is the process of obtaining information and using it to form judgments that in turn will be used in decision making". Thus, evaluation is a systematic and intentional process that produces information and generates knowledge with a feedback character, since it makes it possible to show aspects that otherwise would not be possible to see. Therefore, "every evaluation process contributes to "give an account" and to "realize" changes and appropriations, achievements and deficiencies, in order to make informed decisions for the future" so that "evaluation assumes two interactive functions, feedback and giving an account" (Toranzos, 2014, p. 11).

Another position to take into account is that of García-Jiménez (2015) who, although focused on the field of higher education, makes a contribution to the evaluative process by referring specifically to the passage from feedback to self-regulation. He establishes in the first instance that the importance of evaluation lies in the information it yields and that it should serve as a basis for decision-making by the different actors involved in

the teaching-learning process. Feedback is defined as "that information that is used to reduce the difference between the learning results obtained by the student and the expected learning results" (p. 8). It takes up the classification of feedback made by Laurillard (2002), identifying on the one hand that which is intrinsic to the process, i.e., in which doubts are clarified to the student during the execution of the evaluation. And on the other hand, the extrinsic one, which is given to the student after the analysis of the results obtained in the evaluation.
According to Shute (2008), in the feedback process there are certain cognitive mechanisms that allow it to be effective: [1] to reduce students' uncertainty about the results of their evaluation, encouraging them to reduce the gap between the results obtained and the expected or reference results. [2] ability to motivate students who may feel discouraged by the reference level of demand, and [3] help to correct both conceptual and procedural errors, encouraging students to modify their learning strategies. The latter favors the student's self-regulation of learning, which is understood as the control of strategies that allow the achievement of previously established objectives. (Cfr. Ibid., 2015).
Likewise, as well as the correction of errors by teachers, feedback allows the collection of evidence on student learning, which facilitates the adjustment and adaptation of pedagogical strategies and measures (Canabal & Margalef, 2017). The implication of having knowledge of their learning process, to eventually make decisions that help the achievement of the stated objectives(Sánchez & Manrique, 2018).
Finally, Alvarado (2014), takes up what Barberá (2006) stated, for whom "These "Feed - Back" processes progressively adapt and readapt knowledge by adjusting it in a correct way" (p. 61). Alvarado (2014) states that for there to be adequate feedback it is necessary to take into account first the "Feed Up" considered as the path that the student is taking, if he is clear about where he is going and what are the objectives of the activities and the course itself. Second, the "Feed Foward" which consists of the student establishing a route for improvement based on the teacher's feedback, and finally, the "Feed Back" which is the feedback that helps the student to be aware of his performance.
to be aware of their performance.

1.4 From the pedagogical point of view

At this point, the postulates of Herbart, Ausubel and Vygotsky are brought up as pedagogical support. In this way, in relation to Herbart, it should be said that he conceives morality as "the supreme end of education" (Herbart, 1983), so that the concept of formation within his pedagogical postulates takes precedence. Thus, and according to Bedoya (2009), for Herbart the formative process must be carried out in an integral, critical, reflexive and systematic manner; therefore, "to form is to help, to guide the other, the child, the adolescent, the individual, to be formed, showing him how to become aware of the apparently dispersed elements with which he arrives at the formative process" (Bedoya, 2009).
(Bedoya, 2009)
For this, it should be taken into account that "character is only formed by the action of one's own will" (Herbart, 1983), therefore, it is of utmost importance that little by little the student takes charge of his own process; thus, "what is needed for this to be achieved is to encourage or awaken a real interest in the learner himself so that he initiates or continues his own formative project by himself, avoiding any kind of imposition, inhibition, repression or obligation. No violent, forced or imposed procedures under the authoritarian gaze of a teacher who pretends to direct the whole process".
process." (Bedoya, 2009)
Under this scenario, the purpose of the research is to generate in students a formative, critical and reflective process, through which they can identify their own mistakes, so that, from this, they can solve them and improve their own learning process. Therefore, the development of their mathematical thinking will occur as a consequence of a more active and autonomous attitude in their formative process, becoming more aware of their actions.
The next pedagogue cited is Ausubel and his postulates related to Meaningful Learning, understood as "the type of learning that occurs when the student is able to relate and integrate new information, new content, within the knowledge structures he previously possessed" (Villar, 2003). Thus, "both the new knowledge and the previous knowledge are transformed to give rise to a new integrated structure" (Ibid.: 361).
Finally, the last pedagogue who is related is Vygotsky, and from whom the concept of Zone of Proximal Development is taken up, understood as the "space of interaction in which a certain psychological function is executed before being internalized and therefore capable of being executed on an individual level" (Villar, 2003).
Also, for Vygotsky's theory, learning is primordial to development, "the only good teaching is that which is ahead of development" (Ibid.: 407). Hence, learning as a social construction is what drives cognitive development, so that development ceases to be universal to be situated in a given sociocultural context. This implies that "the individual develops thanks to his or her active participation in social interaction and culturally significant activities, contributing in a decisive way to the interaction with cultural agents and to the development of the individual.
that he establishes with cultural agents and through the instruments of mediation, also cultural, to the determination of his own development" (Ibid. 381).

Therefore, given that mathematical knowledge is a social construction, in which the imaginary of the difficulty in the process of acquiring it predominates and in which it is accepted that its development is not adequate, the need arises to encourage the student to be more active and autonomous in his formative process, that by carrying it out in a critical and reflective way and making use of collaborative learning among peers and thanks to the teacher's mediation, it is possible to show an improvement with respect to his own learning.

1.5 From the mathematically operational

Basic operations are not only fundamental to a student's academic environment and intellectual development, but also have practical applications in everyday life. These range from calculating prices at the school or neighborhood store to managing their finances responsibly. In addition, secondary education lays the groundwork for students' future professional or academic development leading them to engineering, administrative careers or jobs permeated by street vending, entrepreneurship where a solid command of basic mathematical operations is crucial for good professional and job performance.

In terms of short- and long-term academic development, mastery of basic operations contributes significantly to students' educational success. It provides a solid foundation for understanding more complex concepts in mathematics and fosters skills such as discipline, concentration, and problem solving, key aspects in both educational settings and daily life (Pérez, 2017). Therefore, focusing efforts on ensuring that students acquire a solid knowledge in addition, subtraction, multiplication, and division during their elementary education is crucial for their academic and personal development, in addition to their life as a high school and middle school student (Pérez, 2017).

The mastery of basic operations in mathematics is not only a fundamental academic skill, but also has a positive impact on many aspects in a person, generating in him/her security in the processes and in the optimal or adequate intellectual development related to numerical and variational thinking in favor of an adequate form of decision making, strengthening logical reasoning in various situations (MEN).

2. METHODOLOGY

The research approach, the definition of variables, population, sample and instruments applied are listed below.

2.1 Research approach

Although the central research is assumed from the mixed approach, for the elaboration of this text only the qualitative aspect is taken into account since, as previously stated, its objective is to account for the conceptions - that both students and teachers have - of evaluation, error and feedback, in the area of mathematics, and this obeys the qualitative part of the research.

Thus, the participatory action research (PAR) approach is chosen, given the need to adopt a strategy of the "learning by doing and evaluating what has been done" type, to guide the possibility of a systematic work, in which surveys, logs and protocols related to the process are recorded (Sandoval, 1996) Likewise, in this aspect, we take up again what Erickson proposed, for whom qualitative methodology focuses its attention on classroom teaching and the impact of the events that take place there on each of the actors participating in the educational process (Erickson, 1989). (Erickson, 1989)

1.1 Definition of variables, population and sample

According to Briones, independent variables are those that modify other variables -which are designated as dependent- with which they are related (Briones, 1996). Thus, this classification was used for the present study, insofar as the variables to be studied are the feedback of error in the evaluation of problem solving and its impact, or better, how it influences the development of mathematical thinking. This classification is detailed below:

TABLE 1.CLASSIFICATION OF VARIABLES

Tipo de Variable	Definition
Dependent	Development of mathematical thinking
Independent	Error feedback in the evaluation of problems, focused on problem solving.

Thus, this text shows the results related to the independent variable, on the conceptions that both teachers and students have of evaluation, error and feedback, in the area of mathematics. On the other hand, with regard to the population, it must be said that the IEMFS - made up of three primary and one secondary school - has 2409 students (of which 55% are male and 45% are female) and 97 teachers (74% female and 26% male). As shown in the following table:

TABLE 2. GENERAL DATA

Educational level	Nro. Student	Nro. Teacher

Preschool and Elementary Elementary	1420	49
Secondary and Middle School	850	48
TOTAL	227	97

Thus, since the research was carried out with high school students, the population was taken as 850 students belonging to the Stadium Campus, 191 in sixth grade, 180 in seventh grade, 108 in eighth grade, 146 in ninth grade, 124 in tenth grade and 101 in eleventh grade. Therefore, 170 students are taken as a sample: 110 belonging to the ninth grade and 60 belonging to the eleventh grade. Thus, it can be said that this sample represents 20% of the population, which is composed of 88 males and 82 females whose average age is 16 years old and the average socioeconomic stratum in which they live is 2.
As for the teachers, of the total of 97, 48 are secondary school teachers and 49 are elementary school teachers; and since the research was carried out specifically in the area of mathematics, it should be said that all elementary school teachers are in charge of teaching all areas, including mathematics. Of the 48 secondary and middle school teachers, 7 are appointed for the area of mathematics, so the survey was applied to each of them and 7 were chosen at random from elementary school in order to have a comparison with the same number of teachers from elementary school and secondary and middle school.
The characterization of the surveyed teachers is presented below:

TABLA 3. CARACTERIZACIÓN MUESTRA - DOCENTES DE PRIMARIA

Profesor	Género	Edad (años)	Años de experiencia	Título pregrado	Estudios de posgrado	
					Si	No
1	Masculino	28	3	Licenciado en Educación: Artes Plásticas		X
2	Femenino	28	10	Licenciada en Lengua Castellana		X
3	Femenino		20	Licenciada en Ciencias Políticas	X	
4	Femenino		20	Licenciada en Administración Educativa		
5	Femenino		24	Licenciada en Prescolar	X	
6	Femenino	38	12	Psicóloga		X
7	Masculino	44	11	Licenciado	X	

From the table above, it can be seen that the initial training of teachers corresponds to different areas that are not directly related to teaching in primary education (political science, psychology, etc.). A clarification that should be made is that primary school teachers, in the Colombian case, are in charge of being in charge of all the subjects corresponding to the grade in which they are and that their training does not always corresponds to the areas they should be working on.
As for the characterization of secondary school teachers, all of them have training in the specific area of mathematics, except one of them whose training corresponds to another specialty (mechanical technology). The experience of all teachers exceeds ten years, the longest being 40 years. This is listed below:

TABLA 4. SAMPLE CHARACTERIZATION - SECONDARY SCHOOL TEACHERS

Teacher	Genre	Age Years	years of Experience	Undergraduate degree	Graduate studies Si	No
1	Female	64	32	Degree in Mathematics	X	
2	Masculine	64	40	and Physics	X	
3	Masculine	55	26	Bachelor's Degree in Education with specialization in Mechanical Technology		X
4	Masculine	51	27	Bachelor of Science in Mathematics	X	
5	Masculine	43	12	and Physics		X
6	Masculine	38	15	Bachelor of Science in Mathematics		X
7	Female	36	19	Degree in Mathematics and Physics	X	

2.3 Instruments applied

In the first instance, surveys were applied to both teachers and students to identify the conceptions and the importance that each of them give to error, evaluation and feedback in the teaching and learning process. In addition, tests on basic operations were carried out and feedback was given over several years in the same

TABLA 5.SAMPLE OF QUESTIONS ASKED TO TEACHERS AND STUDENTS

QUESTIONS TO TEACHERS	QUESTIONS TO STUDENTS
Do you consider that you can learn from mistakes? What mistakes do your students make when solving mathematical problems? What strategy do you use to help students correct the mistakes they make on assessments? Why do you think students make mistakes in mathematics? Do you think students are aware of the mistakes they make? What do you think the assessment is for? According to the students, what is the best way to evaluate them? What do you think is the best way to assess student learning? In your pedagogical practice, do you give feedback on the evaluations? YES - NO: How do you do it?	Do you think we can learn from our own mistakes? Why do you think mistakes are made when solving problems in mathematics? How do you seek to solve the mistakes you make when solving problems in mathematics? What does it mean to you to make a mistake in mathematics? What do you think the assessment is for? How would you like to be assessed in mathematics? When you do poorly on a math assessment, why do you think that is? When you miss an assessment, do you review it and solve it again to see where you went wrong?

grades, and then the information collected was tabulated and analyzed to finally identify some descriptors that would allow making generalizations that would serve as a basis to promote a change of mentality with respect to the treatment of errors in the area of mathematics and correct the most common errors when performing basic operations such as addition, subtraction, multiplication and division. Thus, the following is a sample of the questions applied in the survey:

2.4 Methodological design

In order to identify the conceptions that teachers and students had about the treatment of error and feedback in mathematics assessments, three stages were taken into account: [1] Planning and design of surveys, [2] Application and [3] Analysis of results.

3 RESULTS

The present analysis was applied to 280 students per year, in an educational institution in the city of Medellin, where more than 40% of them present deficiencies in this aspect and were sought to overcome through feedback processes and the results were significant.

3.1 Errors detected in the application of the addition algorithm.

- In the imagination of some youngsters, when the given problem consists of positive and negative quantities, the positive quantities are added and the negative quantities are subtracted.
- Some youngsters are not clear whether an initial unsigned number is positive or negative. Therefore, they end up asking whether it is positive or negative.
- Some people make frequent errors when accumulating by not completing tens when adding. They organize the numbers in columns, but when adding the units and moving to the tens, they lose the amount that goes from one column to the other. This happens with the thousands and with the other columns, which generates an error in the final result.
- A small percentage of people make the mistake of arranging the numbers in columns from left to right, instead of matching ones to ones, tens to tens and so on. This misses the logic of the process, as the numbers are not ordered according to their actual value.
- When it is their turn to arrange the negative numbers in columns, they have the need to subtract without knowing how to do it. They relate the negative sign with the subtraction operation directly.
- Some youngsters make the mistake of placing the result of a sum of negative numbers with a positive sign. This is because they unconsciously believe that, when performing an addition, the result must be a positive number.
- They present limitations when accumulating numbers arranged in columns because they accumulate with their fingers and any distraction stops them or leads them to lose count. They lose verticality when accumulating numbers arranged in columns because they do not have an adequate spelling or simply because they are not used to carry out these processes.
- Almost 80% of the students either do not know the multiplication tables, do not handle the concept of completing tens or simply do not understand what is colloquially taught in elementary school "adding by subtracting".

- It is a mistake not to have mechanisms to verify each step of the addition process in order to reach adequate levels of certainty and confidence. Many do not do it because every time they try it they get different results or ambiguous answers.
- A certain number of students do not write the quantities adequately, presenting syntax errors when forming the columns, generating transcription or addition errors when confusing columns. 40% or more students add up rudimentarily with the help of their fingers, leaving aside more effective and agile processes such as multiplicative principles or only completing tens, which reflects the poor management of the addition algorithm.
- Not using a verification mechanism such as estimating an approximate result at the beginning and verifying it at the end.

3.2 Errors detected in the application of the subtraction algorithm.

- - One of the most recurrent errors of the students is that when they see a number with a minus sign they assume that it is subtracting. Then when they have two negative numbers they feel the need to subtract them.
- - When students must compare two quantities of opposite sign or subtract them and the larger absolute value quantity is the negative one, they tend to place it as a subtrahend and perform the operation forcing it to give a result or in the best case they reach a dead end. This procedure is a sequel of the primary education where they always put the positive quantity on top and the negative one on the bottom, as they all said, arguing that they did not know any other way to do it.
- - Some students accumulate with their fingers picking them up inside and to do the subtraction they pick them up again, this process is a little rudimentary since with large quantities it becomes unmanageable making it a little dispersed by the little numerical mastery of some students.
- - When several quantities are assigned where some are positive and others negative, they tend to solve by subtracting in pairs, and then put the results together with additions or subtractions.
- - It is not clear to them that to subtract they must have two positive or negative quantities; or the combination of them. The process consists of comparing the quantities trying to establish whether I have more than I owe or I owe more than I have and thus predict the sign of the result.
- - It is not clear the procedure for them when a quantity must borrow to achieve the subtraction, also some do not borrow in the quantities because they do not perceive that the minuend must be greater than the subtrahend.
- - Very few students organize the quantities from left to right and pair them with zeros to perform the subtraction. This error is conceptual and procedural reflecting a high lack of foundation.
- - Not knowing that subtraction of two positive numbers is a positive number and not knowing that subtraction of two negative numbers is a negative number or that subtraction of two equal numbers yields zero.
- - Errors generated by not knowing how to subtract multi-digit numbers, not knowing how to subtract numbers that have zeros in between or simply losing the sign when performing the operation.
- - It can also be considered as a recurring error the lack of a valid mechanism for verifying the result, such as adding the subtrahend with the difference that the minuend should give.

Conclusions and recommendations on how to avoid errors when performing addition, subtraction, multiplication and division.

The learning of the basic addition operation is a fundamental stage in the development of mathematical thinking that begins almost from when the child begins to be aware of aspects such as quantity or the ability to accumulate in his or her thinking. However, errors in the performance of these operations are common among students of all ages, to such an extent that many of them reach eleventh grade with deficiencies in this aspect; these can be generally procedural and conceptual.
Errors in addition can have several causes, such as lack of understanding of the mathematical concepts involved, lack of mastery of the procedures to perform the operations, lack of attention or concentration, lack of practice and personal abandonment of students who learned over time that they could lose mathematics and pass the following year having an acceptable performance in other less demanding areas.
To avoid errors in operations in the addition algorithm, it is important that students understand the mathematical concepts involved such as the ability to accumulate, such as the actions of completing tens or applying criteria such as adding by subtracting and having the concentration, practice on a regular basis. All this is reflected in the feedback processes carried out in the present research project, which has allowed them to retake the processes and gain confidence in themselves.

Recommendations

Addition and subtraction:

- Ensure that students understand the concepts of addition and subtraction, such as the meaning of the + and - signs, and the absolute value of a number. It is recommended to use the mirror coloring book for young people to create a visual concept of negative numbers. This consists of making a number line on the floor with paper tape and where the zero goes, place a vertical mirror and from there create a review of the concepts related to everything that has to do with integers and especially negative numbers.
- Teach students the procedures for addition and subtraction in a clear and concise manner. Starting with small quantities, then larger quantities, then negative quantities, and then combine them.
- Provide students with opportunities to practice addition and subtraction on a regular basis, teaching them to check their results and identify possible errors.
- Correct students' errors constructively through feedback mechanisms of propose similar activities.

General recommendations:

- Use a variety of methods and strategies to teach basic operations.
- Tailor instruction to individual student needs.
- Create a positive and stimulating learning environment.
- Involve parents and guardians in the learning process.

It is important to remember that mistakes are a natural part of learning. Students need to make mistakes in order to learn from them and improve their skills. However, mistakes can also be demotivating for students, generating fear of rejection, fear of correction, or simply a sense of disengagement from learning. Therefore, it is important for teachers and parents to address student errors constructively, focusing on learning and not on judgment; learning to feed back errors in a natural way, emphasizing how to correct them. In addition, it is important for teachers and tutors to be attentive to these common errors and to provide constructive, targeted guidance and stimulate practice processes necessary for students to improve their skills in performing basic operations and in the development of mathematical thinking in each learner. Constant practice, reinforcement of key concepts and solving various problems can help to overcome these errors and acquire the required thematic mastery.

- Therefore, it is important to identify the types of errors that students make in order to design appropriate and, if possible, personal teaching and learning strategies. In addition, students who present learning problems and manifest them in the instability of learning the multiplication tables should be taught the tables by the Doman method in mathematics, which consists of assuming that mathematics is a language, thus exposing students who were not able to learn the tables by themselves to exhibit the different multiplication tables every day and make them repeat what they see, for a period of a month and a half or until they express the acquisition of learning, for example 9x4= 36. In addition, perform auditory exercises associated with writing, such as mentioning a table and writing it down, to work on auditory, visual and mechanical memory until they learn them and with them improve the ability to add or subtract properly.

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