Educational Administration: Theory and Practice
2024, 30(5), 13122-13128
ISSN: 2148-2403
https://kuey.net/
Research Article

Comparative Analysis Of Roman And Inverse Roman Domination Numbers Across Graph Families.

J Jannet Raji[1*], Dr. S Meenakshi[2]

[1*]Research Scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai -600117, Tamil Nadu, India. mail: jefylivya@gmail.com
[2]Associate Professor, Department of Mathematics, Vels Institute Science, Technology and Advance Studies, Chennai-600117, Tamil Nadu, India.

Citation: J Jannet Raji et al. (2024), Comparative Analysis Of Roman And Inverse Roman Domination Numbers Across Graph Families., *Educational Administration: Theory and Practice*, *30* (5), 13122 - 13128
Doi: 10.53555/kuey.v30i5.3630

ARTICLE INFO **ABSTRACT**

This research paper delves into the intriguing realm of Roman domination and its inverse counterpart within various graph structures. Initially defining Roman domination as a graph theory concept where vertices are covered by distinct dominating sets, and inversely, inverse Roman domination as a novel extension where vertices outside the dominating set are considered, the study progresses to explore these concepts across a spectrum of graph families. The investigation begins with cubic graphs, exploring their Roman and inverse Roman domination numbers, followed by an analysis of cubic symmetric graphs, Platonic graphs, Holt graphs, Folkman graphs, Frucht graphs, and culminating with the Levi graph. For each graph family, meticulous calculations and comparisons are conducted to ascertain the Roman Domination number and inverse Roman domination numbers, shedding light on their inherent characteristics and relationships.

Keywords: Domination number; Roman domination number, Inverse Roman domination number.

1. Introduction:

Inverse Roman domination is a theoretical framework inside graph concept, focusing on the identity of a minimum subset of vertices which can be unaffected by way of domination from every other vertex within the graph, rather than the conventional approach of identifying a minimum dominating set. The usage of this concept extends throughout numerous domain names including network safety, social community evaluation, and Social networks.

Definition 1:
In ordinary Roman domination, for instance, each vertex is assigned a weight of zero, 1, or 2. The constraint is that any vertex with a weight of zero need to be adjacent to a vertex with a weight of two. The minimum general weight required to fulfill this condition is the Roman domination range of the graph.
Cockayne Cockayne, Dreyer, Hedetniemi, Hedetniemi, and Mcrae (n.d.) defined Roman
dominating function of a graph G = (V,E) as a function $f : V \rightarrow \{0, 1, 2\}$ where each vertex u, with f(u) = zero, is connected to at least one vertex v with f(v) = 2. the weight of a
actual-valued function $f : V \rightarrow R$ is calculated because the sum of f(v) for all $v \in V$. The Roman domination number (RDN), represents the minimum weight manageable among all
RDFs in G. it is denoted by $\gamma_R(G)$, In other words, a Roman dominating feature corresponds to a vertex coloring of a graph using the colours {0, 1, 2} in this kind of way that every vertex coloured as "0" shares an side with at the least one vertex colored as "2".

Definition 2:
Inverse Roman dominating feature is the function equal to a Roman dominating feature Coxeter (1973), below the situation that the set V − D includes a Roman dominating characteristic $f_1 : V \rightarrow \{0, 1, 2\}$, in which D represents the vertices v, that satisfy f(v) > zero. in the end, f1 is recognized as an Inverse Roman Dominating feature (IRDF) on a
graph Kumar and Murali (2014) G with respect to f. The inverse Roman domination wide variety (IRDN),

symbolized as γ' IR(G), corresponds to the least weight among all IRDF in G

2. Cubical graph

The Platonic graph that corresponds to the cube's connectedness is called the cubical graph. It's far isomorphic to the subsequent graphs: crown graph , grid graph G2,2,2, prism graph, hypercube graph Q3, bipartite Kneser graph H(1,four), and generalized Petersen graph GP(4,1). Several embeddings (e.g., Knuth 2008, p. 14) illustrate it above. Twelve specific (directed) Hamiltonian cycles make up its particular order of four. There are 12 edges, 3 vertex and 8 nodes connections, three edge connections, three graph diameters, three graph radiuses, and 4 girths within the cubical graph Grunbaum (1967).

Lemma 2.1:
Let G be the cube then $\gamma_R(G) = \gamma'_{IR}(G) = 4$.
Proof:
Let G represent the stable platonic cube [13], with 8 vertices and 12 edges, G is a three normal graph. allow D^1 = (V-D) be the Inverse Roman dominating set of G. Let D be the Roman dominating set. in keeping with Fig. 1, in order to dominate G, each the Roman dominating set and the inverse Roman dominating set require vertices.

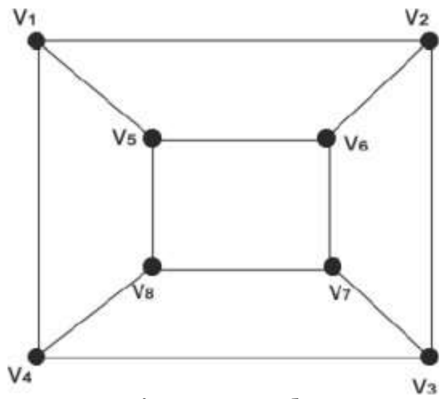

Figure 1 : Cube

Dominating set D ={v_1,v_7} has 2 vertices that have degree three. So label the vertices of D with f(v_1)=2 and f(v_7)=2. With the aid of definition of RDF its adjoining vertices will have label zero. As a result $\gamma_R(G)$ = 2(2)+0=4. Let D^1 ={v_2,v_6} has 2 vertices that have degree three. So label the vertices of D with f(v_2)=2 and (v_6)=2. By definition of IRDF, its neighbouring vertices might have label zero. So f(v')=2. Subsequently $\gamma'_{IR}(G)$=2(2)+0=4.
Hence, γ_R (G)= γ'_{IR}(G) = 2.

Lemma 2.2:
If G is a cubic symmetric graph. Then $\gamma_R(G) = \gamma'_{IR}(G) = 4$.
Proof :
The cubic symmetric graph Harary (1975) G, with 8 vertices of degree three each, is tested. To dominate all of the other vertices in G, only vertices are required. Whilst D is the Roman dominant set and V is the set of all of its vertices, then those 2 vertices may have label
2. By means of definition of Roman dominating function its adjoining vertices may have label

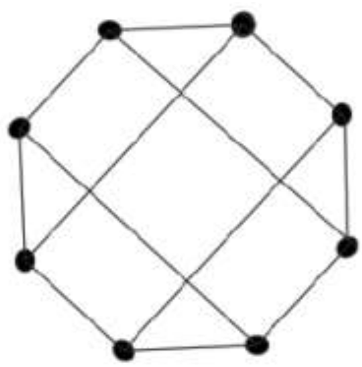
FIGURE 2: Cubic symmetric graph

zero. For this reason f(v)= 2 and its adjoining vertices could have label zero. consequently f(u)=0. So γ_R(G) = 2 (2)+0 =4. Any other set, D^1, nevertheless has dominance over G's vertices. With cardinality 2, D^1 is the Inverse Roman dominating set. D's vertices will bear label 2. So f(v)=2 and its adjoining vertices can have label 0. As a result γ'_{IR}(G) = 2(2)+0=4.

Consequently γ_R(G) = γ'_{IR}(G) = 4.

3. Platonic graph:

A graph with the skeleton of one of the Platonic solids is called a Platonic graph in the subject of graph idea. There exist five Platonic graphs, all of which are polyhedral, ordinary, and Hamiltonian graphs Harary (1975) (by necessity, vertex-transitive, area-transitive, additionally three-vertex-connected and planar graphs).

- Tetrahedral graph: 4 edges and 6 vertices
- Octahedral graph: 12 edges and six vertices
- Cubical graph: 12 edges and 8 vertices
- Icosahedral graph with 30 edges and 12 vertices
- A dodecahedral graph with 30 edges and 20 vertice

Lemma 3.1:

Let graph G be the octahedron, then γ_R(G) = γ'_{IR}(G) = 3.

Proof:

Consider the graph G, the octahedron of platonic solid graph. G is then a four-regular graph with twelve edges and six vertices Harary (1975),

Consider the dominating set D. It has 2 vertices, which has max degree 4. Label these vertices with f(v)=2 , by definition of RDF its adjacent vertices will be f(u)=0 and one more vertex will be f(v)=1 , so γ_{IR} function has |V2| =2 and |V1| =1. Hence γ_R (G) =2(1)+1=3.

Consider the inverse dominating set of G, D^1 = (V - D). It has 2 vertices which has max degree 4. Label these vertices with f(v')=2, by definition of IRDF its adjacent vertices will be f'(u)=0 and one more vertex will be f'(v)=. so γ_{IR} function has |V2| =2 and |V1| =1.

So γ'_{IR}(G) = 2(1)+1=3. Hence γ_R(G) = γ'_{IR}(G) = 3.

Lemma 3.2:

If G is the dodecahedron. Then γ_R(G) = γ'_{IR}(G) = 12.

Proof:

Consider the graph G, the dodecahedron platonic solid. G is a three-dimensional regular graph with 20 vertices and 30 edges [2]. permit D constitute the dominant set. Six vertices[5] make up the dominating set D in Figure 4.

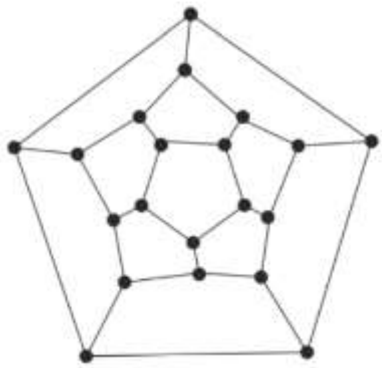

FIGURE 4: Dodecahedron

D has 6 vertices which has degree three. Label those vertices with f(v)=2 subsequently by definition of RDF its adjacent vertices might be f(u)=0 so γIR function has |V2| =6 and |V1| =0. So γ_R (G) = 2(6)+0=12. Assume that G's inverse dominant set is D^1 = (V - D). Six vertices also are required for D^1, the inverse dominating set, to dominate G. Label these vertices with f'(v)=2. For this reason by definition of IRDF its adjoining vertices can be f'(u)=0 so γ_{IR} function has |V2| =6 and |V1| =0. So γ'_{IR}(G) = 2(6)+0=4, for this reason γ_R(G) = γ'_{IR}(G) = 12.

Lemma 3.3:

Let G be a Icosahedral graph, then γ_R(G) = γ'_{IR}(G) = 4

Proof:

Consider an icosahedral graph G, which has 12 vertices of degree 5, each. Given two sets of vertices, V = {v1, v2,..., v6} and U = { u1, u2,..., u6}, represent G. We require just two vertices to dominate G. As seen in Figure 5, all it takes to dominate the entire set of vertices is two.

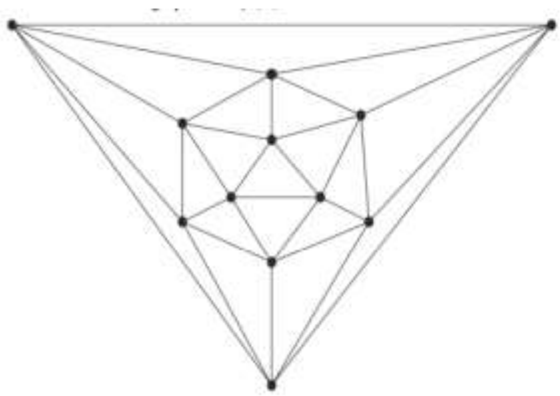

FIGURE 5: Icosahedral graph

Let D be the dominating set. It has 2 vertices with label 2 with f(v)=2 by definition of RDF and its adjacent vertices will be f(u)=0 so a γ_{IR} function has |V2| =2 and |V1| =0.So γ_R (G) = 2(2)+0=4. D^1 be the inverse dominating set .D' has 2 vertices with label 2 with f(v)=2. Hence by definition of IRDF f'(v)=2 and its adjacent vertices will be f'(u)=0. so a γ_{IR} function has |V2| =2 and |V1| =0. Therefore $\gamma'_{IR}(G)$ = 2(2)+0=4. Hence
$\gamma_R(G) = \gamma'_{IR}(G) = 4$.

Lemma 3.4:
If G is the Cayley graph, then $\gamma_R(G) = \gamma'_{IR}(G) = 6$.
Proof:
The truncated tetrahedron, sometimes known as the Cayley graph [4], is a graph with 12 degree three vertices and 18 edges total. Suppose V is the set of all vertices, the three vertices lying between the inner cycle and outer cycle dominates the entire graph.

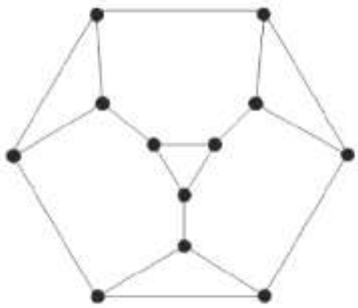

FIGURE 6: Cayley graph

Let D be the Roman dominating set, Label the vertices of the set with 2. Then f(v)=2 and its adjacent vertices will have f(u)=0. So $\gamma_R(G)$ = 2(3)+0=6. There exists another set D' which have one vertex in the inner cycle and two vertices at the outer cycle continues to control V-D's vertices. Thus D^1 is the Inverse Roman Dominating set of cardinality again 3. These vertices of D^1 will have label 2. So f(v)=2 and its adjacent vertices will have label 0. So f(u)=0. So γ_{IR} (G) = 2(3)+0=6. Thus $\gamma_R(G) = \gamma'_{IR}(G) = 6$.

Proposition3.1:
A graph with 27 vertices is called the Holt graph [6]. Another name for the Holt graph is the Doyle graph. Assume that C_n is a Doyle graph. After that, $\gamma(C_n) = 9$.

Lemma 3.5:
Let G be a Doyle graph, then $\gamma_R(G) = \gamma_{IR}(G) = 18$
Proof:
Consider a Doyle graph G with 27 vertices. Let V = {v_1, v_2... v_n}, be the set of vertices of the graph.

FIGURE 7: Doyle graph

Three vertex sets (V1,V2,V3), each with nine vertices and four degrees per vertex, can be formed from the vertex set V. Figure 7, displays the graph. $\gamma(G) = 9$ is obtained if V1 is considered as the dominating set, with its vertices dominating the other two sets. Vertices of V1 take the label 2. So f(V1)=2, and its adjacent vertices will have label 0. Hence $\gamma_R(G) = 9(2)=18$. Then (V - D) contains another two vertex sets V2 and V3 which dominates the remaining vertices of (V - D). Consider V2 as the inverse roman dominating set. Hence $\gamma'(G) = 9$. Vertices of V2 take the label 2. So f(V2)=2, and its adjacent vertices will have label 0. Hence $\gamma_{IR}(G) = 9(2)=18$.

Lemma 3.6:
Let G be a Folkman graph, then $\gamma_{IR}(G) =12$ and $\gamma_R(G) =12$.

Proof :
Examine the Folkman graph [4] G, which has 20 vertices having degree 4. Let V = {$a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4, c_0, c_1, c_2, c_3, c_4, d_0, d_1, d_2, d_3, d_4$} be the graph's vertices. To control the complete graph, we require six vertices. A Roman dominant set of G is formed by the vertices {$a_1, a_2, b_0, b_3, c_3, d_2$}, as seen in Fig 7. These vertices will have label 2.

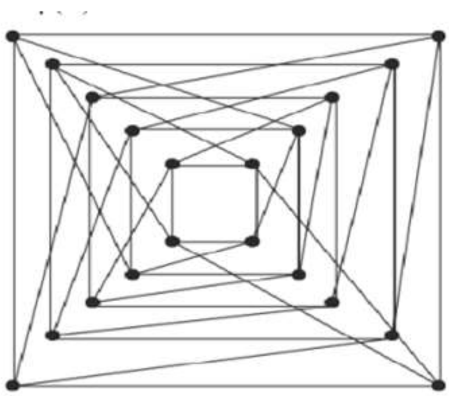

FIGURE 8: Folkman graph

So there exists a γ_R function such that f (a_1)= f(a_2)=f(b_0)=f(b_3) =f(c_3)=f(d_2)= 2. Hence $\gamma_R(G) = 6(2)=12$. An inverse roman dominating set of G is formed by another dominating set, D^1, which has the same number of vertices as V-D and These vertices will have label 2. so there exists a γ_R function such that f(v)=2 at these vertices and f(u)=0 at the adjacent vertices. Hence $\gamma'_{IR}(G) = 6(2)=12$

Proposition3.2:
The Frucht Graph has 12 vertices and 18 edges, making it a 3-regular graph. The graph is Hamiltonian and planar cubic. Consider the Frucht graph as G. Consequently, $\gamma(G) = 3$.

Lemma 3.7:
Let G be the Frucht graph. Then γ_R (G) =6 and γ'_{IR}(G) =8.
Proof :
Let G represent the Frucht graph [4], a cubic graph with 18 edges and 12 vertices. Let V = {v_1, v_2, ... , v_7} and U = {u_1, u_2, ... ,u_5} be the two sets of vertices of G.

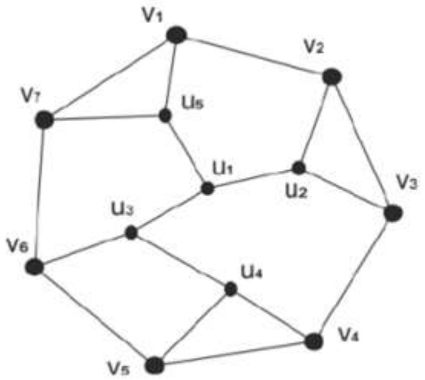

FIGURE 9: Frucht Graph

Assume that the dominating set is D, and its inverse is D^1. After that, there exists one D^1, resulting in one inverse roman dominating set in (V - D). To dominate G, we need three vertices which can take label 2. So γ_R (G) = 3(2)=6. Figure 8 shows that for D^1 to dominate the complete collection of vertices, four vertices are required. So these vertices take label 2. Hence γ'_{IR}(G) = 4(2)=8.

Proposition3.3:
In the Levi graph , there are 30 nodes and 45 edges.
Lemma 3.8:
Let G be a Levi Graph, then γ_R(G) = γ'_{IR}(G) = 20.
Proof:
Let G be a Levi graph, with 30 vertices and forty five edges. Let V = {v_1, v_2, ... , v_{10}}, U = { u_1, u_2, ..., u_{10}} and W = {w_1, w_2, ..., w_{10}} are the vertices of V,U and W with degree 3. To dominate the graphG, 10 vertices are needed. Figure 10, shows U = {u_1, u_2, ..., u_{10}} are the vertices adjacent to V and W.

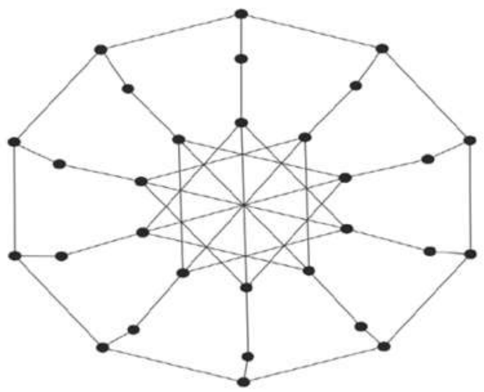

FIGURE 10: Levi graph

Label these vertices as 2. Hence f(u)=2. By definition of Roman dominating function its adjacent vertices could have label 0. Subsequently γ_R(G) = 10. Allow D^1 has 5 vertices + five vertices in V-D. Those vertices could have label 2. Its adjoining vertices can have label 0. So γ'_{IR}(G) = 20. For this reason γ_R(G) = γ'_{IR}(G) = 20.

Conclusion:

When the number of elements in the dominating set and its inverse dominating set are equal, so too are their Roman domination numbers and Inverse Roman domination numbers. The outcomes of this investigation unveil captivating observations regarding the interaction between Roman and inverse Roman domination numbers within diverse graph families, offering a comprehensive comprehension of their structural attributes and intrinsic intricacies. These insights not only enrich the theoretical foundations of graph theory but also have practical implications in various domains, such as network design, optimization and algorithm development.

REFERNCES:

1. E. J. Cockayne, P, A. Dreyer Jr,, S. M. Hedetniemi, S. T. Hedetniemi, Roman Domination in Graphs, Discrete Mathematics 278(2004),11-22.
2. H. S. M. Coxeter, (1973). Regular Polytopes (third ed.). New York: Dover Publications.
3. B. Grunbaum, (1967), "Convex Polytopes". New York: John Wiley & Sons.
4. F. Harary, (1975), Graph Theory, Addison Wiley, Reading Mass.
5. W. Haynes, S. T. Hedetniemi, & P. J. Slater, (1997), Basics of Domination in Graphs.
6. W. Haynes, S. T. Hedetniemi, & P. J. Slater, (1998), Domination in Graphs: Advanced topics.
7. M. A. Henning, & S. T. Hedetniemi, (2003), Defending the Roman Empire – A new strategy. 266, 239–251.
8. M. Kumar, & R, Murali, (2014), Inverse Roman domination in some classes of graphs. 4(4), 219–238.
9. K. Kumar, & S. Reddy, (2013), Inverse Roman domination in graphs. 5(3), 1–4.

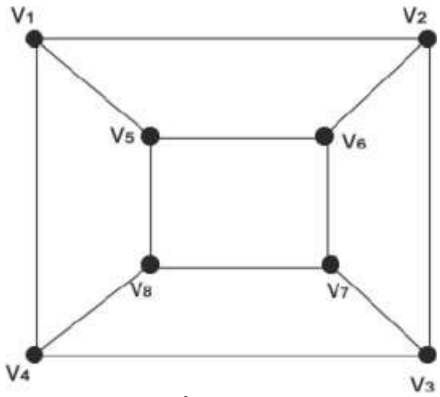

Figure 1.

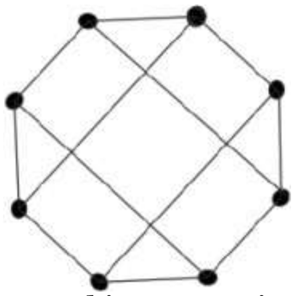

Figure 2: cubic symmetric graph

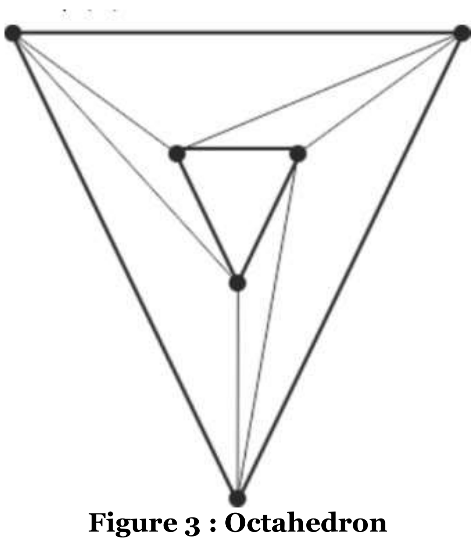

Figure 3 : Octahedron