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Comparison of Scholastic Attainment in English and Math amongst Other Studies at the Higher Secondary Level: A Study using Mahalanobis Distance

Eusob Ali Ahmed [1], *Mohammad Rezaul Karim* [2*], *Munmun Banerjee* [3], *Subir Sen* [4]

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Abstract

The Mahalanobis Distance (MD) is applied in this investigation in order to analyze the dynamical character of academic attainment in five different disciplines including Mathematics, English, Biology, Physics, and Chemistry between two student groups of higher secondary class in the Bodoland Territorial Region (BTR) regions in the state of Assam, India. There are five categories of groupings taken into account such as students under tribal and non-tribal backgrounds, boys and girls, urban and rural, urban boys and rural boys, urban girls and rural girls. In five disciplines, the MD is employed to compute the variance in the dynamical nature of attainment between two sections of learners. Despite the fact that urban students received better conditions than rural students, there is no substantial distinction in the dynamical character regarding attainment between urban and rural learners. Similar findings are reported for urban boys and rural boys, as well as urban girls and rural girls. It is also found that there is no statistically substantial variation between tribal and non-tribal students, and between boys and girls.

Keywords: Mathematics; English; BTR; Mahalanobis Distance; Higher Secondary Level

[1]*Assistant Professor, Department of Mathematics, Sapatgram College, Sapatgram, India, eusob1974@gmail.com*

[2*]*Assistant Professor, Department of English, College of Science and Humanities, Prince Sattam bin Abdulaziz University, Al Kharj, Saudi Arabia, karimrezaul318@gmail.com*

[3]*Assistant Professor, Department of Education, Sapatgram College, Sapatgram, India, mnmn.banerjee123@gmail.com*

[4]*Associate Professor, Department of Education, Sidho-Kanho-Birsha University, Purulia, India, subirsenmath@gmail.com*

Introduction

Mahalanobis Distance (MD) has played an important role in separating traits in domains like clustering, categorization, image managing, neuro computing, precision medicine, and so on during the last few decades. While examining the issue like hypothesis assessing, goodness of fit experiment, categorization procedures, grouping testing, outlier noting, and density calculation procedure, statistical distances play a vital role. We can quantify the proximity of two statistical objects by using distance measurements (or similarities). The MD is one of many statistical distance measures (Venturin, 2015), and it has the benefit of being able to identify two or many variable outliers. An estimate of division or distance between groups in terms of various traits is employed in MD. This metric was proposed by Mahalanobis in 1936 (Mahalanobis, 1936). It played a keypart in data processing and statistics involving multiple measurements. Mahalanobis Distance (Mahalanobis D2 statistics) is an admired and helpful measure of "closeness" of two or more variable examination that was devised by P.C. Mahalanobis (1927, 1936). It was an influential contribution, and many researches have been done about it according to statistics and by means of theory (Rao, 1963; 1973; Rudra et al., 1996).

Scholastic attainment in Mathematics and English in combination with other disciplines at the higher secondary level represents the level of achievement in Mathematics and English in combination with other disciplines. When comparing achievement in Mathematics and English with other disciplines among different groups with diverse learners, several problems arise. For example – Are the intelligence levels of the learner the same? Is the difficulty level in the same for different groups? Is the socioeconomic condition of different learners in different groups the same? etc. As a result, it is believed that the MD is a more generalized concept for comparing the substantial variance in dynamical nature for a lot of subjects in terms of attainment between two groups of learners with different learning styles.

The current study compared the achievement in five disciplines, namely Mathematics, English, Biology, Physics, and Chemistry, amongst tribal and non-tribal, boys and girls, rural and urban higher secondary level students from BTR, Assam, India, using Mahalanobis Distance (MD).

Purpose of the Investigation

The key objective of this investigation is to compare the dynamic characteristics of the attainment of different subjects such as English, Mathematics, Biology, Physics, and Chemistry summarized as a set between different dichotomous variables. The main purposes of this investigation are as follows.

1. To determine the difference in the attainment of 12th-grade tribal and non-tribal students who combine English, Mathematics, Biology, Physics, and Chemistry as one unit.
2. To determine the difference in the attainment of 12th-grade boy and girl students who combine English, Mathematics, Biology, Physics, and Chemistry as one unit.
3. To determine the difference in the attainment of 12th-grade rural and urban students who combine English, Mathematics, Biology, Physics, and Chemistry as one unit.
4. To determine the difference in the attainment of 12th-grade rural and urban boys who combine English, Mathematics, Biology, Physics, and Chemistry as one unit.
5. To determine the difference in the attainment of 12th-grade rural and urban girls who combine English, Mathematics, Biology, Physics, and Chemistry as one unit.

Research Hypotheses

To test the objectives, the following null hypotheses were explored for the current study.

H01: There is no substantial difference between tribal and non-tribal students in respect to attainment in group of subjects.

H02: There is no substantial difference between boy and girl students in respect to attainment in group of subjects.

H03: There is no substantial difference between rural and urban students in respect to

attainment in group of subjects.

H04: There is no substantial difference between rural and urban boy students in respect to attainment in group of subjects.

H05: There is no substantial difference between rural and urban girl students in respect to attainment in group of subjects.

The subjects in this group are English, Mathematics, Biology, Physics, and Chemistry at a higher secondary level.

Literature Review

The MD, as described by Xiang et al. (2008), is a computation betwixt two statistic sets in the volume explained by related attributes. It properly examined the distance by giving various weights or important elements to the attributes of data points because it takes into consideration differently not only in variances but also in correlations between characters. They claim that it can improve the performance of clustering or classification techniques such as the KNN classifier. If a proper MD metric is provided, such advantages can be exploited to perform particular tasks on a given data set.

According to Bedrick et al. (2000), the MD is the usual and accustomed calculation of distance between two samples while the perceived data is quantitative, and no other effective techniques are available to calculate the distance when the fact is a combination of quantitative and qualitative features. MD was also weighed up by Rubin (1976; 1979; 1980) and Cochran and Rubin (1973). Here, the variance-covariance matrix of variables is calculated and it is denoted by S. The square of MD between x1 and x2 is expressed by

$$M(x1, x2) = (x1-x2)T\ S\text{-}1\ (x1-x2).$$

where x1 and x2 are the covariates (actually mean column vectors for two sets of variables).

MD is employed in categorization problems (Mclachlan, 1999), where there are numerous groups and the investigation is concerned with group similarity. The purpose of the study could be to create a group of attributes that are alike to one another, possibly in a hierarchical structure. The issue of sample identification or discriminant probe, as well as the difficulties of medical diagnosis, are two more situations in which MD is relevant.

The MD and related ideas have been manifested to be useful in devising non-stationarity and dependence in time series and geographic data (Robinson, 2014). Marty et al. (2007) claimed that MD can furnish a scale of multivariable effect while two clusters of the study samples are estimated in two or more dependent variables.

Rosenbaum (2015) designed a study to detect the most likely hidden biases. Diedrichsen et al. (2016) used MD to investigate the scientific assertions of the media as well as the covariance of the specimen allocation. Muralidhar and Domingo-Ferrer (2019) examined the use of Mahalanobis distance for disclosure risk assessment. It is also utilized by Human Medicines Research and Development Support (2018) to evaluate the comparability of drug dissolution characteristics. It also applies to the re-identification of Cristani and Murino (2018). Bailey et al. (2019), in their study, presented a novel calculation of cross-sectional dependency strength in panel data, as well as asymptotic and finite sample performance, and a financial application. MD statistic is related to the suggested measure because it is founded on pair-wise cross-section correlations. Toma (2019) used MD to investigate the vibration and sound waveforms of a motor fan. Etherington (2019) calculates MD and shows how to appropriately create probability using a virtual ecology experiment, as well as discusses the implications of the inaccuracy for prior MD explored by others. Balakrishna et al. (2019) also applied MD in their research study for the time series model. Testing a parametric null hypothesis against an omnibus alternative is a popular use of MD and related divergence metrics, which contrasts nicely with the current technique. For dependent data, Cai et al. (2019) proposed a Lasso-based model selection process. Geographic or geodesic distances are used to model spatial dependence in this case, which has an intriguing link to MD. Lee et al. (2019) did a speculative contribution to improve bias correction. This has some excellent risk-measuring applications, but it also has an association with MD in terms of its popular usage

for finding outliers. Imani (2019) proposed methods for detecting targets based on differences using the MD.

Ahmed et al. (2020; 2021) employed MD to tackle the difference in dynamical nature-related mathematical attainment in the sphere of education. Sen and Pal (2020) looked at the VII and VIII grade students' performance on the Unit Test and Annual Examination in three distinct types of schools. For three different disciplines, Mathematics, Physical Science, and Life Science, they employed MD and discovered a significant difference in the majority of cases.

Mahato and Sen (2021) conducted a research on educational psychology. MD is used for two groups of higher secondary learners to determine the contrast between dependent variables. For different groupings of independent variables, there is no substantial variation in the progressive properties of the three dependent variables.

Gorain et al. (2021) used MD to make a comparison between the different psychological features of PG-level students. This study took into account five important aspects of personality, civic separation and internet dependence. Many sections, for this examination, have been developed, including boys and girls, streams like science and arts. The MD is used to measure internet dependence, civic separation, extraversion, agreeableness, openness, neuroticism, and conscientiousness. It was found that there were no remarkable variances in dynamical nature between the learners among boys and girls, and between learners studying the streams like science and arts. Ahmed et al. (2022) employed MD to make a relative research comparing the scholastic attainment in the subjects like Mathematics and English along with other disciplines of tenth-grade students in the region of BTR of Assam, India ,and an insignificant difference was observed in the dynamical character of five dependent variables for different groupings of independent variables.

Methodology

Population

All the students studying in class XII of senior secondary level schools, junior colleges, and colleges of government and private management in BTR of Assam constitude the population.

Sampling

The study is based on a sample of 1504 class XII students from tribal and non-tribal backgrounds, as well as boys and girls from rural and urban areas, who took English, Mathematics, Biology, Chemistry, and Physics subjects and passed in the year 2020, and 32 higher secondary schools, junior colleges, and colleges are chosen using a stratified random sampling technique. The sample distribution is represented in the figures 1

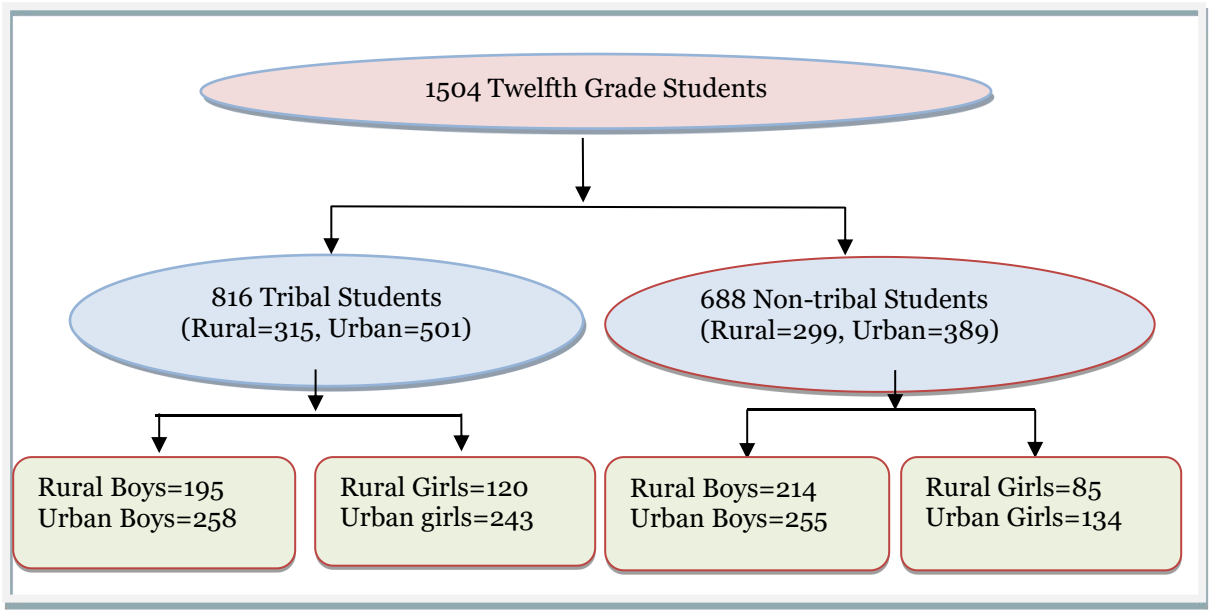

Figure 1. Frame of the sampling distribution.

Arrangement of Data

The MD is calculated using two data sets. The first data set represents tribal students' attainment in English, Mathematics, Biology, Physics, and Chemistry, whereas the second dataset represents non-tribal students' attainment in the same subjects. Other categories for comparison of attainment include boys and girls, students from rural and urban backgrounds, rural and urban boys, and rural and urban girls has been also investigated.

Statistics used

Mean, Standard Deviation (SD), covariance, and so on are computed for finding MD.

The mathematical formula for calculating Mahalanobis Distance is given by:

$\Delta^2 = (X-Y)^T \Sigma^{-1} (X-Y)$....................................(Equation-1)

Where X and Y represent the column vectors of means of the first and second groups of data and Σ represent the pooled covariance matrix of both groups.

Again, the pooled covariance matrix Σ is obtained from the following formula:

$$\Sigma = \frac{N_1\Sigma_1 + N_2\Sigma_2}{N} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (Equatio - 2)$$

Where N1 represents the sample size of the first group, N2 represents the sample size of the second group, N=N1+N2, and Σ_1 and Σ_2 are the covariance matrices for the corresponding groups.

Therefore, the MD = $[(X-Y)^T \Sigma^{-1} (X-Y)]^{\frac{1}{2}}$......................(Equation − 3)

It is to be noted that MD is a more effective and reliable metric to compare two groups of data since it uses covariance and pooled covariance. A single dimensionless number is represented in this distance.

Statistical Analysis

Data were analyzed in terms of Mean, Standard Deviation (SD), and Mahalanobis Distance (MD) method. The results have been presented in the following tables hypothesis-wise.

Hypothesis: H01. There is no substantial difference between tribal and non-tribal students in respect to attainment in group of subjects (Table 1).

Table 1. Mean and SD of tribal and non-tribal twelfth-grade students

Category	Statistics	English	Biology	Physics	Chemistry	Mathematics
Total	N=N1+N2	1504	1504	1504	1504	1504
	Mean	57.26	62.79	52.81	56.12	35.79
	SD	15.868	11.647	11.157	10.789	13.318
Tribal	N1	816	816	816	816	816
	Mean	56.82	62.70	52.15	55.10	35.54
	SD	15.083	11.159	10.485	9.759	12.847
Non-Tribal	N2	688	688	688	688	688
	Mean	57.78	62.90	53.60	57.33	36.09
	SD	16.748	12.207	11.865	11.790	13.859

Table 2. Covariance of tribal twelfth-grade students

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	227.496	81.485	69.510	76.556	51.062
Biology	81.485	124.525	63.453	68.967	67.115
Physics	69.510	63.453	109.940	69.046	78.141
Chemistry	76.556	68.967	69.046	95.232	78.476
Mathematics	51.062	67.115	78.141	78.476	165.056

Table 3. Covariance of non-tribal twelfth-grade students

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	280.499	119.580	99.654	108.489	62.828
Biology	119.580	149.021	80.998	94.133	60.095
Physics	99.654	80.998	140.771	103.258	85.170
Chemistry	108.489	94.133	103.258	139.006	96.099
Mathematics	62.828	60.095	85.170	96.099	192.076

The pooled covariance matrix for tribal and non-tribal students is calculated using the above-mentioned Tables 2 and 3 as well as equation-2, which is given by the following matrix

$$\begin{bmatrix} 251.7421 & 98.91144 & 83.29928 & 91.16365 & 56.44432 \\ 98.91144 & 135.7306 & 71.4789 & 80.47911 & 63.90372 \\ 83.29928 & 71.4789 & 124.0435 & 84.69617 & 81.35639 \\ 91.16365 & 80.47911 & 84.69617 & 115.2563 & 86.53759 \\ 56.44432 & 63.90372 & 81.35639 & 86.53759 & 177.4162 \end{bmatrix}$$

Interpretation: By using equation-3, it is found that when all five subjects are taken together, the MD between tribal and non-tribal students is 0.2791, which shows that there is no substantial difference between tribal and non-tribal students in respect to attainment in group of subjects and hence the null hypothesis H01 is accepted.

Hypothesis:H02. There is no substantial difference between boy and girl students in respect to attainment in the group of subjects (Table 4).

Table 4. Mean and SD of twelfth-grade boy and girl students

Category	Statistics	English	Biology	Physics	Chemistry	Mathematics
Total	N=N1+N2	1504	1504	1504	1504	1504
	Mean	57.26	62.79	52.81	56.12	35.79
	SD	15.868	11.647	11.157	10.789	13.318
Boys	N1	922	922	922	922	922
	Mean	54.59	61.81	52.58	55.47	36.28
	SD	15.566	11.513	11.362	10.705	13.651
Girls	N2	582	582	582	582	582
	Mean	61.48	64.34	53.18	57.15	35.01
	SD	15.433	11.699	10.824	10.852	12.746

Table 5. Covariance of twelfth-grade boy students

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	242.309	86.649	89.510	85.716	61.117
Biology	86.649	132.543	67.765	73.851	63.497
Physics	89.510	67.765	129.099	85.141	83.724
Chemistry	85.716	73.851	85.141	114.588	91.675
Mathematics	61.117	63.497	83.724	91.675	186.346

Table 6. Covariance of twelfth-grade girl students

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	238.174	107.774	71.821	94.072	54.764
Biology	107.774	136.877	76.628	88.669	66.603
Physics	71.821	76.628	117.155	85.445	78.585
Chemistry	94.072	88.669	85.445	117.765	80.496
Mathematics	54.764	66.603	78.585	80.496	162.456

The pooled covariance matrix for twelfth-grade boys and girls can be obtained by using the

equation-2 with the help of Tables 5 and 6 as indicated above, and the pooled covariance matrix is given by

$$\begin{bmatrix} 240.7089 & 94.8237 & 82.66492 & 88.94951 & 58.65859 \\ 94.8237 & 134.2201 & 71.1947 & 79.58509 & 64.69892 \\ 82.66492 & 71.1947 & 124.4771 & 85.25864 & 81.73537 \\ 88.94951 & 79.58509 & 85.25864 & 115.8174 & 87.34908 \\ 58.65859 & 64.69892 & 81.73537 & 87.34908 & 177.1013 \end{bmatrix}$$

Interpretation: By using equation-3, it is found that when all five disciplines as mentioned above are taken together, the MD between twelfth-grade boy students and twelfth-grade girl students is 0.5179. Since the MD is less than 1, there is no substantial difference between boy and girl students in respect to attainment in the group of subjects and therefore the null hypothesis H02 is accepted.

Hypothesis:H03. There is no substantial difference between rural and urban students in respect to attainment in group of subjects (Table 7).

Table 7. Mean and SD of twelfth-grade rural and urban students

Category	Statistics	English	Biology	Physics	Chemistry	Mathematics
Total	N=N1+N2	1504	1504	1504	1504	1504
	Mean	57.26	62.79	52.81	56.12	35.79
	SD	15.868	11.647	11.157	10.789	13.318
Rural	N1	614	614	614	614	614
	Mean	52.85	61.22	51.48	55.55	37.70
	SD	14.821	11.176	10.480	10.665	12.141
Urban	N2	890	890	890	890	890
	Mean	60.30	63.88	53.73	56.51	34.47
	SD	15.862	11.846	11.517	10.863	13.928

Table 8. Covariance of twelfth-grade rural students

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	219.665	81.691	83.127	90.546	75.287
Biology	81.691	124.899	58.378	77.690	55.920
Physics	83.127	58.378	109.832	74.957	66.375
Chemistry	90.546	77.690	74.957	113.736	78.332
Mathematics	75.287	55.920	66.375	78.332	147.402

Table 9. Covariance of twelfth-grade urban students

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	251.605	102.781	77.142	89.561	53.476
Biology	102.781	140.328	78.188	81.549	72.958
Physics	77.142	78.188	132.644	91.879	94.992
Chemistry	89.561	81.549	91.879	118.005	93.977
Mathematics	53.476	72.958	94.992	93.977	193.993

Again the pooled covariance matrix for rural and urban students is calculated by applying the afore-mentioned Tables 8 and 9 as well as equation-2, and the pooled covariance matrix is given by

$$\begin{bmatrix} 238.5657 & 94.17112 & 79.58534 & 89.96312 & 62.38022 \\ 94.17112 & 134.0292 & 70.10067 & 79.97358 & 66.00233 \\ 79.58534 & 70.10067 & 123.3311 & 84.97068 & 83.30926 \\ 89.96312 & 79.97358 & 84.97068 & 116.2622 & 87.59001 \\ 62.38022 & 66.00233 & 83.30926 & 87.59001 & 174.9725 \end{bmatrix}$$

Interpretation: In this case also, the MD between rural and urban students is 0.6780. So, there is no substantial difference between rural and urban students in respect to attainment in the group of subjects as the value of MD is less than 1 and therefore the null hypothesis H03 is accepted.

Hypothesis:H04. There is no substantial difference between rural and urban boy students in respect to attainment in the group of subjects (Table 10).

Table 10. Mean and SD of twelfth-grade rural and urban boys

Category	Statistics	English	Biology	Physics	Chemistry	Mathematics
Total	N=N1+N2	922	922	922	922	922
	Mean	54.59	61.81	52.58	55.47	36.28
	SD	15.566	11.513	11.362	10.705	13.651
Rural Boys	N1	409	409	409	409	409
	Mean	51.29	60.77	51.19	55.26	37.84
	SD	14.625	10.975	10.728	10.268	11.689
Urban Boys	N2	513	513	513	513	513
	Mean	57.23	62.65	53.69	55.64	35.04
	SD	15.804	11.869	11.736	11.048	14.928

Table 11. Covariance of twelfth-grade rural boys

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	213.882	72.400	85.382	85.776	70.287
Biology	72.400	120.457	50.945	65.660	46.049
Physics	85.382	50.945	115.085	70.615	60.506
Chemistry	85.776	65.660	70.615	105.423	68.579
Mathematics	70.287	46.049	60.506	68.579	136.641

Table 12. Covariance of twelfth-grade urban boys

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	249.755	93.224	86.370	84.837	61.311
Biology	93.224	140.870	79.216	80.207	79.855
Physics	86.370	79.216	137.738	96.462	105.498
Chemistry	84.837	80.207	96.462	122.052	110.728
Mathematics	61.311	79.855	105.498	110.728	222.846

As in above, the pooled covariance matrix (using Table 11 and 12, and equation-2) for rural and urban boy students is given by

$$\begin{bmatrix} 233.8417 & 83.98646 & 85.93172 & 85.25354 & 65.29276 \\ 83.98646 & 131.8148 & 66.67496 & 73.75394 & 64.85863 \\ 85.93172 & 66.67496 & 127.6891 & 84.99625 & 85.53951 \\ 85.25354 & 73.75394 & 84.99625 & 114.6754 & 92.03067 \\ 65.29276 & 64.85863 & 85.53951 & 92.03067 & 184.6054 \end{bmatrix}$$

Interpretation: The value of MD between students in the twelfth grade from rural and urban boys is 0.5825, which is obtained by using equation (3) when all five disciplines are considered together. Thus, there is no substantial difference between rural and urban boy students in respect to attainment in the group of subjects because the value of MD is less than 1 and therefore the null hypothesis H04 is accepted.

Hypothesis:H05. There is no substantial difference between rural and urban girls students in respect to attainment in the group of subjects (Table 13).

Table 13. Mean and SD of twelfth-grade rural and urban girls.

Category	Statistics	English	Biology	Physics	Chemistry	Mathematics
Total	N=N1+N2	582	582	582	582	582
	Mean	61.48	64.34	53.18	57.15	35.01
	SD	15.433	11.699	10.824	10.852	12.746
Rural Girls	N1	205	205	205	205	205
	Mean	55.98	62.12	52.05	56.13	37.41
	SD	14.751	11.540	9.968	11.420	13.020
Urban Girls	N2	377	377	377	377	377
	Mean	64.47	65.55	53.79	57.70	33.70
	SD	14.983	11.622	11.227	10.505	12.417

Table 14. Covariance of twelfth-grade rural girls

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	217.602	96.449	76.310	97.792	86.986
Biology	96.449	133.182	72.749	101.342	76.319
Physics	76.310	72.749	99.365	83.503	78.684
Chemistry	97.792	101.342	83.503	130.409	98.470
Mathematics	86.986	76.319	78.684	98.470	169.528

Table 15. Covariance of twelfth-grade urban girls

Covariance	English	Biology	Physics	Chemistry	Mathematics
English	224.489	103.893	64.374	87.599	48.569
Biology	103.893	135.072	76.831	80.124	66.018
Physics	64.374	76.831	126.056	85.766	81.016
Chemistry	87.599	80.124	85.766	110.349	73.016
Mathematics	48.569	66.018	81.016	73.016	154.179

The pooled covariance matrix between students in the twelfth grade from rural and urban girls can be obtained by using equation (2) with the help of Tables 14 and 15 as indicated above, and the pooled covariance matrix is given by

$$\begin{bmatrix} 222.0632 & 101.271 & 68.57826 & 91.18932 & 62.10076 \\ 101.271 & 134.4063 & 75.39318 & 87.59769 & 69.64636 \\ 68.57826 & 75.39318 & 116.6545 & 84.9689 & 80.19459 \\ 91.18932 & 87.59769 & 84.9689 & 117.4148 & 81.98176 \\ 62.10076 & 69.64636 & 80.19459 & 81.98176 & 159.5854 \end{bmatrix}$$

Interpretation: By using equation-3, it is found that when all five disciplines are taken together, the MD between rural and urban girls is 0.8006. So, there is no substantial difference between rural and urban girl students in respect to attainment in the group of subjects as the value of MD is less than 1 and hence the null hypothesis H05 is accepted.

Results

Based on the stats above, the values of MD for all five subjects are put together and reflected in table 16:

Table 16. MD for different pairs of variables when analyzed together.

Considering Variables	**MD**
Between tribal and non-tribal	0.2791
Between boys and girls	0.5179
Between rural and urban	0.6780
Between rural and urban boys	0.5825
Between rural and urban girls	0.8006

Table 16 shows that all MD are less than one. This means that in BTR, the students associated with tribal and non-tribal backgrounds, boys and girls, rural and urban, rural and urban boys, and rural and urban girls of higher secondary schools have a similar level of attainment in the dynamical character of a group of subjects. This is the beauty of MD that a single number represents the measure of difference for a group of variables (here attainment of different disciplines).

Discussions

It is stated previously that MD is a strong measure of the difference in terms of distance because it uses covariance matrices among several variables. A single variable like achievement in mathematics does not describe the dynamical character of the achievement of a learner. When more variables are taken into account, the result becomes more meaningful. Let us consider the present situation, here five variables in terms of achievement are considered and we get a more meaningful result about the dynamic nature of the achievement. In BTR, urban areas offer more study opportunities; yet, the performance of students in different social categories in terms of sex, location, and community is not considerably different. These results are similar to the results of Ahmed et al. (2022). It may be opined that if a better opportunity to study in a rural area is provided, the dynamical character may be changed for rural students. Actually, most of the rural students are coming from very poor families and their socio-cultural and economic environments are not up-to-date. As a result, a large number of talented students are lost due to poverty and the lack of favorable socio-cultural status.

Limitations

The investigation of the study is limited to the students of the BTR of Assam, India. The investigation is also limited to the achievements of the students in 12th grade, Moreover, this investigation is limited to five subjects viz. English, Mathematics, Biology, Physics, and Chemistry.

Conclusions

This study provides how to calculate MD and how to use it in combination with attainment. Researchers can use the distance to compare two sets of recorded responses. MD has a substantial lead in this area. When the five disciplines such as English, Mathematics, Biology, Physics, and Chemistry are studied as a set of academic disciplines in the current study, there is no substantial difference between tribal and non-tribal students together with sex and location in respect to attainment in the group of disciplines. The result of this test is a single number correctly reflects the dynamic nature of the group.

Recommendations

The following recommendations may be considered for further studies:

1. The study may be extended by incorporating achievement in more subjects.
2. MD may be applied for not only achievements for a group of subjects; it may be applied for computing the distance for psychological characteristics like intelligence, personality factors, interest, etc.
3. The study may be extended to the Assam state where several regions like BTR are situated. A comparison among different regions may be made.
4. An experimental design may be considered by creating an experimental group by providing necessary facilities for rural students to compare with urban students.

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