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Research Article

Decomposition Of Jump Graph Of Cycles Into Paths, Cycles, Complete Bipartite Graphs And Banner Graphs

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ARTICLE INFO | **ABSTRACT**

The Jump graph J(G) of a graph G is the graph whose vertices are edges of G and two vertices of J(G) are adjacent iff they are not adjacent in G. In this paper, we present necessary and sufficient condition for the decomposition of jump graph of cycles into various graphs such as paths, cycles, stars and complete bipartite graphs. Also, we give necessary and sufficient condition for the decomposition of [J(C_n)-e] into banner graphs and cycles.

Keywords and Phrases: Decomposition of graphs, Jump graph, Path, Cycle, Complete Bipartite graph, Banner graph.

1 Introduction

Let $G = (V, E)$ be a simple undirected graph without loops or multiple edges. A path on n vertices is denoted by P_n. A Cycle on vertices is denoted by C_n. The graph $K_{1,r}$ is called a star and is denoted by S_r. Let $\{x_n : x_1, x_2, \dots, x_r\}$ denotes a star S_r with x_n as its center. The undefined terms are used in the sense of Harary[3].

A decomposition of a graph G is a family of edge disjoint subgraphs $\{G_1, G_2, \dots, G_k\}$ Such that $E(G) = E(G_1) \cup E(G_2) \cup \dots \cup E(G_k)$. If each G_i is isomorphic to H for some subgraph H of G, then the decomposition is called a H- decomposition of G.

The Jump graph J(G) of a graph G is the graph whose vertices are edges of G and two vertices of J(G) are adjacent iff they are not adjacent in G. This concept was introduced by Chartrand in [1]. Also, complement of the line graph L(G) is the Jump graph J(G) of G.

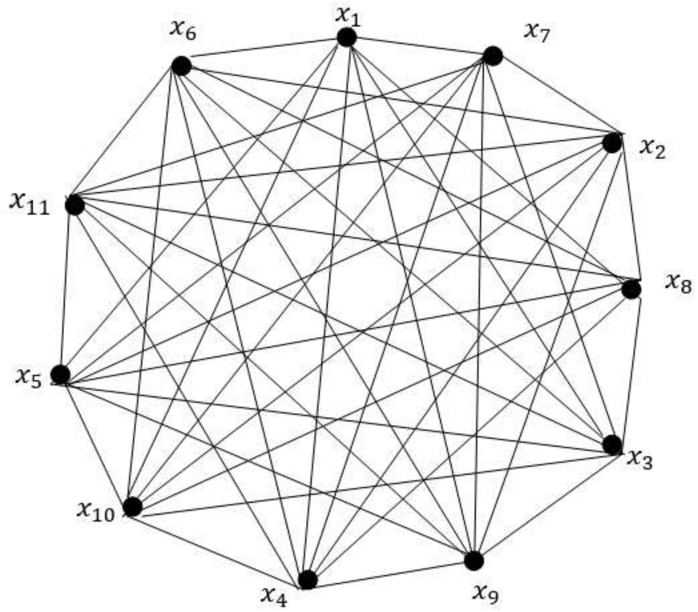
In[2], N. Gnanadhas and J. Paulraj Joseph discussed Continuous Monotonic Decomposition of Graphs. M. Jenisha and P. Chithra Devi inquired Decomposition of Jump Graph of Cycles in [4]. R. Vanitha, D. Vijayalakshmi and G. Mohanapriya dealed with P_4 Decomposition of Line and Middle Graph of some graphs in [6]. In this paper, we discuss about various decomposition of jump graph of cycles.

2 Main Result

Definition 2.1. Let $J(C_n)$ denote the jump graph of cycle C_n. $J(C_n)$ is connected iff $n \geq 5$. Here we consider only connected jump graph of cycles. The number of vertices of $J(C_n)$ is n because the number of edges of C_n is n. Let the edges of cycle C_n be labelled as $x_1, x_2, \dots, x_n$. So, the vertices of $J(C_n)$ are labelled as $x_1, x_2, \dots, x_n$.

The number of edges of $J(C_n)$ is $\frac{n^2-3n}{2}$.

Example 2.2. Jump graph of cycle $J(C_{11})$ is given below. It has 11 vertices and 44 edges.



Definition 2.3. n - Pan graph is the graph obtained by joining a cycle C_n to K_1 with a bridge. The 3-pan graph is often known as the Paw graph and the 4-pan graph as the banner graph.
In this paper, we denote the banner graph as H. It is given below.

Denote this graph as $H = \{x_1\, ;\, x_2\, x_3\, x_4\, x_5\, x_2\}$. Here x_1 is a pendent vertex, x_2 has degree 3 and all other vertices have degree 2.

Theorem 2.4. Let $n \geq 6$ be an even positive integer with $q = \frac{(n-2)(n-4)}{8}$, $r = \frac{(n-2)}{2}$ and $t = n - 3$. Then $J(C_n)$ is decomposed into q copies of P_5, one copy of S_r and one copy of S_t iff $4q + r + t = \frac{n^2-3n}{2}$.

Proof. Let $n \geq 6$ be an even positive integer with $q = \frac{(n-2)(n-4)}{8}$, $r = \frac{(n-2)}{2}$ and $t = n - 3$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
(Necessity) Suppose that there exists a decomposition of $J(C_n)$ into q copies of P_5, one copy of S_r and one copy of S_t.
Since $|E(J(C_n))| = \frac{n^2-3n}{2}$, we have $4q + r + t = \frac{n^2-3n}{2}$.

(Sufficiency) Suppose $4q + r + t = \frac{n^2-3n}{2}$.
For $i = 1, 3, 5, \dots, n-5$, $\{x_k\, x_i\, x_{k+1}\, x_{i+1}\, x_{k+2} \,/\, K = i+2, i+4, \dots, n-3\}$ forms P_5 in $J(C_n)$.
From this, we get $\frac{(n-2)(n-4)}{8} = q$ copies of P_5.
For $i = n-1$, $\{x_i : x_1, x_3, x_5, \dots, x_{n-3}\}$ forms a star S_r with x_i as its center where $r = \frac{(n-2)}{2}$.
Also, for $i = n$, $\{x_i : x_2, x_3, x_4, \dots, x_{n-2}\}$ forms a star S_t with x_i as its center where $t = n - 3$.
Hence $E\left(J(C_n)\right) = \underbrace{E(P_5) \cup E(P_5) \cup \dots \cup E(P_5)}_{q\ times} \cup\ E(S_r) \cup E(S_t)$.
Thus $J(C_n)$ is decomposed into q copies of P_5, one copy of S_r and one copy of S_t.

Theorem 2.5. Let $n \geq 5$ be an odd positive integer with $q = \frac{(n-1)(n-3)}{8}$ and $r = \frac{(n-3)}{2}$. Then *J(Cn)* is decomposed into q copies of P_5 and one copy of S_r iff $4q + r = \frac{n^2-3n}{2}$.

Proof. Let $n \geq 5$ be an odd positive integer with $q = \frac{(n-1)(n-3)}{8}$ and $r = \frac{(n-3)}{2}$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
(Necessity) Suppose that there exists a decomposition of $J(C_n)$ into q copies of P_5 and one copy of S_r.
Since $|E(J(C_n))| = \frac{n^2-3n}{2}$, we have $4q + r = \frac{n^2-3n}{2}$.
(Sufficiency) Suppose $4q + r = \frac{n^2-3n}{2}$.
For $i = 1, 3, 5, \dots, n-4$, $\{x_k\, x_i\, x_{k+1}\, x_{i+1}\, x_{k+2} \,/\, K = i+2, i+4, \dots, n-2\}$ forms P_5 in $J(C_n)$.
Here we obtain $\frac{(n-1)(n-3)}{8} = q$ copies of P_5.
For $i = n$, $\{x_i :\, x_3, x_5, \dots, x_{n-2}\}$ forms a star S_r with x_i as its center where $r = \frac{(n-3)}{2}$.
Hence $E(J(C_n)) = \underbrace{E(P_5) \cup E(P_5) \cup \dots \cup E(P_5)}_{q\ times} \cup\ E(S_r)$.
Thus, $J(C_n)$ is decomposed into q copies of P_5 and one copy of S_r.

Theorem 2.6. [4] Let n be an even positive integer with $p = \frac{n}{2}$ and $q = \frac{n^2-6n}{8}$. Then there exists a decomposition of $J(C_n)$ into p copies of P_4 and q copies of C_4 iff $n \geq 6$ and $3p + 4q = \frac{n^2-3n}{2}$.

Theorem 2.7. Let $n \geq 5$ be an odd positive integer with $q = \frac{(n-3)}{2}$, $r = \frac{(n-3)(n-5)}{8}$ and $t = n-3$. Then $J(C_n)$ is decomposed into $\{qP_4, rC_4, S_t\}$ iff $3q + 4r + t = \frac{n^2-3n}{2}$.

Proof. Let $n \geq 5$ be an odd positive integer with $q = \frac{(n-3)}{2}$, $r = \frac{(n-3)(n-5)}{8}$ and $t = n-3$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
(Necessity) Suppose $J(C_n)$ is decomposed into $\{qP_4, rC_4, S_t\}$.
Since $|E(J(C_n))| = \frac{n^2-3n}{2}$, we get $3q + 4r + t = \frac{n^2-3n}{2}$.
(Sufficiency) Suppose $3q + 4r + t = \frac{n^2-3n}{2}$.
Clearly, for $i = 1, 3, 5, \dots, n-4$, $\{x_{i+2}\, x_i\, x_{i+3}\, x_{i+1}\}$ forms P_4 in $J(C_n)$.
From this, we get $\frac{(n-3)}{2} = q$ copies of P_4.
For $i = 1, 3, 5, \dots, n-6$, $\{x_i\, x_k x_{i+1}\, x_{k+1}\, x_i / k = i+4, i+6, \dots, n-2\}$ forms C_4 in $J(C_n)$.
This gives $\frac{(n-3)(n-5)}{8} = r$ copies of C_4
Also, $\{x_n : x_2, x_3, \dots, x_{n-2}\}$ forms a star S_t with x_n as its center and $t = n-3$.
Therefore $E(J(C_n)) = \underbrace{E(P_4) \cup E(P_4) \cup \dots \cup E(P_4)}_{q\ times} \cup \underbrace{E(C_4) \cup E(C_4) \cup \dots \cup E(C_4)}_{r\ times} \cup E(S_t)$.
Hence $J(C_n)$ is decomposed into $\{qP_4, rC_4, S_t\}$.

Theorem 2.8. Let $n \geq 5$ be an odd positive integer with $q = \frac{(n-5)(n+1)}{8}$ and $m = \frac{(n+1)}{2}$. Then $J(C_n)$ is decomposed into $\{qC_4, P_m, P_4\}$ iff $4q + m = \frac{n^2-3n}{2} - 2$.

Proof. Let $n \geq 5$ be an odd positive integer with $q = \frac{(n-5)(n+1)}{8}$ and $m = \frac{(n+1)}{2}$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
(Necessity) Suppose $J(C_n)$ is decomposed into $\{qC_4, P_m, P_4\}$.
Since, $|E(J(C_n))| = \frac{n^2-3n}{2}$, we have $4q + m = \frac{n^2-3n}{2} - 2$.
(Sufficiency) Suppose $4q + m = \frac{n^2-3n}{2} - 2$.
Clearly, $\{x_i\, x_1\, x_{i+1}\, x_2 x_i / i = 4, 6, \dots, n-3\}$ forms C_4 in $J(C_n)$.
This gives $\frac{n-5}{2}$ copies of C_4.
For $i = 3, 5, 7 \dots, n-4$, $\{x_k\, x_i x_{k+1}\, x_{i+1}\, x_k / k = i+3, i+5, \dots, n-1\}$ forms C_4 in $J(C_n)$.
This gives $\frac{(n-3)(n-5)}{8}$ copies of C_4.
Totally, we get $\frac{(n-5)(n+1)}{8} = q$ copies of C_4 in $J(C_n)$.
Clearly, $x_1\, x_3 x_5 \dots x_n$ forms P_m in $J(C_n)$ where $m = \frac{(n+1)}{2}$.
Next, $x_1\, x_{n-1}\, x_2 x_n$ form one copy of P_4.

Therefore $E(J(C_n)) = \underbrace{E(C_4) \cup E(C_4) \cup \dots \cup E(C_4)}_{q\ times} \cup E(P_m) \cup E(P_4)$.

Hence *J(Cn)* is decomposed into {qC_4, P_m , P_4}.

Theorem 2.9. Let $n \geq 6$ be an even positive integer with $q = \frac{(n-2)(n-4)}{8}$, $m = \frac{n}{2}$ and $t = n-3$. Then *J(Cn)* is decomposed into {qC_4, P_m, S_t} iff $4q + m + t = \frac{n^2-3n}{2} + 1$.

Proof. Let $n \geq 6$ be an even positive integer with $q = \frac{(n-2)(n-4)}{8}$, $m = \frac{n}{2}$ and $t = n-3$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
(Necessity) Suppose *J(Cn)* is decomposed into {qC_4, P_m, S_t}.
Since, $|E(J(C_n))| = \frac{n^2-3n}{2}$, we have $4q + m + t = \frac{n^2-3n}{2} + 1$.
(Sufficiency) Suppose$4q + m + t = \frac{n^2-3n}{2} + 1$.
Clearly, $\{x_i\, x_1\, x_{i+1}\, x_2 x_i / i = 4, 6, \dots, n-2\}$ forms C_4 in *J(Cn)*.
This gives $\frac{n-4}{2}$ copies of C_4.
For $i = 1, 3, 5, \dots, n-5$, $\{x_k\, x_i x_{k+1}\, x_{i+1}\, x_k / k = i+3, i+5, \dots, n-2\}$ forms C_4 in *J(Cn)*.
This gives $\frac{(n-4)(n-6)}{8}$ copies of C_4.
Totally, we get $\frac{(n-2)(n-4)}{8} = q$ copies of C_4 in *J(Cn)*.
Clearly, $x_1\, x_3\, x_5 \dots x_{n-1}$ forms P_m in *J(Cn)* where $m = \frac{n}{2}$.
Next, $\{x_n : x_2, x_3, \dots, x_{n-2}\}$ forms one copy of S_t.
Therefore $E(J(C_n)) = \underbrace{E(C_4) \cup E(C_4) \cup \dots \cup E(C_4)}_{q\ times} \cup E(P_m) \cup E(S_t)$.
Hence *J(Cn)* is decomposed into {qC_4, P_m, S_t}.

Theorem 2.10. [4] Let n be an odd positive integer with $p = \frac{n-3}{2}$ and $q = \frac{n-5}{2}$. Then there exists a decomposition of *J(Cn)* into p copies of C_5 and q complete bipartite graphs of the form $K_{2,2l}; l = 1, 2, \dots, \frac{n-5}{2}$ iff $n \geq 5$ and $5p + 2q(q+1) = \frac{n^2-3n}{2}$.

Theorem 2.11. Let $n \geq 6$ be an even positive integer with $q = \frac{n-4}{2}$ and $t = \frac{n-6}{2}$. Then $[J(C_n) - e]$ is decomposed into $\left\{qC_5, S_3, t\ copies\ of\ K_{2,2r+1}, r = 1, 2, \dots, \frac{n-6}{2}\right\}$ iff $5q + 2t(t+2) = \frac{n^2-3n}{2} - 4$.

Proof. Let $n \geq 6$ be an even positive integer with $q = \frac{n-4}{2}$ and $t = \frac{n-6}{2}$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
Consider $[J(C_n) - e]$ where $e = x_1 x_{n-1}$.
(Necessity) Suppose $[J(C_n) - e]$ is decomposed into
$\left\{qC_5, S_3, t\ copies\ of\ K_{2,2r+1}, r = 1, 2, \dots, \frac{n-6}{2}\right\}$.
Since $|E(J(C_n))| = \frac{n^2-3n}{2}$, we get $5q + 2t(t+2) = \frac{n^2-3n}{2} - 4$.
(Sufficiency) Assume $5q + 2t(t+2) = \frac{n^2-3n}{2} - 4$.
For $i = 3, 5, \dots, n-3$, $\{x_1\, x_i\, x_{i+2}\, x_2 x_{i+1} x_1\}$ forms C_5 in *J(Cn)*.
This gives $\frac{n-4}{2} = q$ copies of C_5.
For $i = 3, 5, \dots, n-5$, x_i and x_{i+1} are non-adjacent and they are adjacent with $x_{i+3}, x_{i+4}, \dots, x_n$.
This gives $\frac{n-6}{2} = t$ complete bipartite graphs of the form $K_{2,2r+1}$, $r = 1, 2, \dots, \frac{n-6}{2}$.
Next, $\{x_n : x_2, x_{n-3}, x_{n-2}\}$ forms S_3 in *J(Cn)*.

Hence $E[J(C_n) - e] = \underbrace{E(C_5) \cup E(C_5) \cup \dots \cup E(C_5)}_{q\ times} \cup E(S_3) \cup E(K_{2,3}) \cup E(K_{2,5}) \cup \dots$
$\cup E(K_{2,n-5})$.
Therefore $[J(C_n) - e]$ is decomposed into $\left\{qC_5, S_3, t\ copies\ of\ K_{2,2r+1}, r = 1, 2, \dots, \frac{n-6}{2}\right\}$.

Theorem 2.12. Let $n \geq 6$ be an even positive integer with $m = \frac{n}{2}$, $l = n-4$ and $t = \frac{n-6}{2}$. Then $[J(C_n) - e]$ is decomposed into $\left\{P_m, S_2, K_{2,l}, t\ copies\ of\ K_{2,2r+1}, r = 1, 2, \dots, \frac{n-6}{2}\right\}$ iff $m + 2l + 2t(t+2) = \frac{n^2-3n}{2} - 2$.

Proof. Let $n \geq 6$ be an even positive integer with $= \frac{n}{2}$, $l = n-4$ and $t = \frac{n-6}{2}$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
Consider $[J(C_n) - e]$ where $e = x_2x_n$.
(Necessity) Suppose $[J(C_n) - e]$ is decomposed into
$\left\{P_m, S_2, K_{2,l}, t\ copies\ of\ K_{2,2r+1}, r = 1, 2, \dots, \frac{n-6}{2}\right\}$.
Since $|E(J(C_n))| = \frac{n^2-3n}{2}$, we have $m + 2l + 2t(t+2) = \frac{n^2-3n}{2} - 2$.
(Sufficiency) Suppose $m + 2l + 2t(t+2) = \frac{n^2-3n}{2} - 2$.
$x_1\, x_3\, \dots\, x_{n-1}$ forms P_m where $m = \frac{n}{2}$.
x_1 and x_2 are non-adjacent and they are adjacent with $x_4, x_5, \dots, x_{n-1}$. This gives $K_{2,l}$ where $l = n-4$.
For $i = 3, 5, \dots, n-5$, x_i and x_{i+1} are non-adjacent and they are adjacent with $x_{i+3}, x_{i+4}, \dots, x_n$.
This gives $\frac{n-6}{2} = t$ complete bipartite graphs of the form $K_{2,2r+1}, r = 1, 2, \dots, \frac{n-6}{2}$.
Clearly, $\{x_n : x_{n-3}, x_{n-2}\}$ forms star S_2 with x_n as its center.
Hence $E[J(C_n) - e] = E(P_m) \cup E(S_2) \cup E(K_{2,l}) \cup E(K_{2,3}) \cup E(K_{2,5}) \cup \dots \cup E(K_{2,n-5})$.
Therefore $[J(C_n) - e]$ is decomposed into $\left\{P_m, S_2, K_{2,l}, t\ copies\ of\ K_{2,2r+1}, r = 1, 2, \dots, \frac{n-6}{2}\right\}$ iff $m + 2l + 2t(t+2) = \frac{n^2-3n}{2} - 2$.

Theorem 2.13. Let $n \geq 5$ be an odd positive integer with $m = \frac{n+1}{2}$, $l = n-4$ and $t = \frac{n-5}{2}$. Then $[J(C_n) - e]$ is decomposed into $\left\{P_m, K_{2,l}, t\ copies\ of\ K_{2,2r}, r = 1, 2, \dots, \frac{n-5}{2}\right\}$ iff $m + 2l + 2t(t+1) = \frac{n^2-3n}{2}$.
Proof. Let $n \geq 5$ be an even positive integer with $m = \frac{n+1}{2}$, $l = n-4$ and $t = \frac{n-5}{2}$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
Consider $[J(C_n) - e]$ where $e = x_2x_n$.
(Necessity) Suppose $[J(C_n) - e]$ is decomposed into
$\left\{P_m, K_{2,l}, t\ copies\ of\ K_{2,2r}, r = 1, 2, \dots, \frac{n-5}{2}\right\}$.
Since $|E(J(C_n))| = \frac{n^2-3n}{2}$, we have $m + 2l + 2t(t+1) = \frac{n^2-3n}{2}$.
(Sufficiency) Suppose $m + 2l + 2t(t+1) = \frac{n^2-3n}{2}$.
$x_1x_3 \dots x_n$ forms P_m where $m = \frac{n+1}{2}$.
x_1 and x_2 are non-adjacent and they are adjacent with $x_4, x_5, \dots, x_{n-1}$. This gives $K_{2,l}$ where $l = n-4$.
For $i = 3, 5, \dots, n-4$, x_i and x_{i+1} are non-adjacent and they are adjacent with $x_{i+3}, x_{i+4}, \dots, x_n$.
This gives $\frac{n-5}{2} = t$ complete bipartite graphs of the form $K_{2,2r}$, $r = 1, 2, \dots, \frac{n-5}{2}$.
Hence $E[J(C_n) - e] = E(P_m) \cup E(K_{2,l}) \cup E(K_{2,2}) \cup E(K_{2,4}) \cup \dots \cup E(K_{2,n-5})$.
Therefore $[J(C_n) - e]$ is decomposed into $\left\{P_m, K_{2,l}, t\ copies\ of\ K_{2,2r}, r = 1, 2, \dots, \frac{n-5}{2}\right\}$.

Theorem 2.14. Let $n \geq 6$ be an even positive integer with $q = \frac{n-4}{2}$, $r = \frac{(n-4)(n-6)}{8}$ and $t = n-3$. Then $[J(C_n) - e]$ is decomposed into $\{qH, rC_4, S_t\}$ iff $5q + 4r + t = \frac{n^2-3n}{2} - 1$.
Proof. Let $n \geq 6$ be an even positive integer with $q = \frac{n-4}{2}$, $r = \frac{(n-4)(n-6)}{8}$ and $t = n-3$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
Consider $[J(C_n) - e]$ where $e = x_{n-3}x_{n-1}$.
(Necessity) Suppose $[J(C_n) - e]$ is decomposed into $\{qH, rC_4, S_t\}$.
Since $|E(J(C_n))| = \frac{n^2-3n}{2}$, $5q + 4r + t = \frac{n^2-3n}{2} - 1$.
(Sufficiency) Suppose $5q + 4r + t = \frac{n^2-3n}{2} - 1$.
For $i = 3, 5, 7, \dots, n-3$, $\{x_i ; x_{i-2}\, x_{i+1}\, x_{i-1} x_{i+2} x_{i-2}\}$ forms H in $J(C_n)$.
This gives $\frac{n-4}{2} = q$ copies of H.
Also, for $i = 1, 3, 5, \dots, n-7$, $\{x_i\, x_k x_{i+1}\, x_{k+1}\, x_i / \ k = i+5, i+7, \dots, n-2\}$ forms C_4 in $J(C_n)$.
From this, we get $\frac{(n-4)(n-6)}{8} = r$ copies of C_4.
Next, $\{x_n : x_2, x_3, \dots, x_{n-2}\}$ forms a star S_t where $t = n-3$ with x_n as its center.
Hence $E[J(C_n) - e] = \underbrace{E(H) \cup E(H) \cup \dots \cup E(H)}_{q\ times} \cup \underbrace{E(C_4) \cup E(C_4) \cup \dots \cup E(C_4)}_{r\ times} \cup E(S_t)$
Therefore $[J(C_n) - e]$ is decomposed into $\{qH, rC_4, S_t\}$.

Theorem 2.15. Let $n > 5$ be an odd positive integer with q = $\frac{n-3}{2}$, $r = \frac{(n-1)(n-7)}{8}$ and $t = n - 3$. Then $[J(C_n) - e]$ is decomposed into $\{qH, rC_4, P_4\}$ iff $5q + 4r = \frac{n^2-3n}{2} - 4$.

Proof. Let $n > 5$ be an odd positive integer with $q = \frac{n-3}{2}$, $r = \frac{(n-1)(n-7)}{8}$ and $t = n - 3$.
Let $V(J(C_n)) = \{x_1, x_2, \dots, x_n\}$.
Consider $[J(C_n) - e]$ where $e = x_{n-2}x_n$.
(Necessity) Suppose $[J(C_n) - e]$ is decomposed into $\{qH, rC_4, P_4\}$.
Since $|E(J(C_n))| = \frac{n^2-3n}{2}$, $5q + 4r = \frac{n^2-3n}{2} - 4$.
(Sufficiency) Suppose $5q + 4r = \frac{n^2-3n}{2} - 4$.
For $i = 3, 5, 7, \dots, n-2$, $\{x_i\,; x_{i-2}\, x_{i+1}\, x_{i-1} x_{i+2} x_{i-2}\}$ forms H in $J(C_n)$.
This gives $\frac{n-3}{2} = q$ copies of H.
$\{x_1 x_k x_2\, x_{k+1}\, x_1 / \ k = 6, 8, \dots, n-3\}$ forms C_4 in $J(C_n)$.
This gives $\frac{n-7}{2}$ copies of C_4.
Also, for $i = 3, 5, \dots, n-6$, $\{x_i\, x_k x_{i+1}\, x_{k+1}\, x_i / \ k = i+5, i+7, \dots, n-1\}$ gives C_4 in $J(C_n)$.
This gives $\frac{(n-5)(n-7)}{8}$ copies of C_4.
Totally, we get $r = \frac{(n-1)(n-7)}{8}$ copies of C_4
Next, $\{x_1\, x_{n-1}\, x_2\, x_n\}$ forms P_4 in $J(C_n)$.
Hence $E[J(C_n) - e] = \underbrace{E(H) \cup E(H) \cup \dots \cup E(H)}_{q\ times} \cup \underbrace{E(C_4) \cup E(C_4) \cup \dots \cup E(C_4)}_{r\ times} \cup E(P_4)$
Therefore $[J(C_n) - e]$ is decomposed into $\{qH, rC_4, P_4\}$.

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