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Research Article

Optimal Analysis Of Bulk Arrival Queueing System With Server Startup, Breakdown And Timeout

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ABSTRACT

The Probability Generating Functions (PGFs) are used for the proposed batch arrival N-policy vacation two phase service queuing model with server startup, timeout and breakdowns. In this paper, we obtained system state equations and its probability distribution to get operating characteristics, optimum threshold and cost. Proposed an algorithm to obtain Preliminary numerical experiments reported to show the performance of the characteristics and total cost of the proposed queuing Model.

Key words: *N-policy, two-phase, queuing model, timeout and breakdown.*

1 INTRODUCTION

This paper analyzes the optimal strategy analysis has been carried out for an infinite capacity two-phase $M^x/M/1$ vacation queuing system with an N-policy, server start-up, time-out and breakdown. In the proposed model, the arrivals in the system are considered to be in the batch mode. The waiting units in the system will receive the service in batch mode in first phase and service in individual mode in second phase. The server does the service for the waiting units exhaustively without gating. After completed of the first phase first phase service, every one of this batch receives second phase second phase service. Assume that the server is considered to be breakdown at any point of time in second phase, it can be instantly repaired and resumes service immediately. After completing second phase service the server returns to the first phase to serve if any unit in the system. If there is no unit waiting in first phase, the server waits for a fixed time for units before vacation which is called server Time-out. If no unit found in the system even after timeout period is completed serve takes vacation. The server needs random start-up time after waitingunits or units reaches a threshold value before providing service to the waiting customer in the system.
The proposed system assumptions are as follows:
Arriving units are assumed to follow Compound Poisson Process with parameter λ. Units will get service in FIFO. The service is provided in two phases (batch and second phase service). First phase-first phase service and second phase-second phase services are assumed to be exponentially distributed with mean service rates $1/\beta$ and $1/\mu$ respectively. Assume that the server may fail with a failure rate α is assumed to be exponentially distributed in the first phase service, and it can be instantly repaired with a repair rate γ, which is exponentially distributed. If no waiting unit find in the system, the server waits for some fixed time C, called server time-out. Vacation period is assumed exponentially distributed. The server returns from the vacation only after waiting units reaches to size N (≥1) and then the server immediately begins a random start-up. Start-up times or pre-service times are assumed tofollow an exponential distribution with mean $1/\theta$. After this startup time the server begins service to all units waiting units in the system.

These models can be observed in many real life service applications like machine production, maintenancein automobile industry, computer network models and inventory systems etc. Some important literature is presented as follows: Levy and Yechiali (1975) [5], Tony T. Lee (1982) [9] were studiedqueuing system with idle time or vacation times of server with random length and also generalization of queuing system of similar nature. Jianjun Li and Liwei Lu (2017) [4] studied performance analysis of a complex queuing system with vacations in random environment. The concept of N-policy was introducedby Yadin and Naor (1963) [14]. Firstly, Baker (1973) [2] was proposed N-policy M/M/1 queuing system with exponential startups. Hyo-Seong Lee and Mandyam M. Srinivasan (1989) [3] studied the $M^x/G/1$ queuing system control policies. Wei Li (1997) [13] studied reliability analysis of the M/G/1 queueing system with server breakdowns and vacation. V. Vasanta Kumar et al. (2010) [12], V.Vasanta Kumar and T.Srinivasa Rao (2013) [11] studied Two-phase N-policy $M^x/M/1$ queuing systems with server startup, breakdown and repairs. V.N.Rama Devi et al. (2019) [10] studied the M/M/1 queuing system with two-phase N-policy, server breakdowns of customer impatient behavior. Oliver C.Ibe (2007) [6] and Ramesh Kumar, E. and Praby Loit, Y. (2016) [7] were studied vacation queuing system with server timeout. K.Satish Kumar et al. (2017) [8], A.Ankamma Rao et al. (2019) [1] studied the N-policy M/M/1 vacation queuing systemswith server startupand timeout. V.N. Rama Devi, et.al. (2020) studied theM/G/1 queue with vacation, two cases of repair facilities and server timeout. A.P.Panta et.al., (2021) reviewed vacation queueing modelsin differentframeworks. Kalyanaraman. R. and Sundaramoorthy *A. (*2022), studied A Multi Server MarkovianWorking Vacation queue With Breakdown, N-policy and with Server State Dependent Rates.

The findings of the paper organized as follows: The system size distribution by using PGFs is derived (section 2). Obtained the system characteristics (section 3) and expected system length, total cost function per unit time is considered to obtain the optimal threshold policy (N*) (see section 4). Considered the specific batch size distribution and illustrated the sensitivity analysis for optimal system length and cost (see section 5 and 6) has been carried out to examine the effect of different parameters in the system.

2. STAEDY STATE ANALYSIS

Steady state probabilities for the proposed system are defined as follows:
$P_{(0,1,2,3,4,5),i,j}$: Probability of the server is in different states like 0-vacation, 1-startup, 3-first phase service, 4-breakdoen, 5-second phase service and there are i number of units in first phase and j number of units in second phase.
$P_{0,i,0}$ (for i=0,1,2,3,…,N-1) - probability of the server is in vacation
$P_{1,i,0}$ (for i=N,N+1,N+2,…) - probability of the server is in Startup
$P_{2,i,0}$ (for i=0,1,2,3,…) - probability of the server is in Timeout
$P_{3,i,0}$ (for i=1,2,3,…) - probability of the server is in First phase service
$P_{4,i,j}$ (for i=0,1,2,3,… and j=1,2,3,…) - probability of the server is in second phase service
$P_{5,i,j}$ (for i=0,1,2,3,… and j=1,2,3,…) - probability of the server is in Breakdown

Steady state equations are derived as given below:

$$\lambda P_{0,0,0} = CP_{2,0,0} \quad (1)$$

$$\lambda P_{0,i,0} = \lambda \sum_{k=1}^{i} a_k P_{0,i-k,0} \,; \quad 1 \le i \le N-1 \quad (2)$$

$$(\lambda+\theta)P_{1,N,0} = \lambda \sum_{k=1}^{N} a_k P_{0,N-k,0} \quad (3)$$

$$(\lambda+\theta)P_{1,i,0} = \lambda \sum_{k=1}^{i-N} a_k P_{1,i-k,0} + \lambda \sum_{k=i-(N-1)}^{i} a_k P_{0,i-k,0} \,; \quad i \ge N+1 \quad (4)$$

$$(\lambda+C)P_{2,0,0} = \mu P_{4,0,1} \quad (5)$$

$$(\lambda+\beta)P_{3,1,0} = \lambda a_1 P_{2,0,0} + \mu P_{4,1,1} \quad (6)$$

$$(\lambda+\beta)P_{3,i,0} = \lambda \sum_{k=1}^{i} a_k P_{3,i-k,0} + \mu P_{4,i,1} \,; \quad 2 \le i \le N-1 \quad (7)$$

$$(\lambda+\beta)P_{3,i,0} = \lambda \sum_{k=1}^{i} a_k P_{3,i-k,0} + \mu P_{4,i,1} + \theta P_{1,i,0} \,; \quad i \ge Nk \quad (8)$$

$$(\lambda+\alpha+\mu)P_{4,0,j} = \mu P_{4,0,j+1} + \beta P_{3,j,0} + \gamma P_{5,0,j} \,; \quad j \ge (9)$$

$$(\lambda+\alpha+\mu)P_{4,i,j} = \mu P_{4,i,j+1} + \lambda \sum_{k=1}^{i} a_k P_{4,i-k,j} + \gamma P_{5,i,j} \,; \quad i,j \ge 1 \quad (10)$$

$$(\lambda+\gamma)P_{5,0,j} = \alpha P_{4,0,j} \,; \quad j \ge 1 \quad (11)$$

$$(\lambda+\gamma)P_{5,i,j} = \alpha P_{4,i,j} + \lambda \sum_{k=1}^{i} a_k P_{5,i-k,j} \,; \quad i,j \ge 1 \quad (12)$$

To solve the above steady state equations, we use the following Probability Generating Functions for each individual state separately:

$$G_0(z) = \sum_{i=0}^{N-1} P_{0,i,0} z^i \,, \quad G_1(z) = \sum_{i=N}^{\infty} P_{1,i,0} z^i \,, \quad G_2(z) = P_{2,0,0} \,, \quad G_3(z) = \sum_{i=1}^{\infty} P_{3,i,0} z^i \,,$$

$$G_4(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} P_{4,i,j} z^i y^j \,, \quad G_5(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} P_{5,i,j} z^i y^j \,, \quad R_i(z) = \sum_{i=0}^{\infty} P_{4,i,j} z^i \,,$$

$$S_j(z) = \sum_{i=0}^{\infty} P_{5,i,j} z^i \,, \; A(z) = \sum_{i=0}^{\infty} a_i z^i \,, \; |z|,|y| \le 1 \text{ and for } y_i = \sum_{k=1}^{i} a_k y_{i-k} \,, \; Y_N(1) = \sum_{i=0}^{N-1} y_i$$

G(z,y) - probability generating function for the entire system is given by
$G(z,y) = G_0(z) + G_1(z) + G_2(z) + G_3(z) + G_4(z,y) + G_5(z,y)$
The normalizing condition -

$$G(1,1) = G_0(1) + G_1(1) + G_2(1) + G_3(1) + G_4(1,1) + G_5(1,1) = 1 \quad (13)$$

After solving the equations (1) – (12), we get

$$G_0(1) = Y_N(1) P_{0,0,0} \quad (14)$$

$$G_1(1) = \frac{\lambda}{\theta}P_{0,0,0} \quad (15)$$

$$G_2(1) = \frac{\lambda}{C}P_{0,0,0} \quad (16)$$

$$G_3(1) = \frac{\lambda^2(a_1-1)}{\beta C}P_{0,0,0} + \frac{\mu}{\beta}R_1(1) \quad (17)$$

$$G_4(1,1) = \frac{\gamma\lambda}{[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]}\left[\frac{\lambda(2a_1-1)}{C} + A'(1)\left(\frac{\lambda^2(a_1-1)}{\beta C} + \frac{\lambda}{\theta} + Y_N(1)\right)\right]P_{0,0,0} + \frac{\lambda\gamma\mu A'(1)R_1(1)}{\beta[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]} \quad (18)$$

$$G_5(1,1) = \frac{\alpha}{\gamma}G_4(1,1) \quad (19)$$

Where

$$P_{0,0,0} = \frac{1-\left[\frac{\lambda A'(1)}{\mu}\left(1+\frac{\alpha}{\gamma}\right)+\frac{\lambda A'(1)}{\beta}\right]}{\left[Y_N(1)+\frac{\lambda}{\theta}+\frac{\lambda}{C}\right]+\lambda^2\left[\frac{a_1-1}{\beta C}-\frac{(\alpha+\gamma)(A'(1)-2a_1+1)}{\mu\gamma C}\right]} \quad (20)$$

Normalizing condition (13) gives, $R_1(1) = \frac{\lambda A'(1)}{\mu}$

3. CHARACTERISTICS

In this section, the average number of units in the system at various states is presented by using probability generating functions. Expected number of units in the system when the server is in different states are assumed as L_0, L_1, L_2, L_3, L_4 and L_5 and are given as

$$L_0 = G_0'(1) = Y'_N(1)P_{0,0,0} \quad (21)$$

$$L_1 = G_1'(1) = \frac{\lambda A'(1)}{\theta}\left[Y_N(1) + \frac{\lambda}{\theta}\right]P_{0,0,0} \quad (22)$$

$$L_2 = G_2'(1) = 0 \quad (23)$$

$$R_1'(1) = \frac{\lambda A'(1)}{\mu}\left(1+\frac{\alpha}{\gamma}\right)G_4(1,1)$$

$$L_3 = G_3'(1) = \frac{\lambda}{\beta}\left[\frac{\lambda a_1}{C} + A'(1)\left(\frac{\lambda^2(a_1-1)}{\beta C} + \frac{\lambda}{\theta} + Y_N(1)\right)\right]P_{0,0,0} + \frac{\mu}{\beta}\left[\frac{\lambda A'(1)}{\beta}R_1(1) + R'_1(1)\right] \quad (24)$$

$$\begin{aligned} L_4 = G_4'(1,1) = {} & \frac{[2\lambda A'(1)(\lambda\alpha A'(1)+\alpha\gamma+\gamma^2)+\lambda\gamma^2A''(1)]}{2\gamma[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]}G_4(1,1) + \frac{\lambda\gamma A'(1)\left[\lambda A'(1)\left(\lambda+\theta Y_N(1)\right)+\theta^2Y_N'(1)\right]}{\theta^2[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]}P_{0,0,0} \\ & + \frac{2\lambda^2\gamma(A'(1))^2[\lambda A'(1)+\beta]+\beta\lambda^2\gamma A'(1)A''(1)}{2\beta^2[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]} \\ & + \frac{[\lambda\alpha\beta A''(1)+2\lambda A'(1)(\alpha+\gamma)[\lambda A'(1)+\beta]-2\lambda\beta A'(1)(\alpha+\gamma)]}{2\beta[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]}G_4(1,1) \\ & + \frac{\gamma}{2[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]}\left\{\frac{\lambda^3A''(1)(a_1-1)}{\beta C} + \frac{\lambda A''(1)[\lambda+\theta Y_N(1)]}{\theta}\right. \\ & \left. + \frac{2\lambda[\lambda A'(1)+\beta]}{\beta}\left[\frac{\lambda a_1}{C} + A'(1)\left(\frac{\lambda^2(a_1-1)}{\beta C}+\frac{\lambda}{\theta}+Y_N(1)\right)\right]\right\}P_{0,0,0} \end{aligned} \quad (25)$$

$$L_5 = G_5'(1,1) = \frac{\lambda\alpha A'(1)}{\gamma^2}G_4(1,1) + \frac{\alpha}{\gamma}G_4'(1,1) \quad (26)$$

The expected system length is

$$L(N) = L_0 + L_1 + L_2 + L_3 + L_4 + L_5 \quad (27)$$

4. TOTAL COST STRUCTURE and OPTIMAL THRESHOLD

Let E_0, E_1, E_2, E_3, E_4 and E_5 are idle, startup, timeout, first phase service, second phase service and breakdown state expected lengths of different periods and cycle expected length is given by

$$E_C=E_0+E_1+E_2+E_3+E_4+E_5 \quad (28)$$

The long run fractions of time that the server is in different modes are obtained as follows:

$$\frac{E_0}{E_C} = P_0 = G_0(1) = Y_N(1)P_{0,0,0} \quad (29)$$

$$\frac{E_1}{E_C} = P_1 = G_1(1) = \frac{\lambda}{\theta}P_{0,0,0} \quad (30)$$

$$\frac{E_2}{E_C} = P_2 = G_2(1) = \frac{\lambda}{C}P_{0,0,0} \quad (31)$$

$$\frac{E_3}{E_C} = P_3 = G_3(1) = \frac{\lambda^2(a_1-1)}{\beta C}P_{0,0,0} + \frac{\mu}{\beta}R_1(1) \quad (32)$$

$$\frac{E_4}{E_C} = P_4 = G_4(1,1) = \frac{\gamma\lambda}{[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]}\left[\frac{\lambda(2a_1-1)}{C} + A'(1)\left(\frac{\lambda^2(a_1-1)}{\beta C} + \frac{\lambda}{\theta} + Y_N(1)\right)\right]P_{0,0,0} + \frac{\lambda\gamma\mu A'(1)R_1(1)}{\beta[\mu\gamma-\lambda A'(1)(\alpha+\gamma)]} \quad (33)$$

and

$$\frac{E_5}{E_C} = P_5 = G_5(1,1) = \frac{\alpha}{\gamma} G_4(1,1) \qquad (34)$$

For the idle period, the expected length is

$$E_0 = \frac{Y_N(1)}{\lambda}$$

Substitute this in equation (29) then

$$\frac{1}{E_C} = \frac{\lambda\left[1 - \frac{\lambda A'(1)}{\mu}\left(1+\frac{\alpha}{\gamma}\right) - \frac{\lambda A'(1)}{\beta}\right]}{\left[Y_N(1) + \frac{\lambda}{\theta} + \frac{\lambda}{C}\right] + \lambda^2\left[\frac{a_1 - 1}{\beta C} - \frac{(\alpha+\gamma)(A'(1) - 2a_1 + 1)}{\mu\gamma C}\right]}$$

TOTAL COST and OPTIMAL THRESHOLD (N^*)

In this section, we discussed the total cost function for the proposed queuing model to find a threshold N which minimizes the cost function. Define various costs that incur per unit of time to optimize the cost as shown below:

C_h: holding cost per unit, C_o: operational cost of the server, C_m: pre service cost per cycle, C_t: timeout cost per cycle, C_s: setup cost per cycle, C_b: breakdown cost, C_r: reward for the server being on vacation.

The total expected cost function per unit time is given as

$$T(N) = C_h L(N) + C_o\left[\frac{E_3+E_4}{E_c}\right] + C_m\left[\frac{E_1}{E_c}\right] + C_t\left[\frac{E_2}{E_c}\right] + C_b\left[\frac{E_5}{E_C}\right] + C_s\left[\frac{1}{E_c}\right] - C_r\left[\frac{E_o}{E_c}\right] \qquad (35)$$

Differentiating equation (35) with respect to N and equate to zero, then we will get optimal threshold N^*. It is difficult to get closed form for N^*. Hence, alternatively, we used computer program to find the expected system length and optimum cost function by taking batch size distributions (section 5). By using the cost function, we can identify the optimum threshold N by varying the values of N in both length and cost function. At some value of N, we observe that the resulting cost function is convex. This value is represented as an optimum value N^* for N.

5. SPECIFIC BATCH SIZE DISTRIBUTION

Here we consider the Geometric distribution for batch size with parameter p and the corresponding generating functions is $A(Z)=p(Z^{-1}-(1-p))^{-1}$, which gives $A'(1)=\frac{1}{p}$ and $A''(1)=\frac{2(1-p)}{p^2}$, $y_i = \sum_{k=1}^{i} a_k y_{i-k}$, $Y_N(1)=\sum_{i=0}^{N-1} y_i$ and $B = Y_N'(1) = \sum_{i=0}^{N-1} i y_i$, $y_o=1$, $a_k=p(1-p)^{k-1}$ and $1/p$ is the mean size of arrival batch. Then the expected number of units in the system is

$$L(N) = \left[\frac{\lambda\frac{1}{p}[\lambda+\theta\sum_{i=0}^{N-1} y_i]}{\theta^2} + \sum_{i=0}^{N-1} i y_i\right] P_{0,0,0} + \frac{\lambda}{\beta}\left[\frac{\lambda a_1}{C} + \frac{1}{p}\left(\frac{\lambda^2(a_1-1)}{\beta C} + \frac{\lambda}{\theta} + \sum_{i=0}^{N-1} y_i\right)\right] P_{0,0,0} + \frac{\mu}{\beta}\left[\frac{\lambda\frac{1}{p}}{\beta} R_1(1) + R_1'(1)\right] +$$
$$\frac{(\alpha+\gamma)\left[2\lambda\frac{1}{p}\left[\lambda\alpha\frac{1}{p}+\alpha\gamma+\gamma^2\right]+\lambda\gamma^2\frac{2(1-p)}{p^2}\right]}{2\gamma^2\left[\mu\gamma-\lambda\frac{1}{p}(\alpha+\gamma)\right]} G_4(1,1) + \frac{\lambda\frac{1}{p}(\alpha+\gamma)\left[\lambda\frac{1}{p}(\lambda+\theta\sum_{i=0}^{N-1} y_i)+\theta^2\sum_{i=0}^{N-1} i y_i\right]}{\theta^2\left[\mu\gamma-\lambda\frac{1}{p}(\alpha+\gamma)\right]} P_{0,0,0} + \frac{(\alpha+\gamma)\left[2\lambda^2\left(\frac{1}{p}\right)^2\left[\lambda\frac{1}{p}+\beta\right]+\beta\lambda^2\frac{1}{p}\left(\frac{2(1-p)}{p^2}\right)\right]}{2\beta^2\left[\mu\gamma-\lambda\frac{1}{p}(\alpha+\gamma)\right]} +$$
$$\frac{(\alpha+\gamma)\left[\lambda\alpha\beta\frac{2(1-p)}{p^2}+2\lambda\frac{1}{p}(\alpha+\gamma)\left[\lambda\frac{1}{p}+\beta\right]-2\lambda\beta\frac{1}{p}(\alpha+\gamma)\right]}{2\gamma\beta\left[\mu\gamma-\lambda\frac{1}{p}(\alpha+\gamma)\right]} G_4(1,1) + \frac{(\alpha+\gamma)}{2\left[\mu\gamma-\lambda\frac{1}{p}(\alpha+\gamma)\right]}\left\{\frac{\lambda^3\frac{2(1-p)}{p^2}(a_1-1)}{\beta C} + \frac{\lambda\frac{2(1-p)}{p^2}[\lambda+\theta\sum_{i=0}^{N-1} y_i]}{\theta} + \frac{2\lambda\left[\lambda\frac{1}{p}+\beta\right]}{\beta}\left[\frac{\lambda a_1}{C} +\right.\right.$$
$$\left.\left.\frac{1}{p}\left(\frac{\lambda^2(a_1-1)}{\beta C} + \frac{\lambda}{\theta} + \sum_{i=0}^{N-1} y_i\right)\right]\right\} P_{0,0,0} + \frac{\lambda\alpha\frac{1}{p}}{\gamma^2} G_4(1,1) \qquad (36)$$

Where,

$$P_{0,0,0} = \frac{1-\left[\frac{\lambda 1}{\mu p}\left(1+\frac{\alpha}{\gamma}\right)+\frac{\lambda 1}{\beta p}\right]}{\left[\sum_{i=0}^{N-1} y_i+\frac{\lambda}{\theta}+\frac{\lambda}{C}\right]+\lambda^2\left[\frac{a_1-1}{\beta C}-\frac{(\alpha+\gamma)(\frac{1}{p}-2a_1+1)}{\mu\gamma C}\right]},$$

$R_1(1) = \frac{\lambda}{\mu}\frac{1}{p}$, $R_1'(1) = \frac{\lambda}{\mu}\frac{1}{p}\left(1+\frac{\alpha}{\gamma}\right) G_4(1,1)$, and

$$G_4(1,1) = \frac{\lambda\gamma}{\left[\mu\gamma - \lambda\frac{1}{p}(\gamma+\alpha)\right]}\left[\frac{\lambda(2a_1-1)}{C} + \frac{1}{p}\left(\frac{\lambda^2(a_1-1)}{\beta C} + \frac{\lambda}{\theta} + \sum_{i=0}^{N-1} y_i\right)\right] P_{0,0,0} + \frac{\lambda\gamma\mu\frac{1}{p}}{\beta\left[\mu\gamma - \lambda\frac{1}{p}(\gamma+\alpha)\right]} R_1(1)$$

6. SENSITIVITY ANALYSIS

In this section, sensitivity analysis has been carried out for the proposed model. Numerical results are presented for the different values of monetary and non-monetary parameters to illustrate the validity of the proposed model by taking Geometric distribution for batch size variable to find-out an optimum value N^* by writing a computer program. By using expected system length defined in equation (36), write the cost equation and find-out optimum cost values by varying N^* value.

The sensitivity analyses over fixing Non-monetary and Monitoring parameters are as follows:

Case-I: Effect of Non-monetary parameters (λ=2, m=3, μ=6, α=2, C=0.5, θ=4, β=4, γ=4) on **N*, L(N*) and T(N*)**

Table-1:					
The variation effect of λ					
λ	2	2.2	2.4	2.6	2.8
N*	13	13	17	22	27
L(N*)	8.499993	31.62182	285.4083	436.6824	855.6339
T(N*)	955.5571	5134.821	13678.03	15918.87	27998.99
The variation effect of μ					
μ	4	5	6	7	8
N*	19	13	13	13	13
L(N*)	413.8169	61.264	8.499993	7.549738	11.57891
T(N*)	19716.81	8327.47	955.5571	629.9273	601.5784
The variation effect of α					
α	2	2.5	3	3.5	4
N*	13	13	13	15	16
L(N*)	8.499993	9.800005	18.00013	52.33379	122.7524
T(N*)	955.5571	1391.005	3134.318	3516.7	7115.139
The variation effect of C					
C	0.5	0.55	0.6	0.65	0.7
N*	13	12	12	12	11
L(N*)	8.499993	2.480181	4.136352	5.162567	0.692758
T(N*)	955.5571	748.5158	660.6066	606.1353	493.0122
The variation effect of θ					
θ	4	5	6	7	8
N*	13	13	13	13	14
L(N*)	8.499993	5.014269	2.499976	0.595208	8.026305
T(N*)	955.5571	979.8113	1000.002	1016.722	1049.475
The variation effect of β					
β	4	5	6	7	8
N*	13	13	13	13	13
L(N*)	8.499993	5.030181	2.971421	1.591126	0.596147
T(N*)	955.5571	687.0197	543.4291	453.4322	391.5387
The variation effect of γ					
γ	4	5	6	7	8
N*	13	12	12	12	12
L(N*)	8.499993	2.550842	4.206292	5.377791	6.335347
T(N*)	955.5571	748.1215	687.4276	659.916	647.0733

From **Table-1**, identified the following observations:

- as λ, α increases, optimum threshold, optimum cost and optimum expected system length are also increasing
- as μ increases, optimum threshold, optimum cost and optimum expected system length are decreasing.
- as C increases, optimum threshold and optimum cost are decreases, whereas optimum expected system length is convex.
- as θ increases, optimum threshold and optimum cost are also increases, and optimum expected system length is decreases.
- as β increases, optimum threshold is stable, whereas optimum cos and optimum expected system length aredecreases.
- as γ increases, optimum threshold and optimum cost are decreases and optimum expected system length is increases.

Case-II: Effect of Monetary parameters (for Ch=20, Co=100, Cb=40, Cs=35, Ct=15,Cm=400, Cr=10) on **N*, L(N*) and T(N*).**

Table-2:					
The variation effect of Ch					
Ch	20	30	40	50	60
N*	13	13	13	13	13
L(N*)	8.499993	8.499993	8.499993	8.499993	8.499993

T(N*)	955.5571	1040.557	1125.557	1210.557	1295.557
The variation effect of Co					
Co	100	200	300	400	500
N*	13	13	13	13	13
L(N*)	8.499993	8.499993	8.499993	8.499993	8.499993
T(N*)	955.5571	1975.93	2996.303	4016.676	5037.048
The variation effect of Cb					
Cb	40	80	120	160	200
N*	13	13	13	13	13
(N*)	8.499993	8.499993	8.499993	8.499993	8.499993
T(N*)	955.5571	1094.076	1232.595	1371.114	1509.633
The variation effect of Cs					
Cs	35	40	45	50	55
N*	13	13	13	13	13
L(N*)	8.499993	8.499993	8.499993	8.499993	8.499993
T(N*)	955.5571	942.2237	928.8904	915.557	902.2236
The variation effect of Ct					
Ct	15	20	25	30	35
N*	13	13	13	13	13
L(N*)	8.499993	8.499993	8.499993	8.499993	8.499993
T(N*)	955.5571	928.8904	902.2236	875.5569	848.8901
The variation effect of Cm					
Cm	400	450	500	550	600
N*	13	13	13	13	13
L(N*)	8.499993	8.499993	8.499993	8.499993	8.499993
T(N*)	955.5571	922.2237	888.8902	855.5568	822.2234
The variation effect of Cr					
Cr	10	15	20	25	30
N*	13	13	13	13	13
L(N*)	8.499993	8.499993	8.499993	8.499993	8.499993
T(N*)	955.5571	988.8905	1022.224	1055.557	1088.891

From **Table-2**, identified the following observations:

- as Ch, Co, Cb and Cr are increases, optimum threshold and optimum expected system length are stable and optimum cost increases
- as Cs, Ct and Cm are increases optimum threshold and optimum expected system length are stable, whereasoptimum cost decreases.

7. CONCLUSION

We derived steady state probability distribution and designed a cost structure for the bulk arrival two-phase vacation queueing system with server startup, breakdown and timeout. Explicit expressions like Optimum threshold, Expected system length and Optimum costs are derived for the proposed queuing model. Also Sensitivity analysis has been made for the validity of the proposed model for some fixed values of the parameters.

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