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Research Article

Open Packing Number Of Some Cycle Related Graphs

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ARTICLEINO	ABSTRACT
	A subset S of vertex set of graph G is called a 2-packing if for each pair of closed neighbourhoods of the vertices of S are pairwise disjoint. A 2-packing is called an open packing if open neighbourhoods of the vertices of S are pairwise disjoint. The open packing number, denoted by $\rho^o(G)$, is the maximum cardinality among all open packing sets of S.Here we investigate open packing number of some cycle related graphs.

Definition 1 *A subset S of V (G) is an open packing of G if the open neighborhoods of the vertices of S are pairwise disjoint in G. The maximum cardinality of an open packing set is called the open packing number and is denoted by* ρ^o.
Preposition 1 *[3] The inequality* $\rho(G) \leqslant \rho^o(G) \leqslant 2\rho(G)$ *hold for any graph G.*
Definition 2 *The switching of a vertex v of G means removing all the edges incident to v and adding edges joining v to every vertex which is not adjacent to v in G. We denote the resultant graph by* G_e.

Theorem 1 $\rho^o(\widetilde{C_n}) = \begin{cases} 2 & ;\text{if } n = 4,5 \\ 3 & ;\text{otherwise} \end{cases}$

Proof: Let C_n be a cycle with n-vertices and $\widetilde{C_n}$ be a switching of arbitrary vertex v of C_n with vertex set, $V(\widetilde{C_n}) = \{v_1, v_2, v_3, \ldots, v_n\}; n \geqslant 4$.
Case-1: for $n = 4$
Let $V(\widetilde{C_4}) = \{v_1, v_2, v_3, v_4\}$
Without loss of generality we switch the vertex v_1 then,

$d(v_1) = 1$ and $N(v_1) = \{v_3\}$
$d(v_2) = 1$ and $N(v_2) = \{v_3\}$
$d(v_3) = 1$ and $N(v_3) = \{v_1, v_2, v_4\}$
$d(v_4) = 1$ and $N(v_4) = \{v_3\}$

We claim that, $\rho^o(\widetilde{C_4}) > 1$ as $\widetilde{C_4}$ is not same as K_1 and C_3. Therefore if $S \subseteq V(\widetilde{C_4})$ is an open packing set then $|S| \geqslant 2$.

$$\text{Also } N(v_1) \cap N(v_2) \cap N(v_4) = \{v_3\} \neq \phi$$

Hence atmost one vertex out of these three vertices can belong to S. Thus, $S = \{v_3$, a pendent vertex$\}$ is an open packing set with maximum cardinality.
Consequently, $\rho^o(\widetilde{C_4}) = 2$.
Case-2: for $n = 5$

$$\text{Let } V(\widetilde{C_5}) = \{v_1, v_2, v_3, v_4, v_5\}$$

Without loss of generality we switch the vertex v_1 then,

$d(v_1) = 2$ and $N(v_1) = \{v_3, v_4\}$
$d(v_2) = 1$ and $N(v_2) = \{v_3\}$
$d(v_3) = 3$ and $N(v_3) = \{v_1, v_2, v_4\}$
$d(v_4) = 3$ and $N(v_4) = \{v_1, v_3, v_5\}$
$d(v_5) = 1$ and $N(v_5) = \{v_4\}$

We claim that $\rho^o(\widetilde{C_5}) > 1$, As $\widetilde{C_5}$ is not same as K_1 and C_3. Therefore if $S \subseteq V(\widetilde{C_5})$ is an open packing set then $|S| \geqslant 2$.

$$\text{Also } N(v_3) \cap N(v_4) = \{v_1\} \neq \phi$$

So these two vertices simultaneously can not be in S.
If $v_2 \in S$, then $N(v_2) \cap N(v_3) = \phi$, so $v_3 \in S$
Moreover, $N(v_2) \cap N(v_4) = \{v_3\} \neq \phi$, so $v_4 \notin S$ $N(v_2) \cap N(v_1) = \{v_3\} \neq \phi$, so $v_1 \notin S$ and $N(v_2) \cap N(v_5) = \phi$, so $v_5 \in S$
But $N(v_3) \cap N(v_5) = \{v_4\} \neq \phi$, in this case either v_3 or v_5 is in S.
So either $\{v_2,v_3\}$ or $\{v_2,v_5\}$ is an open packing set S.
By similar course of arguement, if $v_5 \in S$, then either $\{v_2,v_5\}$ or $\{v_4,v_5\}$ is an open packing set S.
Thus in either situation $|S| = 2$ and it is maximum.

Hence $\rho^o(\widetilde{C_5}) = 2$.

Case-3: for $n \geqslant 6$

Let $V(\widetilde{C_n}) = \{v_1, v_2, v_3, ..., v_n\}$
Without loss of generality we switch the vertex v_1 then,

$d(v_1) = n - 3$	and	$N(v_1) = \{v_2,v_3,v_4,...,v_{n-2},v_{n-1}\}$
$d(v_2) = 1$	and	$N(v_2) = \{v_3\}$
$d(v_3) = 3$	and	$N(v_3) = \{v_1,v_2,v_4\}$
$d(v_4) = 3$	and	$N(v_4) = \{v_1,v_3,v_5\}$
$d(v_5) = 3$	and	$N(v_5) = \{v_1,v_4,v_6\}$
...		...
$d(v_{n-1}) = 3$	and	$N(v_{n-1}) = \{v_1,v_{n-2},v_n\}$
$d(v_n) = 1$	and	$N(v_n) = \{v_{n-1}\}$

We claim that $\rho^o(C_{f}n) > 2$, As C_fn is not same as K_1 and C_3 for $n \geqslant 6$.
Therefore if $S \subseteq V(C_{fn})$ is an open packing set then $|S| \geqslant 3$.
Since v_2 and v_n are pendent vertices, moreover $N(v_2) \cap N(v_n) = \phi$

Also $\bigcap_{i=3}^{n-1} N(v_i) = \{v_1\}$

Therefore v_i, (for $i = 3,4,5,...,n-1$) simultaneously cannot be in S. Thus atmost one vertex from v_i, (for $i = 3,4,5,...,n-1$) can belong to set S containing two pendent vertices v_2 and v_n.
If $v_3 \in S$, then $N(v_2) \cap N(v_3) \cap N(v_n) = \phi$, for $n \geqslant 6$. So $\{v_2,v_3,v_n\}$ is an open packing set. ....(1)
By similar course of arguement,
If $v_{n-1} \in S$, then then $N(v_2) \cap N(v_{n-1}) \cap N(v_n) = \phi$, for $n \geqslant 6$. So $\{v_2,v_{n-1},v_n\}$ is an open packing set. ....(2) From (1) and (2) $|S| = 3$, which is maximum for C_{fn}, for $n \geqslant 6$.
Hence $\rho^o(C_{fn}) = 3$, for $n \geqslant 6$.

Definition 3 *The square of a graph G denoted by G^2 has the same vertex set as of G and two vertices are adjacent in G^2 if they are at distance of 1 or 2 apart in G.*

Theorem 2

$$\rho^o(C_n^2) = \begin{cases} 1 & ; \text{if } 3 \leqslant n \leqslant 9 \\ \left\lfloor \frac{n}{5} \right\rfloor & ; n > 9 \end{cases}$$

Proof: For C_n^2, let $V(C_n) = V(C_n^2) = \{v_1, v_2, v_3, ..., v_n\}; n \geqslant 3$
To prove our result we consider following cases. **Case-1:**

Subcase-1: For $n = 3,4,5$
In this case C_3^2, C_4^2 and C_5^2 are complete graphs K_3, K_4 and K_5 respectively and as proved by Slater[1], $\rho^o(K_n) = 1$.

Hence $\rho^o(C_n^2) = 1$, for $n = 3,4,5$.

Subcase-2: For $n = 6,7,8,9$
In this case C_n^2 is a 4-regular graph and $d(v_i,v_j) < 3$, for all $i,j = 1,2,3,...,9$ and $i \neq j$.
Hence $N(v_i) \cap N(v_j) \neq \phi$
Therefore $\rho^o(C_n^2) = 1$, for $n = 6,7,8,9$.

Case-2: For $n > 9$
As, $V(C_n^2)$ (for $n >$9) is a 4-regular graph, all the vertices belong to an open packing set for which, $d(v_i,v_j) = 3$, for all $i,j = 1,2,3,...,n$ and $i \neq j$.
If $S \subseteq V(C_n^2)$ and S is an open packing set.Let $v_i,v_j \in S$ then $N(v_i) \cap N(v_j) = \phi$ happens only if $d(v_i,v_j) = 3$ with $|j - i| = 5$, for all $i,j = 1,2,3,...,n$ and $i \neq j$.
In other words $v_i \in V(C_n^2)$ is any arbitrary vertex in set S then every fifth vertex of $V(C_n^2)$ is in S, in order to satiesfy the conditions,

$N(v_i) \cap N(v_j) = \phi$, for all $i,j = 1,2,3,\ldots,n$ and $i \neq j$, therefore $|S| \leqslant \frac{n}{5}$.

Hence $\rho^o(C_n^2) = \left\lfloor \frac{n}{5} \right\rfloor$, for $n > 9$.

Concluding Remarks

The open packing number of cycle is known, while we investigate the same for the graphs obtained from cycle by means of some graph operations like switching of a vertex, square of a cycle, splitting graph of cycle and shadow graph of cycle.

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