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Research Article

Nig*α -Closed Sets In Nano Ideal Topological Spaces

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ARTICLE INFO	ABSTRACT

The basic objective of this paper is to define and investigate a new class of sets is called NIg***α** -closed sets, NIg***α** -open sets in nano ideal topological spaces. Also define a notions of NIg***α** -continuous functions and NIg***α** -irresolute functions in nano ideal topological spaces, and we study the relationships between the other existing sets in nano ideal topological space. Further we have given an approprite examples to understand the abstract concept clearly.

Keywords: NIg*α -closed sets, NIg*α -open sets, NIg*α -continuous functions, and NIg*α -irresolute functions.

1. INTRODUCTION

In 1970, Levine[6] introduced the concept of generalized closed sets in topological spaces. This concept was found to be useful to develop many results in general topology. In 1991, Balachandran et.al[1] introduced and investigated the notion of generalized continuous functions in topological spaces. In 2000, veerkumar[15] introduced g^* -closed sets in topological spaces. The concept of ideal topological space was introduced by Kuratowski[5]. Further, Jankovic and Hamlett[4] investigated further properties of ideal topological spaces. In 2014, Ravi et.al[11] introduced Ig∗ -closed sets in ideal topological spaces.
In 2013, the notion of nano topology was introduced by Lellis Thivagar [7,8] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established and analyzed tha nano forms of weakly open sets such as nano α -open sets, nano semi open sets and nano pre -open sets. In 2014, Bhuvaneswari and Mythili Gnanapriya[2], introduced and studied the concept of nano generalized closed sets. Lellis Thivagar and Sudha Devi[9] defined nano ideal topological spaces.
The structure of this manuscript is as follows:
In section 2, we recall some fundamental definitions and result which are more useful to prove our main results.
In section 3 and 4, we define and study the notion of NIg*α -closed sets and NIg*α -open sets in nano ideal topological spaces. we also discuss the concept of NIg*α -closed sets and discussed the relationships between the other existing nano ideal sets. In section 5, we define and study the notions of NIg*α -continuous functions and NIg*α -irresolute functions in nano ideal topological spaces. Further we discuss its basic properties and study the relationships between other existing continuous functions in nano ideal topological spaces.

2. PRELIMINARIES

Definition:2.1 [10] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to

be the approximation space.
Let $X \subseteq U$.
1. The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}\}$, where R(x) denotes the equivalence class determined by $x \in U$.
2. The Upper approximation of X with respect to R is the set of all objects, which can be certain classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \varphi\}\}$
3. The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 [7] A topology on a set X is a collection τ of subsets of X having the following properties:
(1) φ and X are in τ.
(2) The union of the elements of any subcollection of τ is in τ.
(3) The intersection of the elements of any finite sub collection of τ is in τ.
A set X for which a topology τ has been specified is called a topological space. Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X)$ as nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. The complement of the nano open sets are called nano closed sets.

Definition 2.3 [4] An ideal I on a topological space (X, τ) is a non-empty collection of subset of X which satiesfies the following properties
(1) $A \in I$ and $B \subseteq A \Rightarrow B \in I$.
(2) $A \in I$ and $B \in I \Rightarrow A \cup B \in I$.
An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I).

Definition 2.4 [9] A nano topological space $\{U, \tau_R(X)\}$ with an ideal I on U is called a nano ideal topological space or nano ideal space and is denoted as $\{U, \tau_R(X), I\}$.

Definition 2.5 [9] Let $\{U, \tau_R(X), I\}$ be a nano ideal topological space. A set operator $(A)^{*N} : P(U) \rightarrow P(U)$ is called the nano local function I on U with respect to I on $\tau_R(X)$ is defined as $(A)^{*N} = \{x \in U : U \cap A \notin I;$ for every $U \in \tau_R(X)\}$ and is denoted by $(A)^{*N}$, where nano closure operator is defined as $Ncl^*(A) = A \cup (A)^{*N}$.

Result 2.6 [9] Let $\{U, \tau_R(X), I\}$ be a nano ideal topological space and let A and B be subsets of U,then
1. $(\phi)^{*N} = \phi$
2. $A \subset B \rightarrow (A)^{*N} \subset (B)^*$
3. For another $J \supseteq I$ on U, $(A)^{*N}(J) \subset (A)^{*N}(I)$
4. $(A)^{*N} \subset Ncl^*(A)$
5. $(A)^{*N}$ is a nano closed set
6. $((A)^{*N})^{*N} \subset (A)^{*N}$
7. $(A)^{*N} \cup (B)^{*N} = (A \cup B)^{*N}$
8. $(A \cap B)^{*N} = (A)^{*N} \cap (B)^{*N}$
9. For every nano open set V, $V \cap (V \cap A)^{*N} \subset (V \cap A)^{*N}$
10. For $I \in I$, $(A \cup I)^{*N} = (A)^{*N} = (A - I)^{*N}$

Result 2.7 [9] Let $\{U, \tau_R(X), I\}$ be a nano ideal topological space and let A and B be subsets of U, If $A \subset (A)^{*N}$, then $(A)^{*N} = Ncl(A^{*N}) = Ncl(A) = Ncl^*(A)$.

Definition 2.8: Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be
1. Nano semi -open [7], if $A \subseteq Ncl(Nint(A))$.
2. Ng -closed [2], if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano - open.
3. Ng^* -closed [13], if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is Ng -open.
4. $Ng^*\alpha$ [12] -closed if $N\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is $Ng\alpha$ -open.

Definition 2.9. A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is called
1. Nano -continuous [8], if $f^{-1}(V)$ is nano -closed in $(U, \tau_R(X))$ for every nano closed set V in $(V, \tau_{R'}(Y))$.

2. Ng -continuous [2], if $f^{-1}(V)$ is Ng -closed in $(U, \tau_R(X))$ for every nano closed set V in $(V, \tau_{R'}(Y))$.
3. Ng* -continuous [13], if $f^{-1}(V)$ is Ng* -closed in $(U, \tau_R(X))$ for every nano closed set V in $(V, \tau_{R'}(Y))$.
4. Ng*α -continuous [14], if $f^{-1}(V)$ is Ng*α -closed in $(U, \tau_R(X))$ for every nano closed set V in $(V, \tau_{R'}(Y))$.

Definition 2.10. A subset A of a nano ideal space. Let $\{U, \tau R(X), I\}$ is said to be
(1) $*^N$ closed [10], if $(A)^{*N} \subseteq A$
(2) $*^N$-dense [10], if $A \subseteq (A)^{*N}$
(3) NIg -closed [10], if $(A)^{*N} \subseteq G$ whenever $A \subseteq G$ and G is nano open.

3. NIg*α -CLOSED SETS

Definition 3.1. A subset A of a nano ideal space $(U, \tau_R(X), I)$ is said to be NIg*α closed, if NαIcl(A) ⊆ G whenever A ⊆ G and G is Ngα -open.

Theorem 3.2. If $(U, \tau_R(X), I)$ is any nano ideal space and A ⊆ U,then the fallowing are equivalent
(1)A is NIg*α -closed.
(2) NαIcl(A) ⊆ G whenever A ⊆ G and G is nano gα -open in U.
(3) For all x ∈ NαIcl(A), Ngαcl({x}) ∩ A ≠ φ.
(4) NαIcl(A) − A contains no nonempty Ngα -closed set.

Proof. (1) ⇒ (2) If A is NIg*α -closed, then NαIcl(A) ⊆ G whenever A ⊆ G and G is Ngα -open in U and NαIcl(A) = A ∪ NαIcl(A) ⊆ G whenever A ⊆ G and G is nano gα -open in U. This proves (2).
(2) ⇒ (3) Suppose x ∈ NαIcl(A). If Ngαcl({x}) ∩ A = φ, then A ⊆ U − Ngαcl({x}). By (2), NαIcl(A) ⊆ U − Ngαcl({x}), a contraction, Since x ∈ NαIcl(A).
(3) ⇒ (4) Suppose F ∈ NαIcl(A) − A, F is nano gα -closed and x ∈ F. SinceF ⊆ U − A and F is nano gα -closed, then A ⊆ U − F and F is nano gα -closed, Ngαcl({x}) ∩ A = φ. Since x ∈ NαIcl(A) by (3), Ngαcl({x}) ∩ A ≠ φ. ThereforeNαIcl(A) − A contains no nonempty nano gα– closed set.
(4) ⇒ (1) Let A ⊆ G where G is Ngα -open set. Therefore U − G ⊆ U − Aand so NαIcl(A) ∩ (U − G) ⊆ NαIcl(A) ∩ (U − A) = NαIcl(A) − A. Therefore NαIcl(A) ∩ (U − A) = NαIcl(A) − A. Since NαIcl(A) is always nano closed set, so NαIcl(A) ∩ (U − G) is a Ngα -closed set contained in NαIcl(A) − A. Therefore NαIcl(A) ∩ (U − G) = φ and hence NαIcl(A) ⊆ G. Therefore A is NIg*α -closed.

Theorem 3.3. Every $*^N$ closed set is NIg*α -closed but not conversely.

Proof. Let A be a $*^N$ -closed, then $(A)^{*N} \subseteq A$. Let A ⊆ G where G is Ng -open. Hence NαIcl(A) ⊆ G whenever A ⊆ G and G is Ngα -open. Therefore A is NIg*α -closed.

Example 3.4. Let U = {a, b, c, d}, with U/R = {{a}, {b, c}, {d}} and X = {b, d}. Then the nano topology $\tau_R(X)$ = {U, φ, {d}, {b, c}, {b, c, d}} with a nano ideal I = {φ, {a}}. Then NIg*α closed sets are {U, φ, {a}, {d}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and $*^N$ closed setsare {U, φ, {a, d}}. It is clear that {a, b} is NIg*α closed set but it is not in $*^N$ closed.

Theorem 3.5. If A and B are NIg*α closed, then A ∪ B is NIg*α closed.

Proof. Let A and B are NIg*α closed in U. Let G be a Ngα -open in U. Then A ⊆ G and B ⊆ G. Since A and B are NIg*α -closed sets, NαIcl(A) ⊆ G and NαIcl(B) ⊆ G. Hence NαIcl(A∪ B) = NαIcl(A) ∪ NαIcl(B) ⊆ G. Therefore A ∪ B is NIg*α closed.

Remark 3.6. The intersection of any two NIg*α closed set is NIg*α closed.
Example 3.7. Let U = {a, b, c, d}, with U/R = {{a}, {b, c}, {d}} and X = {b, d}. Then the nano topology $\tau_R(X)$ = {U, φ, {d}, {b, c}, {b, c, d}} with a nano ideal I = {φ, {a}}. Then NIg*α -closed sets are {U, φ, {a}, {d}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}}. Let A = {a, b}and B = {a, b, d} and A∪B = {a, b, d} is also NIg*α -closed set and A∩B = {a, b} is also NIg*α closed set.

Theorem 3.8. If $(U, \tau_R(X), I)$ is any nano ideal space, then every nano closed set is a NIg*α closed but not conversely.
Proof. Let A be a nano closed set and G be a any Ngα -open set containing A.Then A ⊆ G this implies that Ncl(A) ⊆ G. Also NαIcl(A) ⊆ Ncl(A) ⊆ G. Therefore NαIcl(A) ⊆ G. Hence A is NIg*α

closed.

Example 3.9. Let U = {a, b, c, d}, with U/R = {{a}, {b, c}, {d}} and X = {b, d}. Then the nano topology $\tau_R(X)$ = {U, φ, {d}, {b, c}, {b, c, d}} with a nano ideal I = {φ, {a}} Then $NIg^*\alpha$ closed sets are
{U, φ, {a}, {d}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}} and nano closedsets are {U, φ, {a}, {a, d}, {a, b, c}} and it is clear that {a, b} is $NIg^*\alpha$ closed set but it is not in nano -closed.

Theorem 3.10. Every NαI closed set is $NIg^*\alpha$ closed.
Proof. Let A be a NαI closed set in (U, $\tau_R(X)$) and A ⊆ G, where G is Ngα open. G be a any Ngα -open set containing A. Since A is NαI -closed, we have NαIcl(A) = A ⊆ G. Therefore NαIcl(A) ⊆ G. Hence A is $NIg^*\alpha$ closed.

Example 3.11. Let U = {a, b, c, d}, with U/R = {{a, b}, {c}, {d}} and X = {a, b}. Then the nano topology $\tau_R(X)$ = {U, φ, {a, b}} with a nano ideal I ={φ, {a}}. Then $NIg^*\alpha$ closed sets are {U, φ, {c, d}, {b, c, d}, {a, c, d}} and NαI closed sets are {U, φ, {c, d} and it is clear that {b, c, d} is $NIg^*\alpha$ closed set but it is not in NαI closed.

Theorem 3.12. Let (U, $\tau_R(X)$, I) be a nano ideal space. Then every $Ng^*\alpha$ closed set is a $Ng^*\alpha$ closed set but not conversely.

Proof. Let A be a $Ng^*\alpha$ -closed set. Then Nαcl(A) ⊆ G whenever A ⊆ G and Gis Ngα -open. we have NαIcl(A) ⊆ Nαcl(A) ⊆ G. This implies NαIcl(A) ⊆ G, whenever A ⊆ G and G is Ngα -open. Hence A is $Ng^*\alpha$ closed.

Theorem 3.13. If (U, $\tau_R(X)$, I) is any nano ideal space, then every NI_g -closedset is $NIg^*\alpha$ closed but not conversely.

Proof. Let A be a NI_g -closed set. Then A ⊆ G and G be a nano open set containing A. Then A ⊆ G. Since every nano open set is $NIg^*\alpha$ open set. This implies that $(A)^{*N}$ ⊆ NαIcl(A) ⊆ G and $(A)^{*N}$ ⊆ G and hence NαIcl(A)⊆G.ThereforeA is $NIg^*\alpha$ closed.

Theorem 3.14. If (U, $\tau_R(X)$, I) is any nano ideal space, then every NIg^* -closed set is $NIg^*\alpha$ closed but not conversely.
Proof. Let A be a NIg^* closed set. Then $(A^*)^N$ ⊆ G whenever A ⊆ G and G is Ng -open. We have $(A)^{*N}$ ⊆ NαIcl(A) ⊆ G whenever A ⊆ G and G is Ngα open. Hence A is $NIg^*\alpha$ closed.

Example 3.15. Let U = {a, b, c, d}, with U/R = {{a}, {b, c}, {d}} and X = {b, d}. Then the nano topology $\tau_R(X)$ = {U, φ, {d}, {b, c}, {b, c, d}} with a nano ideal I = {φ, {a}}. Then $NIg^*\alpha$ closed sets are
{U, φ, {a}, {d}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}}. and Ng^* closed sets are {U, φ, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}}. It is clear that {d} is $NIg^*\alpha$ closed set but not $Ng^*\alpha$, NIg and NIg^* closed set.

Theorem 3.16. If A is $NIg^*\alpha$ closed set and A ⊆ **B** ⊆ $NIg^*\alpha$ −Nαcl(A), then **B** is $NIg^*\alpha$ closed.
Proof. Let A be $NIg^*\alpha$ closed and **B** ⊆ G, where **B** is a Ngα -open. Then A ⊆ **B** implies A ⊆ G, since A is $NIg^*\alpha$ closed, $NIg^*\alpha$ Nαcl(A) ⊆ G and **B** ⊆ $NIg^*\alpha$ − Nαcl(A) implies $NIg^*\alpha$ − Nαcl(**B**) ⊆ $NI_g{*}\alpha$ − Nαcl(A). Therefore $NIg^*\alpha$ − Nαcl(**B**) ⊆ G and hence **B** is $NIg^*\alpha$ closed.

Definition 3.17. A subset A of a nano ideal space (U, $\tau_R(X)$, I) is said to be $NIg^*\alpha$ open set and U − A is $NIg^*\alpha$ closed.

Theorem 3.18. Let (U, $\tau_R(X)$, I) be a nano ideal topological space. Then thefollowing statements are hold.
Every nano open set is $NIg^*\alpha$ open.
Every NαI open set is $NIg^*\alpha$ open.
Every $Ng^*\alpha$ open set is $NIg^*\alpha$ open.
Every NIg open set is $NIg^*\alpha$ open.
Every NI_{g^*} open set is $NIg^*\alpha$ open.
Proof. The proof follows from 3.8, 3.10, 3.12, 3.13 and 3.14.

4. $NIg^*\alpha$ -continuous and $NIg^*\alpha$ -irresolute functions

In this section, we define and study the new class of nano functions, namely $NIg^*\alpha$ -continuous and $NIg^*\alpha$ -irresolute functions in nano ideal topological spaces. Also study some of their basic properties. Further we investigated therelationships between the other existing nano continuous functions.

Definition 4.1. A function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_{R'}(Y))$, is said to be $NIg^*\alpha$ continuous, if $f^{-1}(A)$ is $NIg^*\alpha$ -closed in $(U, \tau_R(X), I)$ for every nano closed set A in $(V, \tau_{R'}(Y))$.

Definition 4.2. A function $f : (U, \tau_R(X), I_1) \rightarrow (V, \tau_{R'}(Y), I_2)$ is said to be $NIg^*\alpha$ -irresolute, if $f^{-1}(A)$ is $NIg^*\alpha$ -closed in $(U, \tau_R(X), I_1)$ for every $NIg^*\alpha$ closed set A in $(V, \tau_{R'}(Y), I_2)$.

Theorem 4.3. In a nano ideal topological space $(U, \tau_R(X), I)$, the fallowing state-ments are hold.
1. Every nano continuous functions is $NIg^*\alpha$ -continuous.
2. Every $N\alpha I$ -continuous functions is $NIg^*\alpha$ -continuous.
3. Every $Ng^*\alpha$ -continuous functions is $NIg^*\alpha$ -continuous.
4. Every NI_g -continuous functions is $NIg^*\alpha$ -continuous.
5. Every NI_{g^*} -continuous functions is $NIg^*\alpha$ -continuous.

Proof. 1. Let $f : (U, \tau_R(X), I) \rightarrow (V, \tau_{R'}(Y))$ be nano continuous function and A be any nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(A)$ is nano closedin $(U, \tau_R(X), I)$ as f is nano continuous. Since every nano closed set is $NIg^*\alpha$ -closed set, $f^{-1}(A)$ is $NIg^*\alpha$ -closed set in $(U, \tau_R(X), I)$. Therefore f is $NIg^*\alpha$ -continuous.
2. Let $f : (U, \tau_R(X), I) \rightarrow (V, \tau_{R'}(Y))$ be $N\alpha I$ -continuous function and A be any nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(A)$ is $N\alpha I$ -closed in $(U, \tau_R(X), I)$ as f is $N\alpha I$ -continuous. Since every $N\alpha I$ closed set is $NIg^*\alpha$ -closed set, $f^{-1}(A)$ is $NIg^*\alpha$ -closed set in $(U, \tau_R(X), I)$. Therefore f is $NIg^*\alpha$ -continuous.
3. Let $f : (U, \tau_R(X), I) \rightarrow (V, \tau_{R'}(Y))$ be $Ng^*\alpha$ -continuous function and A be any nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(A)$ is $Ng^*\alpha$ -closed in $(U, \tau_R(X), I)$ as f is $Ng^*\alpha$ continuous. Since every $Ng^*\alpha$ -closed set is $NIg^*\alpha$ -closed set, $f^{-1}(A)$ is $NIg^*\alpha$ -closed set in $(U, \tau_R(X), I)$. Therefore f is $NIg^*\alpha$ -continuous.
4. Let $f : (U, \tau_R(X), I) \rightarrow (V, \tau_{R'}(Y))$ be NIg -continuous function and A be any nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(A)$ is NI_g -closed in $(U, \tau_R(X), I)$ as f is NI_g continuous. Since every NI_g -closed set is $NIg^*\alpha$ closed set, $f^{-1}(A)$ is $NIg^*\alpha$ -closed set in $(U, \tau_R(X), I)$. Therefore f is $NIg^*\alpha$ -continuous.
5. Let $f : (U, \tau_R(X), I) \rightarrow (V, \tau_{R'}(Y))$ be NI_{g^*} -continuous function and A be any nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(A)$ is NIg^* closed in $(U, \tau_R(X), I)$ as f is NIg^* continuous. Since every NIg^* -closed set is $NIg^*\alpha$ -closed set, $f^{-1}(A)$ is $NIg^*\alpha$ -closed set in $(U, \tau_R(X), I)$. Therefore f is $NIg^*\alpha$ -continuous.

Example 4.4. Let $U = \{a, b, c, d\}$, with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$.Then the nano topology $\tau_R(X) = \{U, \varphi, \{d\}, \{b, c\}, \{b, c, d\}\}$ with a nano ideal $I = \{\varphi, \{a\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x, y\}, \{z\}, \{w\}\}$ and $Y=\{x, y\}$. Then the nano topology $\tau_{R'}(Y) = \{V, \varphi, \{x, y\}\}$. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as $f(a) = w$, $f(b) = z$, $f(c) = x$ and $f(d) = y$. Then $f^{-1}(x, y) = \{c, d\}$
That is the inverse image of every nano open set V is $NIg^*\alpha$ -open set in U.Therefore f is $NIg^*\alpha$ -continuous but not nano continuous and $N\alpha I$ -continuous.
Example 4.5. Let $U = \{a, b, c, d\}$, with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$.Then the nano topology $\tau_R(X) = \{U, \varphi, \{d\}, \{b, c\}, \{b, c, d\}\}$ with a nano ideal $I = \{\varphi, \{a\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{y\}, \{z\}, \{x, w\}\}$ and $Y=\{y, w\}$. Then the nano topology $\tau_{R'}(Y)) = \{V, \varphi, \{y\}, \{x, w\}, \{x, y, w\}\}$. Define $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as $f(a) = z$, $f(b) = x$, $f(c) = w$ and $f(d) = y$. Then $f^{-1}(y) = \{d\}$, $f^{-1}(x, w) = \{b, c\}$, $f^{-1}(x, y, w) = \{b, c, d\}$, $f^{-1}(V) = \{U\}$. That is the inverse image of every nano open set is V is $NIg^*\alpha$ -open set in U. Therefore f is $NIg^*\alpha$ -continuous. but not NIg -continuous and NIg^* -continuous.

Theorem 4.6. Every $NIg^*\alpha$ -irresolute function is $NIg^*\alpha$ -continuous function but not conversely.

Proof. Let W be a nano closed set in V which is $NIg^*\alpha$ -closed set, then $f^{-1}(W)$ is $NIg^*\alpha$ -closed in U. Hence f is $NIg^*\alpha$ -continuous.

Theorem 4.7. A function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_{R'}(Y))$ is NIg*α -continuous if and only if the inverse image of every nano closed set in $(V, \tau_{R'}(Y))$ is NIg*α closed set in $(U, \tau_R(X), I)$.

Proof. Necessary part: Let W be a nano open set in $(V, \tau_{R'}(Y)$ Since f is NIg*α continuous, $f^{-1}(W^c)$ is NIg*α -closed set in $(U, \tau_R(X), I)$. But $f^{-1}(W^c) = U - f^{-1}(W)$. Hence $f^{-1}(W)$ is NIg*α closed in $(U, \tau_R(X), I)$.
Sufficency: Assume that the inverse image of every nano closed set in $(V, \tau_{R'}(Y))$ is NIg* closed set in (U, $\tau_R(X), I)$. Let W be a nano open set in $(V, \tau_{R'}(Y))$. By our assumption $f^{-1}(W^c) = U - f^{-1}(W)$ is NIg*-closed in $(U, \tau_R(X), I)$, which implies that $f^{-1}(W)$ is NIg*α -open set in $(U, \tau_R(X), I)$. Hence f is NIg*α continuous.

Theorem 4.8. A function $f : (U, \tau_R(X), I_1) \rightarrow (V, \tau_{R'}(Y), I_2)$ and $g : (V, \tau_{R'}(Y)\ I_2) \rightarrow (W, \tau_{R''}(Z), I_3)$ be any two functions. Then the following statements hold.$g \circ f$ is $NI_{g^*\alpha}$ -continuous if f is $NI_{g^*\alpha}$ -continuous and g is nano continuous. $g \circ f$ is $NI_{g^*\alpha}$ -continuous if f is $NI_{g^*\alpha}$ -irresolute and g is $NI_{g^*\alpha}$ - continuous.$g \circ f$ is $NI_{g^*\alpha}$ -irresolute if f is $NI_{g^*\alpha}$ -irresolute and g is $NI_{g^*\alpha}$ -irresolute.

Proof. (1) Let A be a nano closed set in W. Since g is continuous, $g^{-1}(A)$ is closed in V. Since f is $NI_{g^*\alpha}$ -continuous of f which implies, $g^{-1}(f^{-1}(A))$ is $NI_{g^*\alpha}$ -closed in U and hence $g \circ f$ is $NI_{g^*\alpha}$ -continuous.
Let A be a nano closed set in W. Since g is $NI_{g^*\alpha}$ -continuous, $g^{-1}(A)$ is $NI_{g^*\alpha}$ -closed in V. Since f is $NI_{g^*\alpha}$ -irresolute,$f^{-1}(g^{-1}(A))$ is $NI_{g^*\alpha}$ closed in U and hence $g \circ f$ is $NI_{g^*\alpha}$ -continuous.
Let A be $NI_{g^*\alpha}$ -closed set in W. Since g is $NI_{g^*\alpha}$ -irresolute, $g^{-1}(A)$ is $NI_{g^*\alpha}$ -closed in V. Since f is $NI_{g^*\alpha}$ -irresolute,$f^{-1}(g^{-1}(A))$ is $NI_{g^*\alpha}$ closed in U and hence $g \circ f$ is $NI_{g^*\alpha}$ -irresolute.

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