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Research Article

Soft Eulerian Graphs

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ABSTRACT

Let G^* = (V, E) be a simple graph and A⊆ V (G^*) be any nonempty set of parameters. Let ρ be an arbitrary relation from A to V where (F, A) and (K, A) are soft sets over V and E respectively. H(a) = (F(a), K(a)) is an induced subgraph of G^* for all a ∈ A. (G^*, F, K, A) = {H(a)/ a ∈ A}≅ $\bigcup_{a\in A} H(a)$ is called as the soft graph of G^* corresponding to the parameter set A and the relation ρ. It is said to be a T1- soft graph of G^* only if H(a) is connected ∀ a ∈ A. Otherwise, it is called a T12-soft graph of G^*. Every T1-soft graph is also a T12-soft graph of G^* and not the converse. The geodetic set of the soft graph (G^*, F, K, A) introduced by K Palani et al. [7] is defined as the union of geodetic sets of the induced sub graphs H(a) where a ∈ A. A geodetic set of a T1 or T12- soft graph of G^* of minimum cardinality is said to be a minimum geodetic set of (G^*, F, K, A). The geodetic number of the soft graph (G^*, F, K, A) is the cardinality of a minimum geodetic set of (G^*, F, K, A). A connected graph G is called an Euler graph, if there is a closed trail which includes every edge of the graph. This paper extends the eulerian concept to soft graphs and develops the concept of soft eulerian graphs

Keywords: soft graph, parameter, Eulerian trial, soft eulerian.

1. Introduction

Molodtsov [5] introduced soft set theory in 1999 as a general mathematical tool for dealing with uncertainties. Maji, Biswas and Roy [4] made a theoretical study of the soft set theory in more detail. In 2014, Rajesh K. Thumbakara and Bobin George [8] introduced the new notion soft graph using soft sets. In 2015 Akram M and Nawaz S [1] introduced the concept of soft graphs in broad spectrum.The geodetic sets of a connected graph were introduced by Frank Harary, Emmanuel Loukakis and Constantine Tsoubros [2] ,as a tool for studying metric properties of connected graphs. Soft graphs of some standard and special graphs have been discussed in [6]. Geodetic number of soft graphs of certain graphs has been discussed in [7]. Euler formulated the concept of eulerian trial when he solved the problem of Konigsberg bridges. The concept of eulerian trial mainly deals with the nature of connectivity in graphs. These concepts have applications to the area of puzzles and games. This paper extends the eulerian concept to soft graphs and develops the concept of soft eulerian graphs

2. Preliminaries

2.1 Definition: [8]

Let G^* = (V, E) be a simple graph and A be any non- empty set. Let $R \subseteq$ A × V be an arbitrary relation. A set valued function F: A → P(V) can be defined as F(x) = {y ∈ V/ x Ry}. The pair (F, A) is a soft set over V. Then, (F,A) is said to be a **soft graph** of G^* if the subgraph induced by F(x) in G^* is a connected subgraph of G^* for all $x \in A$. The set of all soft graph of G^* is denoted by SG (G^*)

2.2 Definition: [1]

Let G^* = (V, E) be a simple graph and A be any nonempty set of parameters. Let subset R of A×V be an arbitrary relation from A to V. A mapping F: A → $P(V)$ can be defined as F(x) ={y ∈ V/x R y} and a mapping K: A → $P(E)$ can be defined as K(x) = {uv ∈ E/{u,v } ⊆ F(x)}. A 4-tuple G = (G*, F, K, A) is called a **soft graph** of G if it satisfies the following properties:

1. G*= (V, E) is a simple graph
2. A is a nonempty set of parameters
3. (F, A) is a soft set over V
4. (K, A) is a soft set over E
5. (F(a), K(a)) is a subgraph of G* for all a ∈A

The subgraph (F(a), K(a)) is denoted by H(a).

2.3 Definition:[7]

The geodetic set of the soft graph (G^*, F, K, A) is the union of geodetic sets of the induced subgraphs H (a), where a ∈A. A geodetic set of a soft graph (G^*, F, K, A) of minimum cardinality is said to be a minimum geodetic set of (G^*,, F, K, A). The geodetic number of the soft graph (G^*, F, K, A) is the cardinality of a minimum geodetic set of (G^*, F, K, A).

2.4 Definition: A closed trial containing all points and lines is called as **Eulerian trail**. A graph having Eulerian trail is called as **Eulerian graph.** That is a connected graph G is called an Euler graph, if there is a closed trail which includes every edge of the graph.

3.Main Results

3.1 Definition: A soft graph G = (G^*, F, K, A) = $\bigcup_{x \in A} H(x)$ of a graph G^* is said to be soft Eulerian if H(x) is Eulerian ∀ x ∈ A.

3.2 Example:

Consider the graph G^* = (V, E) in the following figure 3.1.

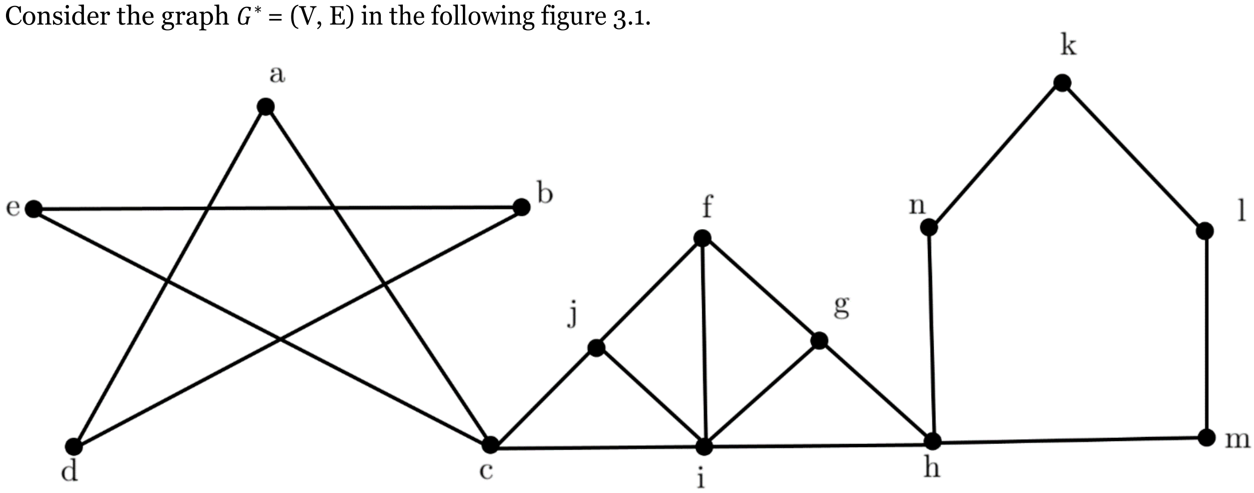
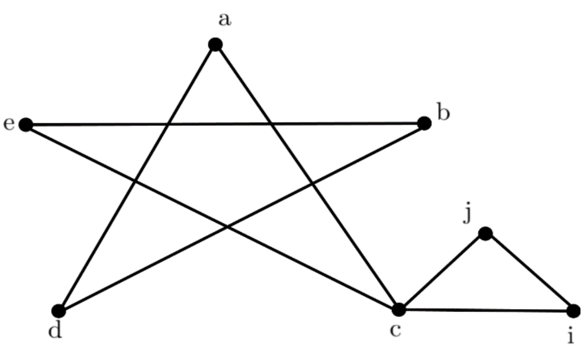
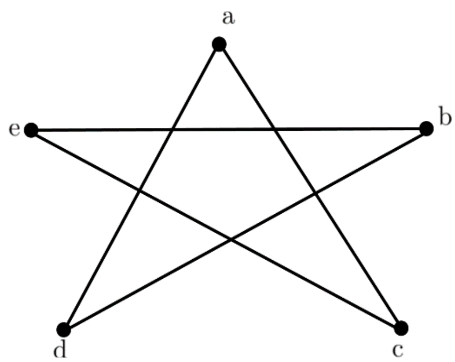


Figure 3.1
G*

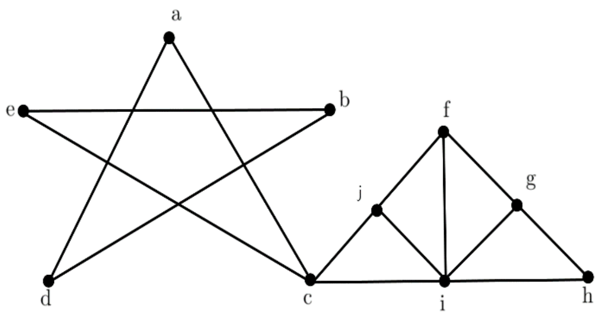
Let A be any singleton parameter set. Let R be a relation on V defined by x R y⟺d (x, y) ≤ 2.
The soft graphs of G* corresponding to each singleton parameter set are as in figures 3.2



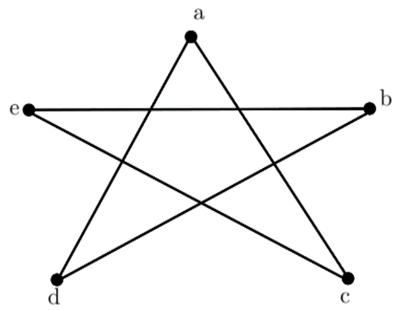
H(a)



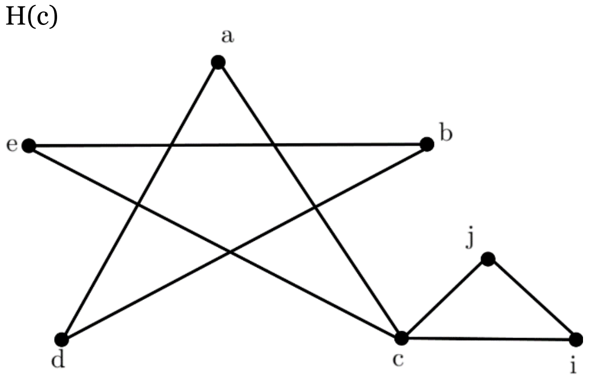
H(b)



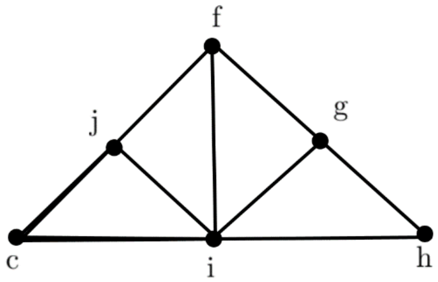
H(c)



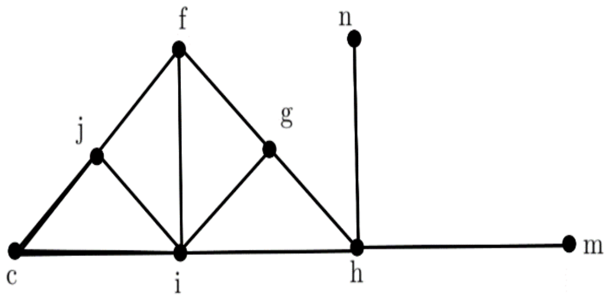
H(d)



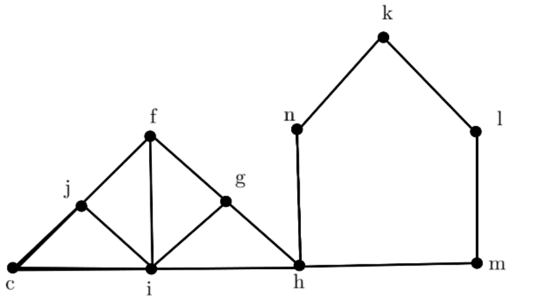
H(e)



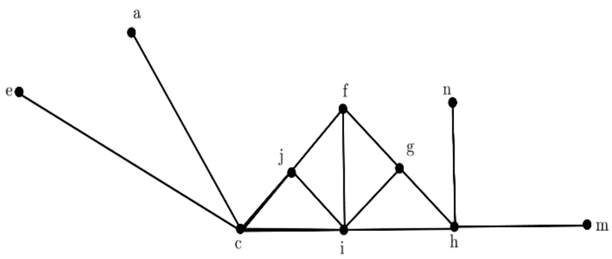
H(f)



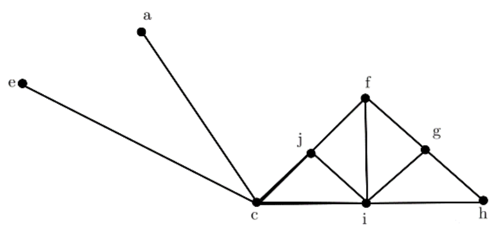
H(g)



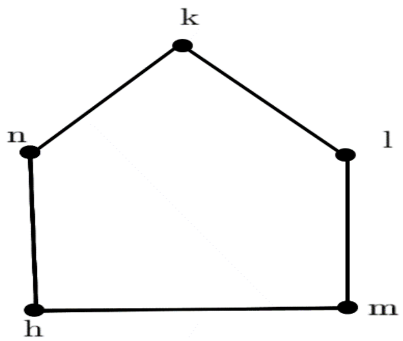
H(h)



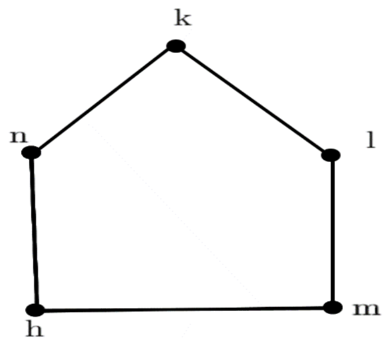
H(i)



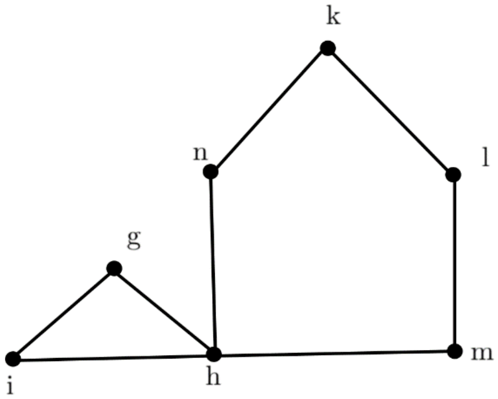
H(j)



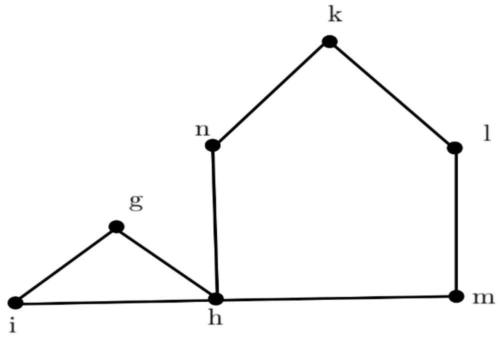
H(k)



H(l)



H(m)



H(n)

Figure 3.2
Soft Graphs of G^*

From the figure, it is clear that the soft graphs from $G^*$are soft eulerian if A is a singleton subset of {a, b, d, e, k, l, m, n}.

3.3 Observation:

1. Let G^* be a simple graph. A T1-soft graph G = (G^*, F, K, A) = $\bigcup_{x \in A} H(x)$ is said to be Eulerian if it has exactly one part H(x) which is Eulerian.ie.,G is Eulerian if and only if A is a singleton set {x} and H(x) is eulerian.
2. Soft eulerian graphs in example 3.2 are eulerian

3.4 Remark: A T1-soft graph is Eulerian ⟹it is soft Eulerian and the converse need not be true.
For example:
Let G^* be a graph in figure 3.1. Let A={a, b}and the relation R be defined by xRy⟺d(x, y) ≤ 2.Here the soft graph (G^*, F, K, A) is soft eulerian but not Eulerian

3.5 Proposition: Let G*= (V, E) be a graph. Suppose A is a singleton parameter set. Then, the soft graph (G*, F, K, A) is soft Eulerian if and only if (G*, F, K, A) is Eulerian
Proof:
Let A={x} .Then, (G*, F, K, A) has exactly one part H(x) and hence (G*, F, K, A) ≅ H(x)
Therefore, (G*, F, K, A) is soft Eulerian if and only if H(x) is Eulerian
Hence, (G*, F, K, A) is soft Eulerian if and only if (G*, F, K, A) is Eulerian

3.6 Theorem: Let G* be a simple graph and G = (G^*, F, K, A) = $\bigcup_{x \in A} H(x)$ be a soft graph of G* in which each H(x) is non-trivial and connected. Then, the following statements are equivalent
(i) G is soft Eulerian
(ii) $d_{H(x)}$(v) is even for all v∈ H(x) and ∀ x∈A
(iii) The set of edges of H(x) can be partitioned into cycles for all x∈ A
Proof:
(i) ⟺(ii)
Let G be soft Eulerian
Then, H(x) is Eulerian for all x ∈ A.Therefore, each H(x) has a closed trail T(x) that includes each edge of H(x) exactly once.Consider H(x) for some x ∈ A.Let T(x) be an Eulerian trail in H(x) with origin and terminus u. Hence, T(x) is a closed trail which contains all points and lines of H(x). Each time a vertex v occurs in this Eulerian trail in a place other than the origin and terminus, two of the edges incident with v are used. At each occurrence of v in T, there are two edges incident with v. At u, every time T(x) moves out of u, it must get back to u. Hence one edge is accounted at origin and the other at terminus and hence d(u) is also even
Hence, every point of H(x) has an even degree. Since H(x) is arbitrary, every point in each H(x) is of even degree
(ii) ⟺(iii)
Consider H(x) for some x ∈ A .Since H(x) is connected and non-trivial, for every v ∈V(G) , $d_{H(x)}$(v) ≥2 in every H(x) containing v. Hence, H(x), contains a cycle J(x). The removal of the lines of J(x) from H(x) results in spanning subgraph say H_1(x) in which again every vertex has an even degree. If H_1(x) has no edges , then all the lines of H(x) forms one cycle and hence (iii) holds.
Otherwise, the degree of each vertex of H_1(x) is even and so H_1(x) contains a cycle say J_1(x). Now, the removal of the lines from $J_1(x)$ from H_1(x) results in a spanning subgraph say H_2(x) in which every vertex has again an even degree. Continuing the above process, when a graph H_n(x) with no edges is obtained, then H(x) can be partitioned into n cycles. Hence, the set of edges of H(x) can be partitioned into cycles. Since x ∈A is arbitrary, the set of edges

of every H(x) can be partitioned into cycles ∀ x ∈ A.
(iii) ⟺(i)
Let x∈ A
By(iii), the set of all edges of H(x) can be partitioned into cycles
If the partition of edges of H(x) has only one cycle, then H(x) is obviously Eulerian.
Otherwise, let J_1(x), $J_2(x)$, ……, J_n(x) be the cycles forming a partition of the edges of H(x).Since H(x) is connected, there exists a cycle say $J_i(x) \neq J_1$(x) having a common point say v_1 with J_1(x).Without loss of generality, let this J_i(x) be J_2(x).Then, the walk beginning at $v_1$and passing through the two cycles J_1(x) and J_2(x) in succession and returning back to v_1 is a close d trail containing the edges of these two cycles J_1(x) and $J_2(x)$. Continuing this process, results in a closed trail containing all the edges of H(x). Therefore, H(x) contains a closed trail consisting of all points and lines of H(x) and hence H(x) is Eulerian. Since x ∈A is arbitrary, H(x) contains a closed trail consisting of all points and lines of H(x) for all x ∈A . Therefore, G is soft Eulerian.
Hence, the theorem.

3.7 Theorem: Let G be a T1-soft graph which is eulerian. Then, g(G) ≤ 2k where E(G) is partitioned into k cycles.
Proof:
Let G be a soft graph. Suppose G is eulerian . Then G has exactly one part say H(x) and H(x) is eulerian . Therefore by theorem 3.6, H(x) is partitioned into cycles.
Let the number of cycles be k and label the cycles as C_1 , C_2,………, C_k in order. Let C_i be a cycle attached to C_{i-1} and C_{i+1}.Selecting one or two points from each cycle, we can form a geodetic set. Hence g(G) ≤ 2k

3.8 Remark: The soft graphs of G^* in figure 3.2 corresponding to any t- element parameter set for 2≤ $t \leq 8$ which is a subset of {a, b, d, e, k, l, m, n } are soft eulerian

3.9 Definition: A soft graph G = (G^*, F, K, A) = $\bigcup_{x \in A} H$(x) is said to be soft arbitrarily traversable from a vertex v if the following procedure always results in an eulerian trail in every H(x) such that v ∈ $H(x)$.Start at v traversing through any incident edge. On arriving at a vertex u, depart through any incident edge not yet traversed and continue until all the lines are traversed

3.10 Definition: A soft graph G=(G^*, F, K, A) is said to be arbitrarily traversable if it is soft traversable from every vertex v ∈ G

3.11 Remark: If a soft graph is arbitrarily traversable, then it is soft eulerian
Proof:
Suppose G= (G^*, F, K, A) is arbitrarily traversable, then it is soft traversable from every vertex v in G^*.Therefore ∀v ∈H(x), there is an Eulerian trail in H(x) at v. Hence, each H(x) is eulerian.
Therefore, G is soft eulerian.

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