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Research Article

4-Total Geometric Mean Cordial Labeling Of Graphs

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ARTICLE INFO | **ABSTRACT**

Let G be a (p, q) graph. Let f : V (G) → {1, 2, 3,..., k} be a function where k∈N and k>1. For each edge uv, assign the label f (uv)= $\left\lceil \sqrt{f(u)f(v)} \right\rceil$. f is called k-Total geometric mean cordial labeling of G if | t_{mf} (i) – t_{mf} (j) | ≤ 1, for all i, j∈{1, 2, 3,..., k}, where t_{mf} (x) denotes the total number of vertices and edges labeled with x, x∈{1, 2, 3,..., k}.A graph that admits the k-total geometric mean cordial labeling is called k-total geometric mean cordial graph. In this paper we investigate 4- total geometric mean cordial labeling of graphs.
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Keywords: H-graph, Pan graph, Bistar, Crown graph, 4-total geometric mean cordial labeling, 4-total geometric mean cordial graphs.

I. Introduction

Finite, simple and undirected graphs are considered here. Cordial labeling was introduced by Cahit [1]. For notations and terminology we follow [2]. Geometric mean cordial labeling of graphs was introduced in [3]. k-total mean cordial labeling of graphs was introduced in [4]. 4-total mean cordial labeling of some graphs derived from H-graph and Star was introduced in [5]. 4-total mean cordial labeling of some graphs derived from path and cycle was introduced in [6]. k-total geometric mean cordial labeling of some graphs was introduced in [7]. In this paper we investigate 4-total geometric mean cordial labeling behavior of H-graph, Pan graph, bistar and crown graphs.

Definition 1.1.
Let G be a (p, q) graph. Let f : V(G) →{1, 2, 3,..., k} be a function where k∈N and k>1. For each edge uv, assign the label f (uv) = $\left\lceil \sqrt{f(u)f(v)} \right\rceil$. f is called k-Total geometric mean cordial labeling of G if | t_{mf} (i) – t_{mf} (j)|≤1, i, j∈{1, 2, 3,..., k} where t_{mf} (x) denotes the total number of vertices and edges labeled with x, x∈{1, 2, 3, ..., k}.A graph that admits the k-total geometric mean cordial labeling is called k-total geometric mean cordial graph.

Definition 1.2.
Let $P_n^{(1)}$: $u_1 u_2 \ldots u_n$ and $P_n^{(2)}$: $v_1 v_2 \ldots v_n$ be any two paths. We join the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ by an edge, if n is odd and join the vertices $u_{\frac{n+2}{2}}$ and $v_{\frac{n}{2}}$ by an edge, if n is even.
Then the resulting graph is called a H-graph on 2n vertices. We denote it by H_n .

Definition 1.3.
The pan graph is the graph obtained by joining a cycle graph to a singleton graph with a bridge. The pan graph is therefore isomorphic with the tadpole graph.

Definition 1.4.
The Bistar $B_{m,n}$ is the graph obtained by joining the two central vertices of $K_{1,m}$ and $K_{1,n}$.

Definition 1.5.
The graph obtained by joining a single pendant edge to each vertex of a path is called a comb $P_n \odot K_1$.

II. Main Results

Theorem 2.1.
Any H_n graph, n ≥ 3 is a 4-total geometric mean cordial labeling.

Proof:
Let H be a graph with 2n vertices and 2n-1edges. Let P_n be a path and $u_1, u_2, u_3, \dots, u_n$ be the vertices of first copy of P_n and $v_1, v_2, v_3, \dots, v_n$ be the vertices of second copy of P_n.

Case (i): when n is odd
Allocate the $\frac{n+1}{2}$ vertices $u_1, u_2, \dots, u_{\frac{n+1}{2}}$ are labeled with 1, the $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \dots, u_n$ are labeled with 2, the $\frac{n+1}{2}$ vertices $v_1, v_2, \dots, v_{\frac{n+1}{2}}$ are labeled with 3. Finally the $\frac{n-1}{2}$ vertices $v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \dots, v_n$ are labeled with 4.

Case (ii): when n is even
Allocate the $\frac{n}{2}$ vertices $u_1, u_2, \dots, u_{\frac{n}{2}}$ are labeled with 1, next we allocate the $\frac{n}{2}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \dots, u_n$ are labeled with 2, then we allocate the $\frac{n}{2}$ vertices $v_1, v_2, \dots, v_{\frac{n}{2}}$ are labeled with 3. Finally the $\frac{n}{2}$ vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \dots, v_n$ are labeled with 4.
The vertex labeling f is a 4 total geometric mean cordial labeling

Nature of n	t_{mf} (1)	t_{mf} (2)	t_{mf} (3)	t_{mf} (4)
n is odd	n	n	n	n-1
n is even	n-1	n	n	n

In the above two cases we see that the function f is 4-total geometric mean cordial labeling. Hence H_n is 4-total geometric mean cordial graph.

Theorem 2.2.
Any Pan graph n ≥ 3 is a 4-total geometric mean cordial graph.

Proof:
Let G be a pan graph with n+1vertices and n+1edges. Let $u_1 u_2 \dots u_n u_1$ be the cycle C_n and v be the pendant vertex and join the pendant vertex to the vertex u_1 .The edges of the pan graph are $vu_1, u_1u_2, u_2u_3, \dots, u_nu_1$. Assign the label 1 to the vertex v.
Case (i): n ≡ 1 (mod 4)
Allocate the $\frac{n-1}{4}$ vertices $u_1, u_2, \dots, u_{\frac{n-1}{4}}$ are labeled with 1, next we allocate the $\frac{n-1}{4}$ vertices $u_{\frac{n+3}{4}}, u_{\frac{n+7}{4}}, u_{\frac{n+11}{4}}, \dots, u_{\frac{n-1}{2}}$ are labeled with 2, now we allocate the $\frac{n+3}{4}$ vertices $u_{\frac{n+1}{2}}, u_{\frac{n+3}{2}}, \dots, u_{\frac{3n-3}{4}}$ *and* u_n are labeled with 3. Finally the $\frac{n-1}{4}$ vertices $u_{\frac{3n+1}{4}}, u_{\frac{3n+5}{4}}, u_{\frac{3n+9}{4}}, \dots, u_{n-1}$ are labeled with 4.
Case (ii): n ≡ 2 (mod 4)
Allocate the $\frac{n-2}{4}$ vertices $u_1, u_2, \dots, u_{\frac{n-2}{4}}$ are labeled with 1, next we allocate the $\frac{n-2}{4}$ vertices $u_{\frac{n+2}{4}}, u_{\frac{n+6}{4}}, u_{\frac{n+10}{4}}, \dots, u_{\frac{n-2}{2}}$ are labeled with 2, then we allocate the $\frac{n+2}{4}$ vertices $u_{\frac{n}{2}}, u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \dots, u_{\frac{3n-2}{4}}$ are labeled with 3. Finally the $\frac{n+2}{4}$ vertices $u_{\frac{3n+2}{4}}, u_{\frac{3n+6}{4}}, u_{\frac{3n+10}{4}}, \dots, u_n$ are labeled with 4.
Case (iii): n ≡ 3 (mod 4)
It is easy to verify for n=3.
If n ≥ 7. Allocate the $\frac{n+1}{4}$ vertices $u_1, u_2, \dots, u_{\frac{n+1}{4}}$ are labeled with 4, the $\frac{n+1}{4}$ vertices $u_{\frac{n+5}{4}}, u_{\frac{n+9}{4}}, \dots, u_{\frac{n+1}{2}}$ are labeled with 1, the $\frac{n+1}{4}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, u_{\frac{n+7}{2}}, \dots, u_{\frac{3n-1}{4}}$ *and* u_n are labeled with 3. Finally the $\frac{n-3}{4}$ vertices $u_{\frac{3n+3}{4}}, u_{\frac{3n+7}{4}}, u_{\frac{3n+11}{4}}, u_{\frac{3n+15}{4}}, \dots, u_{n-1}$ are labeled with 2.

Case (iv): n ≡ 4 (mod 4)
Allocate the $\frac{n}{4}$ vertices $u_1, u_2, \dots, u_{\frac{n}{4}}$ are labeled with 1, next we allocate the $\frac{n}{4}$ vertices $u_{\frac{n+4}{4}}, u_{\frac{n+8}{4}}, \dots, u_{\frac{n}{2}}$ are labeled with 2, then we allocate the $\frac{n}{4}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, u_{\frac{n+6}{2}}, \dots, u_{\frac{3n}{4}}$ are labeled with 3. Finally the $\frac{n}{4}$ vertices $u_{\frac{3n+4}{4}}, u_{\frac{3n+8}{4}}, u_{\frac{3n+12}{4}}, \dots, u_n$ are labeled with 4.
The vertex labeling f is a 4 total geometric mean cordial labeling.

Nature of n	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$	$t_{mf}(4)$
n≡ 1 mod 4	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$
n≡ 2 mod 4	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n+2}{2}$	$\frac{n+2}{2}$
n≡ 3 mod 4	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$
n≡ 4 mod 4	$\frac{n+2}{2}$	$\frac{n+2}{2}$	$\frac{n}{2}$	$\frac{n}{2}$

Hence the graph G is 4- total geometric mean cordial graph.

Theorem 2.3.
The Bistar $B_{n,n}$ is 4-total geometric mean cordial for all n.

Proof:
Let u, v be the centre vertices of the bistar $B_{n,n}$. Let u_i (1≤ i ≤ n) be the pendant vertices adjacent to u and v_i (1≤ i ≤ n) be the pendant vertices adjacent to v. $E(B_{n,n}) = \{uv\} \cup \{uu_i, vv_i : 1 \le i \le n\}$
Allocate the label 3, 4 to the central vertices u, v.
Consider the vertices $u_1, u_2, \dots, u_n$. The n vertices $u_1, u_2, \dots, u_n$ are labeled with 1, the label 2 to the one vertex v_1 and allocate the label 3 to the n-1 vertices $v_2, v_3, \dots, v_n$.
Clearly $t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = n$ and $t_{mf}(4) = n+1$.
Hence the graph $B_{n,n}$ is 4- total geometric mean cordial graph.

Theorem 2.4.
Let G be the graph obtained by attaching a pendant vertex with a vertex of degree two on both sides of a comb graph $P_n \odot K_1$. Then G is the 4- total geometric mean cordial for all n.

Proof:
Comb $P_n \odot K_1$ be the graph obtained from the path $P_n = u_1 u_2 u_3 \dots u_n$ by joining a pendant vertex u_i to v_i(1≤ i ≤ n). Let G be a graph obtained by attaching a pendant vertex x and y to u_1 and u_n respectively.

Case (i): when n is even
Allocate the label 1 to the pendant vertices $v_1, v_2, v_3, v_4, \dots, v_n$, the label 2 to the vertex x, the label 3 to the $\frac{n+2}{2}$ vertices $u_1, u_2, \dots, u_{\frac{n}{2}}$ *and* y. Finally we allocate the label 4 to the $\frac{n}{2}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \dots \dots u_n$.
Case (ii) when n is odd
Allocate the label 1 to the pendant vertices $v_1, v_2, v_3, v_4, \dots, v_n$, the label 2 to the vertex x, the label 3 to the $\frac{n+1}{2}$ vertices $u_1, u_2, u_3, u_4, \dots, u_{\frac{n+1}{2}}$ and the label 4 to the $\frac{n+1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \dots, u_n$ *and* y.
Clearly $t_{mf}(1) = n$ and $t_{mf}(2) = t_{mf}(3) = t_{mf}(4) = n+1$.
Hence the graph G is 4- total geometric mean cordial graph.

Theorem 2. 5:
The Crown $C_n \odot K_1$ is a 4-total geometric mean cordial for all n.

Proof:
Let $u_1 u_2 \dots u_n u_1$ be the cycle C_n. Let $V(C_n \odot K_1) = V(C_n) \cup \{v_i : 1 \le i \le n\}$ and
$E(C_n \odot K_1) = E(C_n) \cup \{u_i v_i : 1 \le i \le n\}$.
Case (i): when n is odd
Allocate the label 1 to the pendant vertices $v_1, v_2, v_3, \dots, v_n$, the label 3 to the $\frac{n+1}{2}$ vertices $u_1, u_2, u_3, u_4, \dots, u_{\frac{n+1}{2}}$ and the label 4 to the $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, u_{\frac{n+7}{2}}, \dots, u_n$.

Case (ii): when n is even
It is easy to verify for n = 4, 6
Subcase (i)
n ≡ 2 (mod 4)
If n ≥10. Allocate the label 1 to the pendant vertices $v_1, v_2, v_3, \ldots, v_n$, the label 3 to the $\frac{n+2}{2}$ vertices $u_1, u_2, u_3, u_4, \ldots, u_{\frac{n-2}{4}}$ *and* $u_{\frac{n}{2}}, u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_{\frac{3n-6}{4}}$ *and* u_{n-1}, u_n and the label 4 to the $\frac{n-2}{2}$ vertices $u_{\frac{n+2}{4}}, u_{\frac{n+6}{4}}, u_{\frac{n+10}{4}}, \ldots, u_{\frac{n-2}{2}}$ *and* $u_{\frac{3n-2}{4}}, u_{\frac{3n+2}{4}}, u_{\frac{3n+6}{4}}, \ldots, u_{n-2}$.
Subcase (ii)
n ≡ 4 (mod 4)
If n ≥8. Assign the label 1 to the $\frac{n+2}{2}$ vertices $u_1, u_2, u_3, u_4, \ldots, u_{\frac{n}{4}}$ *and* $v_1, v_2, v_3, \ldots, v_{\frac{n+4}{4}}$, the label 4 to the $\frac{n}{2}$ vertices $u_{\frac{n+4}{4}}, u_{\frac{n+8}{4}}, u_{\frac{n+12}{4}}, \ldots, u_{\frac{n+2}{2}}$ *and* $v_{\frac{n+8}{4}}, v_{\frac{n+12}{4}}, v_{\frac{n+16}{4}}, \ldots, v_{\frac{n}{2}}$, the label 3 to the $\frac{n}{2}$ vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \ldots, v_n$ and the label 2 to the $\frac{n-2}{2}$ vertices $u_{\frac{n+4}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+8}{2}}, \ldots, u_n$.
Clearly $t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = t_{mf}(4) = n$.
Hence $C_n \odot K_1$ is a 4-total geometric mean cordial graph.

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