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Research Article

Solutions Of Fully Fuzzy Linear Programming Problem Models Using $\vartheta_{\mathcal{R}}$ - Ranking Function

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ABSTRACT

Initially, the paper begins with the introduction of linear programming problem techniques and the literature reviews on fuzzy optimization concepts. Then, it proposes a new ranking function that employs the value of a trapezoidal fuzzy number to solve the fully fuzzy linear programming problems. Furthermore, the paper compares the proposed method with Maleki's ranking function method. Next, this paper discusses the following two models: the shopping model and the power generation model. The first model aims to minimize the costs for consumers who shop either online or offline, while the second model focuses on generating more power to enhance the economy of the country. Through the paper, both the models will be analyzed and then the proposed ranking function will be applied. The goal of the research paper is to contribute to the field of fully fuzzy linear programming problems by suggesting a new ranking function and discussing its applications in two significant models.

Keywords: Fully Fuzzy linear programming problems; Offline shopping; Online shopping; Power generations; Trapezoidal fuzzy number.

1. INTRODUCTION

Linear programming [1] is a technique which is used to solve optimization problems while keeping constraints in mind. It can also be solved using software such as Lindo, AMPL, MPL, etc. However, uncertainty is a common factor in most real-life optimization problems. This is where fuzzy optimization techniques come in handy. Fuzzy linear programming problems are used to make decisions in uncertain situations.

In 1965, Zadeh introduced the concept of fuzzy optimization, which led to the development of optimization problems dealing with uncertainty in 1974. Zimmerman [2] proposed a method for solving linear programming problems that contained fuzzy constraints. Since then, many authors have contributed to the development of fuzzy linear programming based on Bellman and Zadeh [2] principles of decision-making. Ebrahimnejad and Verdegay [3] have developed various solution techniques for solving fuzzy linear programming problems.

This paper covers two topics: shopping and power generation models. Shopping has become a popular activity, and some people now prefer to purchase household items and other necessities online using platforms such as Amazon, Flipkart, etc. Therefore, it is important to analyze the advantages and disadvantages of both online and offline shopping methods to help people make informed choices. Some authors have conducted comparative studies on online and offline shopping through questionnaires and survey reports. In the next section, this paper discusses the shopping model using the collected survey report and obtains the solution of the corresponding formulated fuzzy linear programming problem.

Power generation is critical for India's economic growth, and the majority of electricity in India is generated through coal and other thermal power plants. Fossil fuels, nuclear energy, and renewable energy are the three main categories of energy used to generate electricity. The importance of power generation lies in promoting sustainable development by using renewable sources that lead to cleaner food and energy. Maharashtra is the state that produces the most electricity in India. Later, this paper will deal with power generation from the given data and find the solution to the corresponding fuzzy linear programming problem.

2. MATERIALS AND METHODS

In this section, first, some basic definitions will be discussed.

Definition 2.1

A fuzzy number $\widetilde{A_T} = (p,q,r,s)$ is said to be a trapezoidal fuzzy number [4] if its membership function is given by: $\mu_{\widetilde{A_T}}(x) = \begin{cases} \frac{x-p}{q-p}, & p \le x \le q \\ 1, & q \le x \le r \\ \frac{x-s}{r-s}, & r \le x \le s \\ 0, & else \end{cases}$

Definition 2.2

A trapezoidal fuzzy number $\widetilde{A_T} = (p,q,r,s)$ in which $p \geq 0$ is called as non- negative [5]. If $p \le 0$, then it is called as non- positive [5].

Definition 2.3

Let $\widetilde{A_{T1}} = (p,q,r,s)$ and $\widetilde{A_{T2}} = (w,x,y,z)$ be two arbitrary trapezoidal fuzzy numbers. Then $\widetilde{A_{T1}} < \widetilde{A_{T2}}$ [5] if and only if (q - p) < (x - w) or q < x and q - p = x - w or q = x, q - p = x - w and $\frac{q+r}{2} < \frac{x+y}{2}$ or q = x, q - p = x - w, $\frac{q+r}{2} = \frac{x+y}{2}$ and s - r < z - y.

Definition 2.4

Let $\widetilde{A_{T1}} = (p,q,r,s)$ and $\widetilde{A_{T2}} = (w,x,y,z)$ be two arbitrary trapezoidal fuzzy numbers.
Then the addition of trapezoidal fuzzy numbers $\widetilde{A_{T1}}$ and $\widetilde{A_{T2}}$ [5] is defined as

$$\widetilde{A_{T1}} + \widetilde{A_{T2}} = (p,q,r,s) + (w,x,y,z) = (p+w, q+x, r+y, s+z)$$

Definition 2.5

Let $\widetilde{A_{T1}} = (p,q,r,s)$ and $\widetilde{A_{T2}} = (w,x,y,z)$ be two arbitrary trapezoidal fuzzy numbers, where $p \le q \le r \le s$ and $w \le x \le y \le z$. Then the modified subtraction of trapezoidal fuzzy numbers $\widetilde{A_{T1}}$ and $\widetilde{A_{T2}}$ [5] is $\widetilde{A_{T1}} - \widetilde{A_{T2}} = (p,q,r,s) - (w,x,y,z) = (p-w, q-x, r-y, s-z)$ provided only if $\widetilde{A_{T1}} \geq \widetilde{A_{T2}}$, $D(\widetilde{A_{T1}}) \geq D(\widetilde{A_{T2}})$ or if $\widetilde{A_{T1}} \le \widetilde{A_{T2}}$, $D(\widetilde{A_{T1}}) \le D(\widetilde{A_{T2}})$, where $D(\widetilde{A_{T1}}) = \frac{s-p}{2}$ and $D(\widetilde{A_{T2}}) = \frac{z-w}{2}$.

2.6 Ranking function of Trapezoidal Fuzzy Number

A ranking function of a trapezoidal fuzzy number is a function that assigns a real number to each trapezoidal fuzzy number.
Mary George and Savitha M.T [6] have found the value of a trapezoidal fuzzy number $\widetilde{A_T} = (p,q,r,s)$ using the idea of Adrian.I Ban and Lucian Coroianu [7].
i.e., $\boldsymbol{\vartheta}_{\boldsymbol{\mathcal{R}}}(\widetilde{A_T}) = \frac{p}{6} + \frac{q}{3} + \frac{r}{3} + \frac{s}{6} = \frac{p+2q+2r+s}{6}$.

3. RESULTS

This section will examine the following two models in detail.

Model 3.1: Shopping Model

The following data (Table 3.1.1) has been gathered through an online questionnaire from several experts, including employers, employees, and students. The data pertains to the percentage of customers who prefer online shopping over offline shopping. It includes their requirements in terms of product quality, payment convenience, delivery system, and overall satisfaction with their shopping experience.

Table 3.1.1. Online shopping Vs Offline shopping

	Online shop (%)	Offline shop (%)	Requirement (%)
Quality of the Products	46	54	80
Convenience of Payment	58	42	75
Delivery System	78	22	60
Overall Satisfaction on the shopping	52	48	

Based on the data, there seem to be uncertainties in product quality, payment methods, and door delivery. The objective is to reduce costs to enhance customer benefits.

Let O_n denote the percentage of customers using online shops and O_f denote the percentage of customers using offline shops.
Then a fully fuzzy linear programming problem can be formulated using the provided data to achieve the desired objective.
Min M_c = (50, 51, 52, 53) $\widetilde{O_n}$+ (46, 47, 48, 49) $\widetilde{O_f}$
Subject to
(44, 45, 46, 47) $\widetilde{O_n}$ + (52, 53, 54, 55) $\widetilde{O_f}$ ≥ (78, 79, 80, 81)
(56, 57, 58, 59) $\widetilde{O_n}$ + (40, 41, 42, 43) $\widetilde{O_f}$ ≥ (73, 74, 75, 76)
(76, 77, 78, 79) $\widetilde{O_n}$ + (20, 21, 22, 23) $\widetilde{O_f}$ ≥ (58, 59, 60, 61)
$\widetilde{O_n}, \widetilde{O_f} \geq 0$

Existing Method (Maleki's ranking function [8])

To obtain the solution of this fully fuzzy linear programming problem, it will first be converted into a crisp linear programming problem using Malek's ranking function $\Re(\widetilde{A_T}) = \frac{1}{2}\left(p + s + \frac{1}{2}(r - q)\right)$
Therefore, the crisp linear programming problem is
Min M_c = 51.75 O_n + 47.75 O_f
45.75 O_n + 53.75 O_f ≥ 79.75
57.75 O_n + 41.75 O_f ≥ 74.75
77.75 O_n + 21.75 O_f ≥ 59.75
O_n, O_f ≥ 0
Next, using **Lingo** software, obtain the following output.
Global optimal solution found.
Objective value: 77.25000
Infeasibilities: 0.000000
Total solver iterations: 3
Elapsed runtime seconds: 0.77
Model Class: LP
Total variables: 2
Nonlinear variables: 0
Integer variables: 0
Total constraints: 4
Nonlinear constraints: 0
Total nonzeros: 8
Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
N	0.5764238	0.000000
F	0.9930905	0.000000

Row	Slack or Surplus	Dual Price
1	77.25000	-1.000000
2	0.000000	-0.5000000
3	0.000000	-0.5000000
4	6.666667	0.000000

Hence the optimum solution is M_c = 77.25 when O_n = 0.5764238 and O_f = 0.9930905

Proposed Method ($\vartheta_{\mathcal{R}}$ – ranking function)

To obtain the solution of this fully fuzzy linear programming problem, it will first be converted into a crisp linear programming problem using the following $\boldsymbol{\vartheta_{\mathcal{R}}}$ – ranking function

$$\boldsymbol{\vartheta_{\mathcal{R}}}(\widetilde{A_T}) = \frac{p}{6} + \frac{q}{3} + \frac{r}{3} + \frac{s}{6} = \frac{p+2q+2r+s}{6}.$$

Therefore, the crisp linear programming problem is
Min M_c = 51.5 O_n + 47.5 O_f
Subject to
45.5 O_n + 53.5 O_f ≥ 79.5
57.5 O_n + 41.5 O_f ≥ 74.5
77.5 O_n + 21.5 O_f ≥ 59.5
O_n, O_f ≥ 0
Next, using **Lingo** software, obtain the following output.
Global optimal solution found.
Objective value: 77.00000
Infeasibilities: 0.000000
Total solver iterations: 3

Elapsed runtime seconds: 0.64
Model Class: LP
Total variables: 2
Nonlinear variables: 0
Integer variables: 0
Total constraints: 4
Nonlinear constraints: 0
Total nonzeros: 8
Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
N	0.5778620	0.000000
F	0.9945286	0.000000

Row	Slack or Surplus	Dual Price
1	77.00000	-1.000000
2	0.000000	-0.5000000
3	0.000000	-0.5000000
4	6.666667	0.000000

Hence the optimum solution is $M_c = 77$ when $O_n = 0.5778620$ and $O_f = 0.9945286$.

Model 3.2: Power Generations Model

As per the Ministry of Power, Government of India [9], the majority of power generation in India is dependent on conventional sources such as Thermal, Nuclear, and Hydro. The Ministry also states that thermal power plants play a significant role in the production of electricity using coal in India.
During the years 2017 to 2020, the Ministry of Power, Government of India [10] collected data on the daily power generation status of each state and region in India.
The following table (Table 3.2.1) provides a list of states and their respective regions, along with their national shares.

Table 3.2.1 State and Region wise National share

State / Union territory (UT)	Region	National Share (%)
Rajasthan	Northern	10.55
Madhya Pradesh	Central	9.37
Maharashtra	Western	9.36
Uttar Pradesh	Northern	7.33
Gujarat	Western	5.96
Karnataka	Southern	5.83
Andhra Pradesh	Southern	4.87
Odisha	Eastern	4.73
Chhattisgarh	Central	4.11
Tamil Nadu	Southern	3.95
Telangana	Southern	3.49
Bihar	Eastern	2.86
West Bengal	Eastern	2.7
Arunachal Pradesh	Northeastern	2.54
Jharkhand	Eastern	2.42
Assam	Northeastern	2.38
Ladakh	Northern	1.8
Himachal Pradesh	Northern	1.7
Uttarakhand	Northern	1.62
Punjab	Northern	1.53
Haryana	Northern	1.34
Jammu and Kashmir	Northern	1.28
Kerala	Southern	1.18
Meghalaya	Northeastern	0.68
Manipur	Northeastern	0.68
Mizoram	Northeastern	0.64
Nagaland	Northeastern	0.5
Tripura	Northeastern	0.31
Sikkim	Northeastern	0.21
Goa	Western	0.11
Delhi	Northern	0.04
Dadra and Nagar Haveli and Daman and Diu	Western	0.01
Puducherry	Southern	0.01
Chandigarh	Northern	0.003

The data provided indicates that the actual and estimated power generation from thermal, nuclear, and hydro sources is uncertain. To address this, suppose assumed that the power generation units in lakhs and represented all the data as trapezoidal fuzzy numbers. Based on this approach, a fully fuzzy linear programming problem has been formulated as follows:
Let $\widetilde{t_g}$, $\widetilde{n_g}$ and $\widetilde{h_g}$ denote the number of units estimated in thermal generation, nuclear generation and hydro generation respectively.
Maximize Z = (26, 28, 30, 32) $\widetilde{t_g}$ + (0.8, 1, 1.2, 1.4) $\widetilde{n_g}$ + (3.2, 3.4, 3.6, 3.8) $\widetilde{h_g}$
Subject to the constraints
(5, 5.2, 5.4, 5.6) $\widetilde{t_g}$ + (0.1, 0.3, 0.5, 0.7) $\widetilde{n_g}$ + (0.4, 0.6, 0.8, 1) $\widetilde{h_g}$ ≤ (6.3, 6.5, 6.7, 6.9)
(2, 4, 6, 8) $\widetilde{t_g}$ + (0.1, 0.2, 0.3, 0.4) $\widetilde{n_g}$ + (0, 1, 2, 3) $\widetilde{h_g}$ ≤ (8, 8.2, 8.4, 8.6)
(0.1, 0.2, 0.3, 0.4) $\widetilde{t_g}$ + (0, 0, 0, 0) $\widetilde{n_g}$ + (0, 0.1, 0.2, 0.3) $\widetilde{h_g}$ ≤ (0.1, 0.3, 0.5, 0.7)
(4.3, 4.5, 4.7, 4.9) $\widetilde{t_g}$ + (0, 0, 0, 0) $\widetilde{n_g}$ + (0.1, 0.3, 0.5, 0.7) $\widetilde{h_g}$ ≤ (4.8, 5, 5.2, 5.4)
(8, 10, 12, 14) $\widetilde{t_g}$ + (0.1, 0.2, 0.3, 0.4) $\widetilde{n_g}$ + (0.1, 0.2, 0.4, 0.5) $\widetilde{h_g}$ ≤ (12.2, 12.4, 12.6, 12.8)
$\widetilde{t_g}, \widetilde{n_g}, \widetilde{h_g} \geq 0$

Existing Method (Maleki's ranking function)

To obtain the solution of this fully fuzzy linear programming problem, it will first be converted into a crisp linear programming problem using Malek's ranking function $\Re(\widetilde{A_T}) = \frac{1}{2}\left(p + s + \frac{1}{2}(r - q)\right)$.
Now, the corresponding crisp linear programming problem is:
Max 29.5 t_g + 1.15 n_g + 3.55 h_g
5.35 t_g + 0.45 n_g + 0.75 h_g ≤ 6.65
5.5 t_g + 0.275 n_g + 1.75 h_g ≤ 8.35
0.275 t_g + 0 n_g + 0.175 h_g ≤ 0.45
4.65 t_g + 0 n_g + 0.45 h_g ≤ 5.15
11.5 t_g + 0.275 n_g + 0.35 h_g ≤ 12.55
$t_g, n_g, h_g \geq 0$
Using **Lingo** software, obtain the following output:
Global optimal solution found.

Objective value:	34.62372
Infeasibilities:	0.000000
Total solver iterations:	4
Elapsed runtime seconds:	0.75
Model Class:	LP
Total variables:	3
Nonlinear variables:	0
Integer variables:	0
Total constraints:	6
Nonlinear constraints:	0
Total nonzeros:	16
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
T	1.012681	0.000000
N	1.104670	0.000000
H	0.9800725	0.000000

Row	Slack or Surplus	Dual Price
1	34.62372	1.000000
2	0.000000	2.555556
3	0.7613426	0.000000
4	0.000000	0.6847826
5	0.000000	3.363325
6	0.2573571	0.000000

Hence the optimum solution is Z = 34.62372 when t_g= 1.012681, n_g = 1.104670, h_g= 0.9800725.

Proposed Method ($\vartheta_{\mathcal{R}}$ – ranking function)

To solve this fully fuzzy linear programming problem, it will first be converted into a crisp linear programming problem by using the following ranking function:
$\boldsymbol{\vartheta}_{\mathcal{R}}(\widetilde{A_T}) = \frac{p}{6} + \frac{q}{3} + \frac{r}{3} + \frac{s}{6} = \frac{p+2q+2r+s}{6}$.
Now, the corresponding crisp linear programming problem is:
Maximize Z = 29 t_g + 1.1n_g + 3.5 h_g

5.3 t_g+ 0.4 n_g+ 0.7 h_g $\leq$ 6.6
5 t_g + 0.3 n_g+ 1.5 h_g $\leq$ 8.3
0.3 t_g + 0n_g + 0.2 h_g $\leq$ 0.4
4.6 t_g+ 0 n_g+ 0.4 h_g $\leq$ 5.1
11t_g + 0.3 n_g+ 0.3h_g $\leq$12.5
$t_g, n_g, h_g \geq 0$
Using **Lingo** software, obtain the following output:
Global optimal solution found.
Objective value: 34.26719
Infeasibilities: 0.000000
Total solver iterations: 3
Elapsed runtime seconds: 0.04
Model Class: LP
Total variables: 3
Nonlinear variables: 0
Integer variables: 0
Total constraints: 6
Nonlinear constraints: 0
Total nonzeros: 16
Nonlinear nonzeros: 0

Variable	Value	Reduced Cost
T	1.075000	0.000000
N	1.578125	0.000000
H	0.3875000	0.000000

Row	Slack or Surplus	Dual Price
1	34.26719	1.000000
2	0.000000	2.750000
3	1.870313	0.000000
4	0.000000	1.843750
5	0.000000	3.015625
6	0.8531250E-01	0.000000

Hence the optimum solution is Z = 34.26719 when t_g= 1.07500, n_g = 1.57813, h_g= 0.38750.

4. CONCLUSION

Fuzzy optimization is a valuable method for solving decision-making problems that involve uncertainty. This paper has proposed a new ranking function that utilizes the value of trapezoidal fuzzy numbers. The paper also has discussed two important models namely, shopping and electricity production. The study employs $\boldsymbol{\vartheta}_{\mathcal{R}}$ −ranking function and fuzzy linear programming techniques to obtain the optimal solutions of both models. The results are then compared with Maleki's ranking function. When comparing the two models, it becomes evident that the optimal solution is nearly identical when applying both methods. Additionally, the $\boldsymbol{\vartheta}_{\mathcal{R}}$ −ranking function proves to be easier to use. This approach provides new opportunities for further research in the field of fuzzy optimization, especially for those interested in real-life applications and decision-making.

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