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Two Warehouse Eoq Model For Imperfect Quality And Reworkable Items With Parabolic Time Dependent Demand & Non-Linear Holding Cost Under Trade Credit Financing Policy Subject To Partial Backlogging

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ABSTRACT

A two warehouse (Own warehouse and Rented warehouse) EOQ model for imperfect quality items with parabolic time dependent demand has been developed. It is assumed that the supplier has perfect and imperfect quality items. When the supplier provides lots for sale to his retailer, the retailer separates the whole lot by inspection (Screening) process into perfect and imperfect quality items. The retailer will send the imperfect quality items to repairing shop to make it as a perfect quality item. The cost of holding rises over time as longer storage periods requires more advanced and expansive storage facilities. Furthermore, managing larger inventory volumes often entails securing extra warehouse space (RW), potentially through rental agreements, resulting in increased holding expenses. Additionally, purchasing and storing greater quantities requires more substantial capital investments, typically financed at higher borrowing rates. Therefore, non-linear varying holding Cost in the Rented Warehouse has been considered. Shortage at Own warehouse is allowed subject to partial backlogging. With this assumption, the model has been framed with learning effect and trade credit financing policy. Different cases based on the trade credit period have been considered. The objective of this work is to minimize the total inventory cost and to find the optimal length of replenishment and the optimal order quantity. Computational algorithms are designed to find the optimal order quantity and the optimal cycle time. To elucidate our model, hypothetical numerical examples and sensitivity analysis are carried out to decide the feasibility of renting a warehouse with varying holding cost by availing a trade credit period for imperfect quality items.

Keywords: Two warehouses, Imperfect quality items, Parabolic time dependent demand, Trade credit financing policy, Partial backlogging, Rework, Non-linear holding cost

1. INTRODUCION

Demand is crucial in developing an inventory model. In real-life inventory systems, the demand rate of an item is often influenced by time. Traditional inventory models assume a constant demand rate, which is not always applicable to many items like fashionable clothes and electronic goods, which experience fluctuating demand rates. Many products go through a period of rising demand during the growth phase of their life cycle. The quadratic demand technique is used to determine the optimal ordering policy, as it better represents time-varying market demands. Some researchers suggest that rapidly increasing demand can be modeled by an exponential function of time. However, this assumption may not accurately reflect the fluctuations in real

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market conditions. The accelerated growth in demand for items such as computer chips for machines and spare parts for new airplanes is best represented by a quadratic function of time.

Time-varying demands were first considered by Silver and Meal [36]. Ghosh and Chaudhuri [12], Khanra and Chaudhuri [22] etc., established their models in which quadratic time-varying demand was considered. Sana and Chaudhuri [34] developed production policy for a deteriorating item with time-dependent demand and shortages. Begum et al. [3] has developed an EOQ model with quadratic demand with Weibull distribution deterioration. Khanra et al. [23] considered an EOQ model with stock and price dependent demand rate. In the opinion of many authors, an alternative or perhaps more realistic approach is to consider quadratic time-dependence of demand. This demand may represent all types of time-dependence depending on the signs of the parameters of the time-quadratic demand function.

In real life, not all products are perfect in nature. Sellers' inspection policies help maintain their reputation, meet customer demand, and minimize total costs. Jaggi et al. [4, 20], a mathematical model for the inventory was proposed for the imperfect items with the policy of financing period under shortages and permissible delay on payment. In Jaber and Salameh [19], a mathematical model was derived with shortages and backorder under leaning effect. Further, in Jaber et al. [17], the idea was stretched with the help of learning concepts for the imperfect items. A mathematical idea under impact of learning for the imperfect items was established in Khan et al. [21]. In Jaber and Khan [18], a model was discussed about the order of lots and the number of shipments for the imperfect items using the concept of learning. A model for defective items under learning effect and shortages was offered by Konstantaras et al. [24]. Wahab and Jaber [28] developed a note on Economic order quantity model for items with imperfect quality, different holding costs, and learning effects. Goyal [38] developed Economic order quantity model for imperfect lot with partial backordering under the effect of learning and advertisement dependent imprecise demand. After that, an optimal quantity model of Sangal et al. [35] was proposed with learning impact and shortages where deterioration is a function of time. In De and Mahata [7], an inventory model was investigated for defective items under cloudy fuzzy atmosphere. The commendable work in De and Mahata [7] has been improved by this present paper with the help of a learning effect and the financing period, where defective items follow the S-shape learning curve. Kuppulakshmi et al. [27] considered a fuzzy-based inventory model for defective items under penalty cost. The effects of learning operate as a significant function for reducing the inventory cost and optimizing the total profit of the inventory system. Osama et al. [30] developed a Supply Chain Model with Learning Effect and Credit Financing Policy for Imperfect Quality Items under Fuzzy Environment.

Industrialist must invest their capital not only in the inventory also invest in holding the inventory. For instance, preserving perishable goods like food and pharmaceuticals over extended periods demands refrigeration and specialized handling to prevent deterioration. Likewise, the storage of volatile or flammable liquids requires intensive precautions and stricter safety protocols for prolonged periods. Moreover, the unit holding cost typically escalates with the inventory size, reflecting the increased quantity of items stored. In this aspect we considered the nonlinear variable holding costs in rented warehouse. The research with varying holding cost began with the work of Goh [14] who developed a stock-dependent insist model with variable holding costs, and unspecified that the unit holding cost is a non-linear continuous function of the time the item is in stock or a non-linear continuous function of the inventory level. Giri and Chaudhuri [13] extended this model to account for perishable products. After that, Alfares [2] proposed the inventory model with stock-level dependent demand rate and variable holding cost. Roy [33] developed an inventory model for deteriorating items with time varying holding cost and demand is price dependent. Mishra and Singh [29] developed the inventory model for deteriorating items with time dependent linear demand and holding cost. Tyagi et al. [40] developed an inventory lot-size model for decaying item following the power patterns of demand of item. In this study, shortages were allowed and partial backlogged inversely with the waiting time for the next replenishment. Tyagi and Pandey [41] developed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and variable holding cost.

Many researchers have developed their inventory model for a single warehouse which has unlimited capacity. This assumption is not applicable in real-life situation. When an attractive price discount for bulk purchase, products are seasonal and high demand is available, the management decides to purchase a huge quantity of items at a time. These goods cannot be stored in the existing storage (the owned warehouse with limited capacity). Another equally important aspect associated with inventory management is to decide where to stock the goods. In such scenarios, they might opt to procure large quantities of items, leading to storage challenges. This additional storage capacity may be a Rented Warehouse (RW). Inventory model with double storage facility OW and RW was first developed by Hartley [16] in which he assumed that the holding cost in RW is greater than that in OW, due to the non-availability of better preserving facility which results in higher deterioration rate; therefore, items in RW are first transferred to OW to meet the demand until the stock level in RW drops to zero, and then items in OW are released. Hence to reduce the holding cost, it is more economical to consume the goods of rented warehouse at the earliest. Palanivel M, Sundararajan R, Uthayakumar R [31] Two warehouse inventory model with non-instantaneously deteriorating items, stock dependent demand, shortages and inflation, Sahu and Bishi [37] extended the inventory deteriorating Items under permissible delay in payments. After his pioneering contribution, several other researchers have attempted to extend his work to various other realistic situations.

In the case of perishable products, the retailer may need to backlog demand to avoid costs associated with deterioration. When shortages occur, some customers are willing to wait for back-ordered items, while others may purchase from other sellers. H.J. Chang and Dye [5], as well as Dye, Ouyang, and Hsieh [10], developed an inventory model for deteriorating items with a time-proportional backlogging rate. Park [32] and Wang [44] also studied shortages and partial backlogging of items. Tan and Weng [39] developed a discrete-in-time deteriorating inventory model with time-varying demand, variable deterioration rate and waiting time dependent partial backlogging. Zhou, Wan, Zhang, and Li [46] investigated an optimal quality level, order quantity and selling price for the retailer in a two-level supply chain. Palanivel M, Sundararajan R, Uthayakumar R [31] Two warehouse inventory model with partial backlogging.
In earlier times, payment for items was made at the time of delivery or beforehand. However, in today's complex and expansive business environment, this practice is no longer feasible. Modern retailers are not required to settle their dues at delivery. Trade credit, or permissible delay in payment, has become a standard practice in business. It provides a grace period for retailers to complete their payments, acting as an essential tool for financing growth. The duration of this credit period is determined by mutual agreement between the supplier and the retailer. This extension allows retailers to sell the goods and use the revenue to repay the debt. Trade credit is often extended to boost sales, allowing retailers to accumulate interest on the sales revenue during this period. If the payment period is extended, the supplier charges interest on the unpaid balance, indirectly reducing holding costs. Additionally, trade credit encourages retailers to purchase more products, playing a significant role in inventory control for both suppliers and retailers. Goyal [15] developed an EOQ model under the condition of a permissible delay in payments. Aggarwal and Jaggi [1] then extended Goyal's model to allow for deteriorating items under permissible delay in payments. Uthayakumar and Geetha [42,43] developed a replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging and non-instantaneous deteriorating items with two levels of storage under trade credit policy.

This paper aims to develop a two-warehousing inventory model for imperfect quality reworkable or repairable items with quadratic time dependent (parabolic) demand and non-linear holding cost. The supplier offers the retailer a trade credit period to settle the amount. It is also assumed that the inventory holding cost in RW is higher than that in OW but the deterioration rate in RW is less than that in OW because RW offers better preserving facilities. In addition, shortages are allowed in OW and are partially backlogged. The optimal replenishment schedule has also been proposed. Finally, the numerical examples and managerial insights elucidate the performance of the model.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations have been used in the entire paper.

2.1 ASSUMPTIONS

1. Demand rate is known and which is parabolic time dependent. The Consumption rate $D(t)$ at time t is assumed to be
$D(t) = \begin{cases} a + b\,t + ct^2, & I(t) > 0, \\ B, & I(t) \leq 0 \end{cases}$ where a is the initial rate of demand and a positive constant, b and c are the time-dependent consumption rate parameters, b is the rate at which the demand rate increases and c is the rate at which the change in the demand rate itself increases $0 \leq b, c \leq 1$
2. The owned warehouse OW has limited capacity of W units and the rented warehouse RW has unlimited capacity. For economic reasons, the items of RW are consumed first and continues with those in OW once inventory stored at RW is exhausted. This implies that $t_r < T$.
3. The replenishment rate is infinite and the lead time is zero. The time horizon is infinite.
4. Both screening as well as demand proceeds simultaneously, but the screening rate is assumed to be greater than demand rate.
5. Defective items are all become perfect after rework process
6. Holding cost per unit per unit time in RW is $R + h_r e^{\theta t}, \mathrm{R}, h_r, \theta > 0$ Where R is the fixed rent of the RW per unit. Here, h_r is the holding cost or the cost of storage facilities and services required for keeping items in good conditions. θ represents the variation parameter in the cost of storing.
7. The supplier allows the retailer a grace period to settle the account on credit. During this trade credit period, the revenue is placed in an interest-bearing account. Once the permissible delay ends, the retailer pays for the ordered items and begins paying interest on the remaining stock.
8. Shortages are permitted and are partially backlogged, with the backlog rate defined as $\frac{1}{1+\delta(T-t)}$ when the inventory level is negative. The backlogging parameter δ is a positive constant where $0 < \delta < 1$ and $(T - t)$ is the waiting time $(t_1 \leq t \leq T)$.

2.2 NOTATIONS

In addition, the following notations are used throughout this paper:

OW	-	The owned warehouse
RW	-	The rented warehouse
D	-	The demand per unit time
K	-	The replenishment cost per order ($/order)
p_1	-	The purchasing cost per perfect quality item ($/unit)
p_2	-	The purchasing cost per defective item ($/unit)
c_1	-	The selling price per unit item ($/unit)
c_r	-	The repairing or rework cost per defective item ($/unit)
c_s	-	The screening cost per unit item ($/unit)
S	-	The shortage cost per unit item ($/unit)
h_o	-	The holding cost per unit per unit time in OW
h_r	-	The holding cost per unit per unit time in RW
R	-	The fixed rent of the RW per unit
θ	-	The variation parameter in the cost of storing
π	-	The opportunity cost per unit item ($/unit)
M	-	Permissible delay in settling the accounts
α	-	Percentage of defective items
μ	-	Screening rate per unit item
I_p	-	The interest charged per dollar in stocks per year
I_e	-	The interest earned per dollar per year
$I_o(t)$	-	The inventory level in OW at time t
$I_r(t)$	-	The inventory level in RW at time t
I_m	-	Maximum Inventory level.
I_b	-	Maximum amount of shortage demand to be backlogged
W	-	The storage capacity of OW
Q	-	The retailer's order quantity (a decision variable)
TC_i	-	The total relevant costs
t_s	-	The time at which the screening process end in RW and OW
t_r	-	The time at which the inventory level reaches zero in RW
t_1	-	The time at which the inventory level reaches zero in OW
T	-	The length of replenishment cycle (a decision variable)

3. MODEL FORMULATION

This study develops a two-warehouse inventory model for reworkable imperfect quality items. In some cases, the retailer's own warehouse is not enough to store all goods, necessitating the use of a rented warehouse. To address this scenario, we formulate the replenishment problem for a two-warehousing inventory model for a single imperfect quality item with parabolic time-dependent demand under a trade credit financing policy and partial backlogging. Initially, items are stored in the Owned Warehouse (OW). Once the OW is full, the remaining items are stored in the Rented Warehouse (RW), with the RW items used first to satisfy demand, thereby reducing inventory carrying charges (holding costs). The model considers non-linear time-varying holding costs in the RW. A lot size of Q units enters to the system instantaneously. After meeting the backorders, I_m units enter the inventory system, out of which W units are kept in OW and the remaining $(I_m - W)$ units are kept in the RW. It is assumed that each lot received contains percentage defectives α. The screening (inspection) process for the entire received lot occurs at a rate of μ units per unit time, if the screening rate exceeds the demand rate. During this period, the demand and screening processes run simultaneously, with demand being met by items deemed to be of perfect quality during screening. Defective items are sent to a repair shop to be restored to good condition, and all defective products are considered reworkable within the same cycle. Once reworked, these items are added to inventory to eliminate backorders and are sold at the same price as perfect items. Shortages are permitted and partially backlogged, and are resolved when the reworked items are added to the inventory system at t_1. The behavior of the inventory level is illustrated in Fig. 1, where T is the cycle length αQ is the number of defectives withdrawn from inventory, $t_s = \frac{Q}{\mu}$ is the total screening time of Q units ordered per cycle which is less than cycle time T and t_1, t_r are the inventory level reaches zero in OW and RW respectively.

Let $I_r(t)$ be the inventory level in RW at any time t, in the interval $(0, t_r)$. The change in the inventory level in RW is given by the following differential equation

$$\frac{dI_r(t)}{dt} = -(a + b\,t + ct^2), \qquad 0 \le t \le t_r$$

The solution of this differential equation with the initial condition $I_r(0) = (I_m - W)$ is given by

$I_{r1}(t) = \frac{-1}{3}ct^3 - \frac{1}{2}bt^2 - at + (I_m - W) \qquad 0 \le t \le t_s$

After the screening process, the number of defective items in RW at time t_s is $\alpha(I_m - W)$

Hence, the inventory level in RW during $t_s \le t \le t_r$ is given by

$I_{r2}(t) = \frac{-1}{3}ct^3 - \frac{1}{2}bt^2 - at + (I_m - W) - \alpha(I_m - W)$

From the boundary condition $I_r(t_r) = 0$, the maximum inventory level as

$I_M = W + \frac{t_r(2ct_r^2+3bt_r+6a)}{6(1-\alpha)}$

Let $I_o(t)$ be the inventory level in OW at any time t, in the interval $(0, t_1)$. The change in the inventory level in OW is given by the following differential equation

$$\frac{dI_o(t)}{dt} = -(a + b\,t + ct^2), \qquad 0 \le t \le t_1$$

Since there is no change in the inventory level in OW at the interval $(0, t_r)$ due to demand satisfied from RW, the inventory level in OW is

$I_{o1}(t) = W \qquad 0 \le t \le t_s$

After the screening process, the number of defective items in OW at time t_s is αW

Therefore, the inventory level in OW at the interval $(t_s\,, t_r)$ is given by

$I_{o2}(t) = (1 - \alpha)W \qquad t_s \le t \le t_r$

During the interval (t_r, t_1) the inventory level decreases to zero due to demand and from the boundary condition $I_{o3}(t_1) = 0$, the inventory level in OW for this period is given by

$I_{o3}(t) = \frac{-1}{3}ct^3 - \frac{1}{2}bt^2 - at + \frac{1}{3}ct_1{}^3 + \frac{1}{2}bt_1{}^2 + at_1 - \alpha W$

During the interval (t_1, T), the inventory level is given by the differential equation

$$\frac{dI_{o4}(t)}{dt} = \frac{-B}{1 + \delta(T - t)}, \qquad t_1 \le t \le T$$

Solving the above differential equation with the condition $I_{o4}(t_1) = 0$ is given by

$I_{o4}(t) = \frac{B}{\delta}\{\log(1 + \delta(T - t)) - \log(1 + \delta(T - t_1))\}$

Furthermore, the continuity of $I_{o2}(t_r) = I_{o3}(t_r)$ we get

$W = \frac{1}{3}c(t_1{}^3 - t_r{}^3) + \frac{1}{2}b(t_1{}^2 - t_r{}^2) + at_r$

The maximum backlogging quantity is given by

$I_{o4}(T) = I_b$

$I_b = -\frac{B}{\delta}\{\log(1 + \delta(T - t_1))\}$

Hence the maximum order quantity is $Q = I_m + I_b$

The total inventory cost per cycle consists of the following elements

a) Cost of placing order is K

b) Sales Revenue is the sum of revenue generated by the demand meet during the period $(0, T)$ and by the sale of items is c_1Q

c) Purchase Cost is $p_1(1 - \alpha)Q + p_2\alpha Q$

d) Screening Cost is c_sQ

e) Repairing or Reworking cost is $c_r\alpha Q$

f) Inventory holding cost HC in RW per cycle is given by

$HC = R\left\{\int_0^{t_s} I_{r1}(t)dt + \int_{t_s}^{t_r} I_{r2}(t)dt\right\} + h_r\left\{\int_0^{t_s} e^{\theta t}I_{r1}(t)dt + \int_{t_s}^{t_r} e^{\theta t}I_{r2}(t)dt\right\}$

$HC = R\left\{-\frac{ct_r{}^4}{12} - \frac{bt_r{}^3}{6} - \frac{at_r{}^2}{2} + (1 - \alpha)t_r(I_m - W) + \alpha t_s(I_m - W)\right\} + \frac{h_r}{b^3}\left\{\left(\frac{(I_m-W)e^{\theta t_s}}{\theta}\right) + \frac{1}{6}\frac{e^{\theta t_r}}{\theta^4}\{(-2ct_r{}^3 - 3bt_r{}^2 - 6at_r + 6(1 - \alpha)(I_m - W))\theta^3 + (6ct_r{}^2 + 6a + 6bt_r)\theta^2 + (-12ct_r - 6b)\theta + 12c\} - \frac{(I_m-W)}{\theta} - \frac{a\theta^2-b\theta+2c}{\theta^4}\right\}$

g) Inventory holding cost HC in OW per cycle is given by

$HC = h_o\left\{\int_0^{t_s} I_{o1}(t)dt + \int_{t_s}^{t_r} I_{o2}(t)dt + \int_{t_r}^{t_1} I_{o3}(t)dt\right\}$

$HC = h_o\left\{\frac{ct_r{}^4}{12} + \frac{bt_r{}^3}{6} + \frac{at_r{}^2}{2} + \left(W - \frac{bt_1{}^2}{2} - \frac{ct_1{}^3}{3} - at_1\right)t_r + \frac{bt_1{}^3}{3} + W\alpha(t_s - t_1) + \frac{ct_1{}^4}{4} + \frac{at_1{}^2}{2}\right\}$

h) Shortage cost per cycle SC is given by

$SC = s\int_{t_1}^{T} -I_{o4}(t)dt$

$SC = -\frac{sB}{\delta^2}\{\log(1 + \delta(T - t_1)) - \delta(T - t_1)\}$

i) Opportunity cost per cycle due to lost sales OC is given by

$OC = \pi\int_{t_1}^{T}\left(B - \frac{B}{1+\delta(T-t)}\right)dt$

$OC = \pi B\left\{(T - t_1) - \frac{\log(1+\delta(T-t_1))}{\delta}\right\}$

Based on the assumptions and description of the model, the total annual cost which is a function of t_1, and T is given by

$$TC(t_1,T)=\begin{cases}TC_1(t_1,T), & t_s<M\le t_r\\ TC_2(t_1,T), & t_r<M\le t_1\\ TC_3(t_1,T), & M>t_1\end{cases}$$

Figure 1 depicts the following 3 cases.

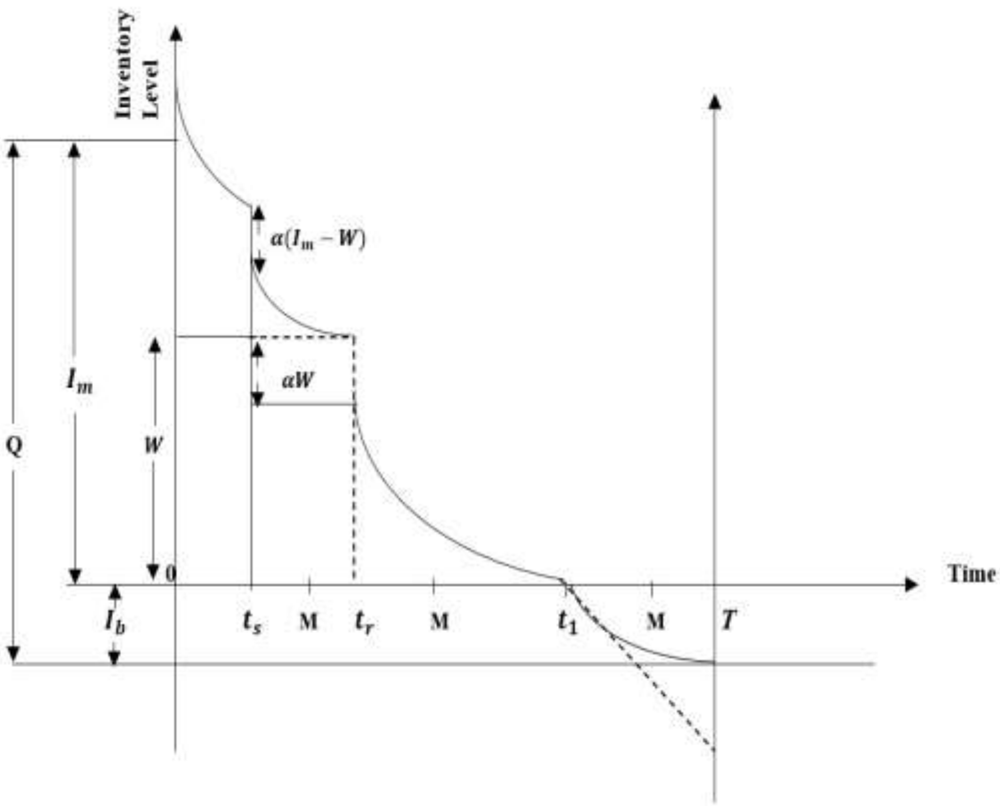

Fig: 1

3.1 CASE 1: $0 < t_s \le M \le t_r$

When the credit period is shorter than or equal to the length of period with positive inventory stock of items ($M \le t_1$), payment for goods is settled and retailer starts paying the interest for the goods still in stocks with annual rate I_p. Thus, the interest payable denoted by IP_1 and it is given by

$$IP_1 = p_1 I_p\left\{\int_M^{t_r} I_{r2}(t)dt + \int_M^{t_r} I_{o2}(t)dt + \int_{t_r}^{t_1} I_{o3}(t)dt\right\}$$

$$IP_1 = p_1 I_p\left\{\left(\frac{-ct_1^3}{3} - \frac{bt_1^2}{2} - at_1 + \alpha W + I_M(1-\alpha)\right)t_r + \frac{ct_1^4}{4} + \frac{bt_1^3}{3} + \frac{at_1^2}{2} - \alpha W t_1 + \frac{M}{12}\left(cM^3 + 2bM^2 + 6aM - 12I_M(1-\alpha)\right)\right\}$$

We assume that during the time when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns the interest with a rate I_e. Thus, the interest earned per cycle is given by IE_1

$$IE_1 = c_1 I_e\left\{\int_0^{t_s}(a+bt+ct^2)tdt + \int_{t_s}^{M}(a+bt+ct^2)tdt\right\}$$

$$IE_1 = c_1 I_e\left\{\frac{M^2}{12}(3cM^2 + 4bM + 6a)\right\}$$

Thus, the total annual cost which is a function of t_1 and T is given by

$$TC_1(t_1,T) = \frac{1}{T}\{K - \text{Sales Revenue} + \text{Purchase Cost} + \text{Screening Cost} + \text{Repairing (Rework)cost} + HC + SC + OC + IP_1 - IE_1\}$$

$$TC_1(t_1,T) = \frac{1}{T}\Bigg\{K - \{c_1\alpha Q\} + p_1(1-\alpha)Q + p_2\alpha Q + c_s Q + c_r\alpha Q + R\left\{-\frac{ct_r^4}{12} - \frac{bt_r^3}{6} - \frac{at_r^2}{2} + (1-\alpha)t_r(I_m - W) + \alpha t_s(I_m - W)\right\} + \frac{h_r}{b^3}\left\{\left(\frac{(I_m-W)e^{\theta t_s}}{\theta}\right) + \frac{1}{6}\frac{e^{\theta t_r}}{\theta^4}\{(-2ct_r^3 - 3bt_r^2 - 6at_r + 6(1-\alpha)(I_m - W))\theta^3 + (6ct_r^2 + 6a + 6bt_r)\theta^2 + (-12ct_r - 6b)\theta + 12c\} - \frac{(I_m-W)}{\theta} - \frac{a\theta^2 - b\theta + 2c}{\theta^4}\right\} + h_o\left\{\frac{ct_r^4}{12} + \frac{bt_r^3}{6} + \frac{at_r^2}{2} + \left(W - \frac{bt_1^2}{2} - \frac{ct_1^3}{3} - at_1\right)t_r +\right.$$

$\frac{bt_1^3}{3} + W\alpha(t_s - t_1) + \frac{ct_1^4}{4} + \frac{at_1^2}{2}\Big\} - \frac{sB}{\delta^2}\{\log(1 + \delta(T - t_1)) - \delta(T - t_1)\} + \pi B\left\{(T - t_1) - \frac{\log(1+\delta(T-t_1))}{\delta}\right\} +$
$p_1 I_p \left\{\left(\frac{-ct_1^3}{3} - \frac{bt_1^2}{2} - at_1 + \alpha W + I_M(1 - \alpha)\right) t_r + \frac{ct_1^4}{4} + \frac{bt_1^3}{3} + \frac{at_1^2}{2} - \alpha W t_1 + \frac{M}{12}\left(cM^3 + 2bM^2 + 6aM - 12I_M(1 - \right.\right.$
$\left.\left.\alpha)\right)\right\} - c_1 I_e \left\{\frac{M^2}{12}(3cM^2 + 4bM + 6a)\right\}\Bigg\}$

3.2 CASE 2: $t_r \leq M \leq t_1$

Interest payable for this period is denoted by IP_2 is given by

$IP_2 = p_1 I_p \left\{\int_M^{t_1} I_{o3}(t)dt\right\}$

$IP_2 = p_1 I_p \left\{\frac{(M-t_1)}{12}(-3ct_1^3 + (cM - 4b)t_1^2 + (cM^2 + 2bM - 6a)t_1) + cM^3 + 2bM^2 + 6aM + 12\alpha W\right\}$

The interest earned from the accumulated sales during this period is

$IE_2 = c_1 I_e \left\{\int_0^{t_s}(a + bt + ct^2)tdt + \int_{t_s}^{t_r}(a + bt + ct^2)tdt + \int_{t_r}^{M}(a + bt + ct^2)tdt\right\}$

$IE_2 = c_1 I_e \left\{\frac{M^2}{12}(3cM^2 + 4bM + 6a)\right\}$

Thus, the total annual cost which is a function of t_1 and T is given by

$TC_2(t_1, T) = \frac{1}{T}\{K - \text{Sales Revenue} + \text{Purchase Cost} + \text{Screening Cost} + \text{Repairing (Rework)cost} + HC + SC + OC + IP_2 - IE_2\}$

$TC_2(t_1, T) = \frac{1}{T}\Bigg\{K - \{c_1\alpha Q\} + p_1(1 - \alpha)Q + p_2\alpha Q + c_s Q + c_r\alpha Q + R\left\{-\frac{ct_r^4}{12} - \frac{bt_r^3}{6} - \frac{at_r^2}{2} + (1 - \alpha)t_r(I_m - W) + \right.$
$\left.\alpha t_s(I_m - W)\right\} + \frac{h_r}{b^3}\left\{\left(\frac{(I_m-W)e^{\theta t_s}}{\theta}\right) + \frac{1}{6}\frac{e^{\theta t_r}}{\theta^4}\{(-2ct_r^3 - 3bt_r^2 - 6at_r + 6(1 - \alpha)(I_m - W))\theta^3 + (6ct_r^2 + 6a + \right.$
$\left.6bt_r)\theta^2 + (-12ct_r - 6b)\theta + 12c\} - \frac{(I_m-W)}{\theta} - \frac{a\theta^2-b\theta+2c}{\theta^4}\right\} + h_o\left\{\frac{ct_r^4}{12} + \frac{bt_r^3}{6} + \frac{at_r^2}{2} + \left(W - \frac{bt_1^2}{2} - \frac{ct_1^3}{3} - at_1\right)t_r + \right.$
$\left.\frac{bt_1^3}{3} + W\alpha(t_s - t_1) + \frac{ct_1^4}{4} + \frac{at_1^2}{2}\right\} - \frac{sB}{\delta^2}\{\log(1 + \delta(T - t_1)) - \delta(T - t_1)\} + \pi B\left\{(T - t_1) - \frac{\log(1+\delta(T-t_1))}{\delta}\right\} +$
$p_1 I_p \left\{\frac{(M-t_1)}{12}(-3ct_1^3 + (cM - 4b)t_1^2 + (cM^2 + 2bM - 6a)t_1) + cM^3 + 2bM^2 + 6aM + 12\alpha W\right\} -$
$c_1 I_e \left\{\frac{M^2}{12}(3cM^2 + 4bM + 6a)\right\}\Bigg\}$

3.3 CASE 3: $t_1 \leq M \leq T$

For this period there is no interest payable, but the interest earned from the accumulated sales during this period is

$IE_3 = c_1 I_e \left\{\int_0^{t_s}(a + bt + ct^2)tdt + \int_{t_s}^{t_r}(a + bt + ct^2)tdt + \int_{t_r}^{t_1}(a + bt + ct^2)tdt\right\} + c_1 I_e\{\alpha Q\} - (M -$
$t_1)\left\{c_1 I_e \left\{\int_0^{t_s}(a + bt + ct^2)dt + \int_{t_s}^{t_r}(a + bt + ct^2)dt + \int_{t_r}^{t_1}(a + bt + ct^2)dt\right\} + c_1 I_e\{\alpha Q\}\right\}$

$IE_3 = c_1 I_e \left\{\frac{t_1^2}{12}(3ct_1^2 + 4bt_1 + 6a) + \alpha Q + (M - t_1)\left\{\frac{t_1}{6}(2ct_1^2 + 3bt_1 + 6a) + \alpha Q\right\}\right\}$

Thus, the total annual cost which is a function of t_1 and T is given by

$$TC_3(t_1, T) = \frac{1}{T}\{K - \text{Sales Revenue} + \text{Purchase Cost} + \text{Screening Cost} + \text{Repairing (Rework)cost} + HC + SC + OC - IE_3\}$$

$$TC_3(t_1,T) = \frac{1}{T}\Bigg\{K - c_1\alpha Q + p_1(1-\alpha)Q + p_2\alpha Q + c_sQ + c_r\alpha Q$$
$$+ R\left\{-\frac{ct_r^{\,4}}{12} - \frac{bt_r^{\,3}}{6} - \frac{at_r^{\,2}}{2} + (1-\alpha)t_r(I_m - W) + \alpha t_s(I_m - W)\right\}$$
$$+ \frac{h_r}{b^3}\Bigg\{\left(\frac{(I_m - W)e^{\theta t_s}}{\theta}\right)$$
$$+ \frac{1}{6}\frac{e^{\theta t_r}}{\theta^4}\{(-2ct_r^{\,3} - 3bt_r^{\,2} - 6at_r + 6(1-\alpha)(I_m - W))\theta^3 + (6ct_r^{\,2} + 6a + 6bt_r)\theta^2$$
$$+ (-12ct_r - 6b)\theta + 12c\} - \frac{(I_m - W)}{\theta} - \frac{a\theta^2 - b\theta + 2c}{\theta^4}\Bigg\}$$
$$+ h_o\left\{\frac{ct_r^{\,4}}{12} + \frac{bt_r^{\,3}}{6} + \frac{at_r^{\,2}}{2} + \left(W - \frac{bt_1^{\,2}}{2} - \frac{ct_1^{\,3}}{3} - at_1\right)t_r + \frac{bt_1^{\,3}}{3} + W\alpha(t_s - t_1) + \frac{ct_1^{\,4}}{4} + \frac{at_1^{\,2}}{2}\right\}$$
$$- \frac{sB}{\delta^2}\{\log(1 + \delta(T - t_1)) - \delta(T - t_1)\} + \pi B\left\{(T - t_1) - \frac{\log(1 + \delta(T - t_1))}{\delta}\right\}$$
$$- c_1I_e\left\{\frac{t_1^{\,2}}{12}(3ct_1^{\,2} + 4bt_1 + 6a) + \alpha Q + (M - t_1)\left\{\frac{t_1}{6}(2ct_1^{\,2} + 3bt_1 + 6a) + \alpha Q\right\}\right\}\Bigg\}$$

The necessary conditions for the total annual cost $\partial TC_i(t_1,T)$ is convex with respect to t_1 and T are $\frac{\partial TC_i(t_1,T)}{\partial t_1} = 0$ and $\frac{\partial TC_i(t_1,T)}{\partial T} = 0$ where $i = 1,2,3$ (1)

Provided they satisfy the sufficient conditions

$$\left.\frac{\partial^2 TC_i(t_1,T))}{\partial t_1^{\,2}}\right|_{(t_1^*,T^*)} > 0, \left.\frac{\partial^2 TC_i(t_1,T))}{\partial T^2}\right|_{(t_1^*,T^*)} > 0 \text{ and}$$
$$\left.\left\{\left(\frac{\partial^2 TC_i(t_1,T)}{\partial t_1^{\,2}}\right)\left(\frac{\partial^2 TC_i(t_1,T))}{\partial T^2}\right) - \left(\frac{\partial^2 TC_i(t_1,T)}{\partial t_1 \partial T}\right)^2\right\}\right|_{(t_1^*,T^*)} > 0 \quad (2)$$

To acquire the optimal values of t_1 and T that minimize $TC_i(t_1,T)$, we develop the following algorithm to find the optimal values of t_1 and T (say, t_1^* and T^*).

ALGORITHM

Step 1: Start

Step 2: Evaluate $\frac{\partial TC_i(t_1,T)}{\partial t_1}$ and $\frac{\partial TC_i(t_1,T)}{\partial T}$ where $i = 1,2,3$.

Step 3: Solve the simultaneous equation $\frac{\partial TC_i(t_1,T)}{\partial t_1} = 0$ and $\frac{\partial TC_i(t_1,T)}{\partial T} = 0$ by fixing M, t_r and initializing the values of K, a, b, c, α, δ, s, π, $p_1, p_2, c_1, c_r, c_s, h_r, h_o, t_s, t_r, I_p, I_e$

Step 4: Choose one set of solution from step 3.

Step 5: If the values in equation (2) are greater than zero, then this set of solution is optimal and go to step 6. Otherwise go to step 4 with next set of solution obtained from step 3.

Step 6: Evaluate $TC_i(t_1^*, T^*)$

Step 7: End

Our aim is to find the optimal values of t_1 and T which minimize $TC(t_1^*, T^*)$

$TC(t_1^*, T^*) = \text{Min}\{TC_1(t_1^*, T^*), TC_2(t_1^*, T^*), TC_3(t_1^*, T^*)\}$

4. NUMERICAL EXAMPLES

The following examples illustrate our solution procedure:

Example 1: Consider an inventory system with the following data: $K = 1000, a = 100, b = 30, c = 20, t_s = 0.0274, t_r = 0.0822, R = 200, h_r = 3, h_o = 1.5, \theta = 0.2, p_1 = 20, p_2 = 15, c_1 = 25, c_r = 5, c_s = 0.5, s = 18, \pi = 12, M = 0.0411, \delta = 0.56, I_p = .13, I_e = 0.12, \alpha = 0.05, W = 50, B = 20$ in appropriate units. In this case, we see that $t_s < M < t_r$. Therefore, applying algorithm for Case 1, we get the optimal solutions, $t_1 = 0.7834$, $T = 1.0932$ the corresponding total cost $TC = 985.24$ and the ordering quantity $Q = 58.61$ units

Example 2: The data are the same as in Example 1 except $M = 0.1644$ in appropriate units. In this case, we see that $t_r < M < t_1$. Therefore, applying algorithm for Case 2, we get the optimal solutions, $t_1 = 0.6559, T = 1.0163$ the corresponding total cost $TC = 875.92$ and the ordering quantity $Q = 59.46$ units.

Example 3: The data are the same as in Example 1 except $M = 0.5753$ in appropriate units. In this case, we see that $M > t_1$. Therefore, applying algorithm for Case 3, we get the optimal solutions, $t_1 = 0.5237, T = 0.9370$ the corresponding total cost $TC = 801.55$ and the ordering quantity $Q = 59.92$ units.

Effect of change in various parameters of the inventory in example 3

Changing parameter	% Change in parameter	Change in parameter	t_1	T	TC	Q
a	-20%	80	0.5835	0.9912	766.33	59.66
	-10%	90	0.5515	0.9619	785.17	59.99
	0%	100	0.5237	0.9370	801.55	60.33
	+10%	110	0.4993	0.9159	815.64	60.67
	+20%	120	0.4779	0.8978	824.64	61.02
b	-20%	24	0.5346	0.9484	792.76	60.34
	-10%	27	0.5290	0.9426	797.22	60.33
	0%	30	0.5237	0.9370	801.55	60.33
	+10%	33	0.5186	0.9317	805.76	60.33
	+20%	36	0.5136	0.9266	809.85	60.33
c	-20%	16	0.5274	0.9409	798.52	60.33
	-10%	18	0.5255	0.9389	800.05	60.33
	0%	20	0.5237	0.9370	801.55	60.33
	+10%	22	0.5219	0.9351	803.03	60.33
	+20%	24	0.5201	0.9333	804.49	60.33
δ	-20%	0.448	0.7685	1.3703	583.89	63.55
	-10%	0.504	0.6529	1.1579	667.21	61.89
	0%	0.560	0.5237	0.9370	801.55	60.33
	+10%	0.616	0.3788	0.7073	1038.73	58.88
	+20%	0.672	0.2183	0.4707	1539.33	57.56
α	-20%	0.040	0.5185	0.9234	789.04	60.17
	-10%	0.045	0.5211	0.9302	795.24	60.25
	0%	0.050	0.5237	0.9370	801.55	60.33
	+10%	0.055	0.5263	0.9438	807.96	60.42
	+20%	0.060	0.5289	0.9506	814.48	60.50
θ	-20%	0.16	0.5235	0.9368	801.53	60.31
	-10%	0.18	0.5236	0.9369	801.54	60.32
	0%	0.20	0.5237	0.9370	801.55	60.33
	+10%	0.22	0.5238	0.9371	801.56	60.34
	+20%	0.24	0.5239	0.9372	801.57	60.35
t_r	-20%	0.0658	0.4679	0.8573	886.47	59.94
	-10%	0.0740	0.4941	0.8943	845.25	60.12
	0%	0.0822	0.5237	0.9370	801.55	60.33
	+10%	0.0904	0.5565	0.9854	756.60	60.58
	+20%	0.0986	0.5922	1.0392	711.49	60.87
M	-20%	0.4602	0.5215	0.9329	806.22	60.30
	-10%	0.5178	0.5226	0.9349	803.88	60.32
	0%	0.5753	0.5237	0.9370	801.55	60.33
	+10%	0.6328	0.5248	0.9391	799.23	60.35
	+20%	0.6904	0.5258	0.9412	796.92	60.36

Graphical Representation of the effect of change in various parameters of the inventory

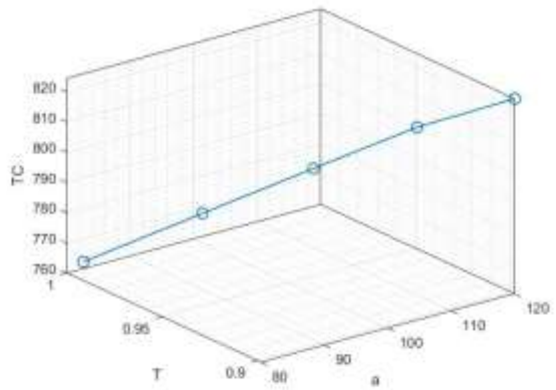

Fig 2. Effect of change of a on Total cost

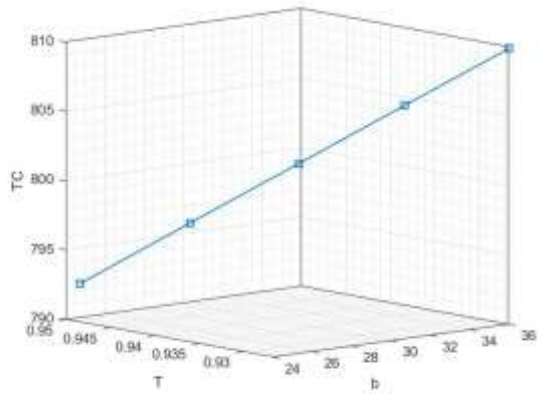

Fig 3. Effect of change of b on Total cost

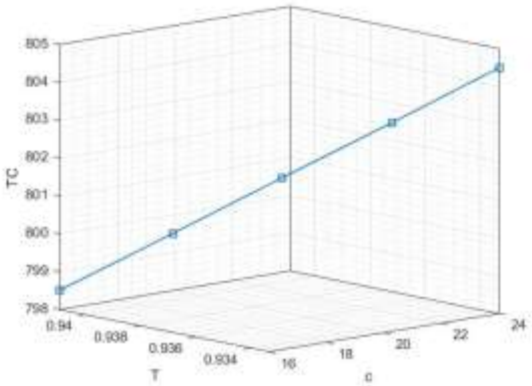

Fig 4. Effect of change of c on Total cost

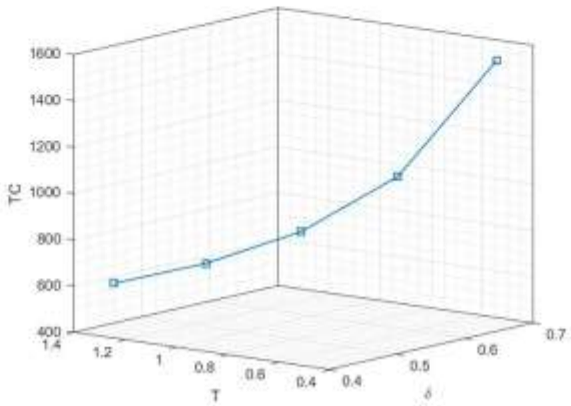

Fig 5. Effect of change of δ on Total cost

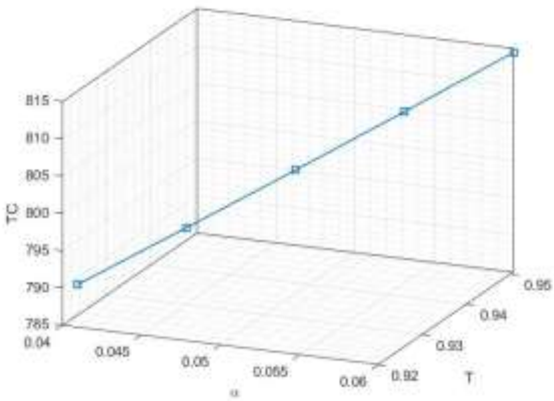

Fig 6. Effect of change of α on Total cost

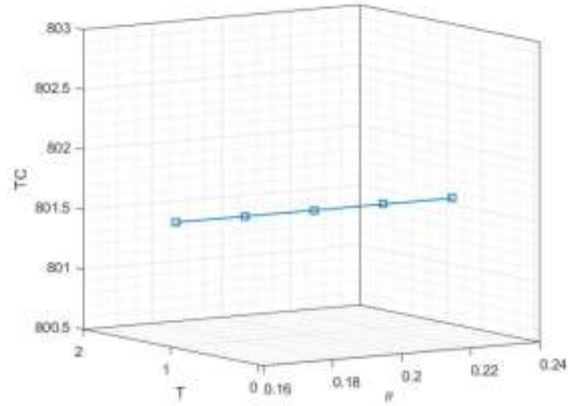

Fig 7. Effect of change of θ on Total cost

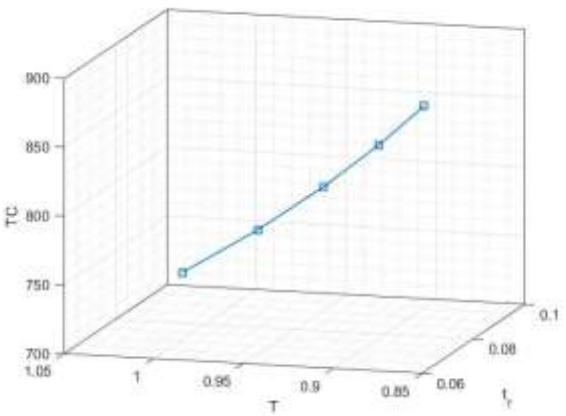

Fig 8. Effect of change of t_r on Total cost

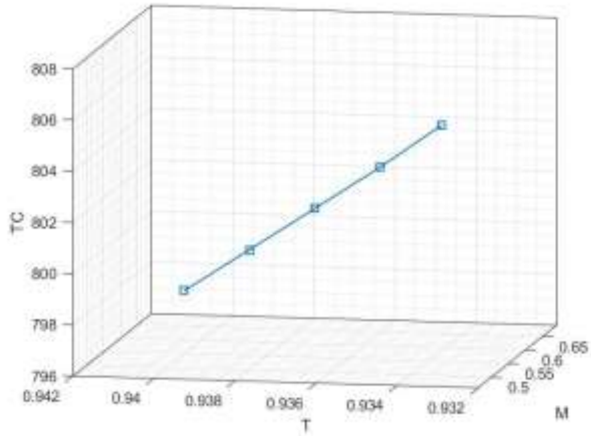

Fig 9. Effect of change of M on Total cost

5. SENSITIVITY ANALYSIS

Parameter values can vary due to uncertainties. To analyze these variations, a sensitivity analysis is essential for decision-making. We will now examine the impact of changes in the system parameters $a, b, c, \delta, \alpha, \theta, t_r$ and M on the optimal replenishment policy of Example 3. Each parameter is adjusted individually while keeping the others constant. The results are detailed in Table 1. Based on our numerical findings, we derive the following managerial implications:

(1) When the parameter a increases, the total optimal cost (TC) and order quantity (Q) increases. But the time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) decreases. That is, increasing of the parameter a will increase the total cost of the retailer.

(2) When the parameter b increases, the total optimal cost (TC) increases. But the time at which the inventory level becomes zero in OW (t_1), the cycle length (T) and the order quantity (Q) decreases. That is, increasing of the parameter b will increase the total cost of the retailer.

(3) When the deterioration rate c increases, the total optimal cost (TC) increases. The time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) are decreasing by small quantity. There is no change in the order Quantity (Q). That is, increasing of the parameter c will increase the total cost of the retailer.
(4) If the backlogging parameter δ increases, the total optimal cost (TC) increases. But the time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) and the order quantity (Q) decreases. That is, to minimize the cost, the retailers should decrease the backlogging parameter.
(5) If percentage of defective items (α) increases, the total optimal cost (TC), the order quantity (Q), the time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) are increasing. That is, increasing percentage of defective items will increase the total cost of the retailer.
(6) Changes in the variation parameter in the cost of storing (θ) the total optimal cost (TC), the order quantity (Q), the time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) are increasing. That is, increasing the variation parameter in the cost of storing will increase the total cost of the retailer.
(7) If the time at which the inventory level becomes zero in RW (t_r) increases, the total optimal cost (TC) decreases. But the time at which the inventory level becomes zero in OW (t_1) and the cycle length (T) and the order quantity (Q) increases. That is, increasing of the parameter t_r will decrease the total cost of the retailer.
(8) If the Credit period M increases, the total optimal cost (TC) decreases. But the time at which the inventory level becomes zero in OW (t_1), the cycle length (T) and the order quantity (Q) increases. That is, to minimize the total cost, the retailers should increase the Credit period M.
The sensitivity analysis shows that increasing the parameters t_r and M reduces the total annual inventory cost, while decreasing the parameters a, b, c, δ, α and θ also leads to a reduction in the total annual inventory cost.

6. CONCLUSION

In this model, a two-warehouse EOQ model for imperfect quality repairable (reworkable) items with parabolic time dependent demand and non-linear holding cost in RW under trade credit period and partial backlogging has been developed. Our model suits well for the retailer in situations involving unlimited storage space. The aim of this paper is to obtain the optimal solution of cycle length, time intervals, order quantity and minimizing the total cost of the retailer. We presented a computational algorithm to find the optimal solution. The necessary and sufficient conditions for the existence and uniqueness of the optimal solutions are also provided. Numerical examples and a sensitivity analysis are carried out. From the managerial insights we could see that the rate of change of the parameters $a, b, c, \alpha, \delta, \theta, t_r$ and M affects the total annual inventory cost and ordering quantity. From the results obtained, an increase in warehouse rent raises the overall relevant costs significantly. This is primarily because each rise in warehouse rent contributes substantially to the total average cost by increasing holding expenses. we see that the retailer can reduce total annual inventory cost by ordering lower quantity when the supplier provides a permissible delay in payments by improving storage conditions for parabolic time dependent imperfect quality repairable (reworkable) items and prioritize adopting superior storage facilities for perishable items to achieve enhanced cost reduction in each replenishment. Therefore, this model provides a new managerial insight which helps the industry to reduce the total inventory cost by renting a warehouse and availing a trade credit period with non-linear holding cost for imperfect quality repairable (reworkable) items.

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