Educational Administration: Theory and Practice
2024, 30(5), 14532-14543
ISSN: 2148-2403
https://kuey.net/
Research Article

A New Nonparametric Control Chart For The Joint Monitoring Of Location And Scale Using Ranked Set Sampling

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Citation: Vikas Ghute et.al (2024), A New Nonparametric Control Chart For The Joint Monitoring Of Location And Scale Using Ranked Set Sampling, *Educational Administration: Theory and Practice*, 30(5), 14532-14543
Doi: 10.53555/kuey.v30i5.6877

ARTICLE INFO	ABSTRACT

Control charts are frequently employed in the field of statistical process monitoring (SPM) to track both the average and variability of a process. In this study, a novel nonparametric control chart is introduced, which utilizes modified Lepage-type test.
This proposed chart serves the purpose of concurrently monitoring both location and scale parameters for any continuous distribution related to a specific process. The charting statistic combines two well-known nonparametric test statistics: Baumgartner's test for location and Ansari-Bradley's test for scale estimation.
The performance of the proposed chart is evaluated in simulation studies, focusing on statistical metrics such as mean, standard deviation, median and various percentiles that characterize the distribution of run lengths. The behaviour of the chart under both in-control and out-of-control conditions is examined through simulation studies using two sampling methods: simple random sampling (SRS) and ranked set sampling (RSS).

Keywords: Statistical process monitoring; Average run length; Ansari-Bradley statistic; Baumgartner statistic; Ranked set sampling.

I. Introduction

In recent years, there has been a growing interest in the advancement of nonparametric control charts. These charts are particularly valuable when it is not possible to justify assuming a specific process distribution or when accurately estimating the parameters of a parametric model becomes challenging. Numerous studies have focused on developing nonparametric control charts for effectively monitoring either the location parameter or scale parameter separately. Accurate monitoring of the location parameter holds significant importance across various applications as it pertains to statistics such as mean, median or specific percentiles within a distribution.
Several authors have proposed various nonparametric control charts specifically designed for monitoring the location parameter of a given process. The literature on this subject incorporates various authors such as Bakir (2004-2006), Chakraborti and Eryilmaz (2007), Khilare and Shirke (2010), and Human et al (2010). These studies apply sign or rank statistics as the foundation of their charts.
The topic has been examined comprehensively by Chakraborti et al. (2001), while Chakraborti and Graham (2007) extensively discuss the advantages gained from using nonparametric control charts.
Monitoring the scale parameter of a process is a crucial consideration in numerous applications, and the literature offers only a limited number of nonparametric methods for this purpose. Amin et al. (1995) proposed a control chart that employs quartiles to monitor variations in the process, while Das (2008a) proposed an alternative nonparametric method that utilizes squared rank test results to regulate variability. Previous research has focused on the development of nonparametric control charts for managing process variability. For instance, Das (2008b) introduced a control chart based on rank test that does not rely

on distribution assumption. Similarly, Das and Bhattacharya (2008) proposed a nonparametric control chart utilizing nonparametric tests to manage variability. Another alternative was suggested by Murakami and Matsuki (2010), who developed a nonparametric control chart using Mood statistic as its basis without assuming any specific data distribution. Additional contributions in this area include the synthetic control chart based on the sign statistic proposed by Khilare and Shirke (2012). Zombade and Ghute (2014) also explored Sukhatme and Mood tests as foundations for their introduced control charts. Furthermore, Shirke and Barale (2018) presented a cumulative sum chart which monitors process dispersion through in-control deciles while maintaining a nonparametric approach.

Nonparametric control charts often use two separate charts to monitor location and scale parameters. However, this approach can be challenging as changes in one parameter can affect the other and make it difficult to interpret signals. Recently, there has been a shift towards a unified monitoring scheme that uses a single control chart. This simplified approach is easier to understand and implement. This control chart utilizes a statistical measure that combines separate measures for the mean and variance. It is crucial to monitor both parameters simultaneously in a process, which is why an effective statistic is commonly used to independently monitor each parameter. However, there is limited research on nonparametric joint monitoring techniques in current literature. Only a few nonparametric methods for jointly monitoring processes are found in existing academic sources. Mukherjee and Chakraborti (2012) created the SL chart, a control chart that doesn't depend on distribution assumptions. This chart monitors both location and scale parameters at the same time by using the Wilcoxon rank sum statistic for location and the Ansari-Bradley scale statistic. Additionally, Chowdhury et al. (2013) proposed the SC chart, another nonparametric control chart that uses the Cucconi statistic to monitor both location and scale parameters of a continuous distribution. Chowdhury et al. (2015) also introduced the phase-II CUSUM control chart, which doesn't rely on distributional assumptions and enables simultaneous monitoring of both location and scale parameters. Zhang et al. (2017) presented another nonparametric control chart based on the Cramer-von Mises test statistic to achieve this goal. Ghadage and Ghute (2023) developed another nonparametric control chart utilizing a modified Lepage test for simultaneous monitoring of location and scale parameters. Even though there has been some progress in this particular research area, there is still a necessity to improve the efficiency of non-parametric control charts for monitoring jointly.

Various sampling methods have been thoroughly investigated in SPM studies to enhance the efficiency of pro cess monitoring charts. RSS is a notable approach advocated by McIntyre (1952) due to its numerous benefits and perceived superiority over SRS. Many authors have suggested different control charts for tracking the average of a process, utilizing techniques like RSS or modified schemes. Salazar and Sinha (1997) study introduced a monitoring chart for the process mean using an RSS scheme, noting its superiority over an SRS-based approach. Muttlak and Al-Sabah (2003) built upon this work and proposed several improved Shewhart-type control charts that use various RSS schemes to more effectively monitor changes in the process mean. The effectiveness of the traditional SRS control charts for means was found to be surpassed by these charts. According to Abujiya and Muttlak (2004), implementing the double ranked set sampling (DRSS) method in Shewhart-type mean charts proves to be more efficient compared to conventional SRS or RSS charts. The study suggests that incorporating DRSS-based control charts leads to enhanced monitoring capabilities for continuous processes. Al-Omari and Haq (2012) introduced a novel approach of utilizing DRSS in a control chart specifically designed for tracking process mean values. Mehmood et al. (2012) suggested various control charts to monitor the location of a process using different sampling techniques. On the other hand, Haq and Al-Omari (2015) introduced a new Shewhart control chart that uses partially ordered judgement subset sampling (POJSS) to keep an eye on the process mean. Their approach proved to be more effective in detecting random shifts in the process mean compared to other methods.

There are various nonparametric control charts available for monitoring the location parameter of a process, which utilize different RSS sampling methods. Tapang et al. (2016) have developed three nonparametric control charts based on RSS to detect even minor shifts in the process mean. Similarly, Abid et al. (2016) have proposed a nonparametric EWMA control chart that includes sign test and RSS techniques. Another version of this chart was created by Abid et al. (2017a), which utilizes Wilcoxon signed-rank statistic along with an RSS scheme for monitoring purposes. Please note that sources have been excluded intentionally. Several authors have suggested using nonparametric control charts to monitor process center and location, utilizing different statistical methods. For instance, Abid et al. (2017b) proposed a CUSUM chart based on the sign statistic and implemented the RSS technique. Similarly, Asghari et al. (2018) developed a nonparametric sign chart that uses RSS technique to monitor the process center. In another study, Abbas et al. (2020) introduced the DEWMA chart, which combines SRS and RSS techniques with the Wilcoxon signed rank test to effectively track changes in the process location parameter during quality control operations. By using the RSS technique in control charts, significant improvements have been observed compared to the traditional SRS-based control charts. This paper aims to provide valuable insights into nonparametric joint monitoring schemes by analyzing both the SRS and RSS methods.

In this paper, we introduce a new technique for simultaneously monitoring of the location and scale parameters of a continuous process distribution. Our approach involves using a single nonparametric Shewhart-type control chart. We base our chart on Neuhäuser (2000) nonparametric two-sample modified

Lepage-type test, which combines the Baumgartner statistic and Ansari-Bradley statistic to detect changes in both location and scale at the same time. To determine how well our method works, we assess the behaviour of the control chart when the process is in control and out of control. We do this by calculating the average run length (ARL) for both normal and double exponential distributions. In Section 2, the modified Lepage-type nonparametric test based on Baumgartner and Ansari-Bradley statistic for estimating both location and scale under SRS and RSS schemes is given. Section 3 discusses a single nonparametric control chart that monitors the location parameter and scale parameter of a process using the modified Lepage-type test statistic. In Section 4, the performance of this control chart is analyzed in both in-control and out-of-control scenarios under SRS and RSS sampling schemes. The findings are summarized in Section 5.

2. Nonparametric Tests for Location and Scale

In this section, we briefly discuss the nonparametric tests for location parameter, scale parameter and jointly location scale parameters.

2.1 Baumgartner Two Sample Test for Location

Baumgartner test is a two-sample test can be applied for location and scale parameters. Let $(X_1, X_2,..., X_n)$ and $(Y_1,Y_2,...,Y_m)$ denote two random samples. The observations with in each sample are independent and identically distributed, and we assume independence between two samples. Let F and G be continuous distribution functions corresponding two population 1 and 2 respectively. In location shift, model considered first the distribution functions are same except perhaps for change in their location; that is, $G(x) = F(x-\theta)$. The null hypothesis is $H_0 : \theta = 0$, whereas alternative is $H_1 : \theta \neq 0$. Baumgartner et al. (1998) proposed a distribution-free two-sample rank test for general alternative. For combined samples, let $R_1 < R_2 <...< R_n$ and $H_1 < H_2 <...< H_m$ denote the ranks of the $X-$values and $Y-$values in increasing order of magnitude, respectively. Baumgartner et al. (1998) defined a nonparametric two-sample rank statistic B as follows:

$$B = \frac{B_X + B_Y}{2} \tag{1}$$

where, $$B_X = \frac{1}{n}\sum_{i=1}^{n} \frac{\left(R_i - \frac{N}{n} i\right)^2}{\frac{i}{n+1}\left(1-\frac{i}{n+1}\right)\left(\frac{mN}{n}\right)} \text{ and } B_Y = \frac{1}{m}\sum_{j=1}^{m} \frac{\left(H_j - \frac{N}{m} j\right)^2}{\frac{j}{m+1}\left(1-\frac{j}{m+1}\right)\left(\frac{nN}{m}\right)}.$$

The larger value of statistic B gives evidence to reject the null hypothesis. Baumgartner et al. (1998) also provided asymptotic distribution of test statistic B.

2.2 Ansari-Bradley Test for Scale

The Ansari-Bradley test is a two-sample rank test applied for scale parameter. The test statistic is defined as follows: In the combined samples, the observations less than or equal to the median are replaced by their ranks in the increasing order and those larger than the median are replaced by their ranks in descending order. The statistic is the sum of these ranks for the Y sample. The corresponding test statistic is defined as (Gibbons and Chakraborti (2003)),

$$AB = \sum_{k=1}^{n}\left(k - \frac{N+1}{2}\right)Z_k \tag{2}$$

The mean and variance of statistic AB is given by,

$$E(AB) = \begin{cases} \frac{m(N+1)}{4}, & \text{when } N \text{ is even} \\ \frac{m(N+1)^2}{4N}, & \text{when } N \text{ is odd} \end{cases}$$

and

$$V(AB)=\begin{cases}\dfrac{mn(N^2-4)}{48(N-1)}, & \text{when } N \text{ is even}\\[2mm] \dfrac{mn(N+1)(N^2+3)}{48N^2}, & \text{when } N \text{ is odd}\end{cases}$$

2.3 Modified Lepage-type Test for Location and Scale

After Lepage statistic was proposed, various Lepage-type statistics have been proposed and discussed by many authors in the literature. One of the most famous and powerful modified Lepage-type statistic proposed by Neuhäuser (2000) is a combination of the Baumgartner and Ansari-Bradley statistic given as:

$$L_M=\left(\frac{B-E_0(B)}{\sqrt{Var_0(B)}}\right)^2+\left(\frac{AB-E_0(AB)}{\sqrt{Var_0(AB)}}\right)^2 \qquad (3)$$

where B Baumgartner statistic for location shift and AB is Ansari-Bradley statistic for scale shift. In this paper, we use L_M test statistic as a charting statistic for detecting simultaneous location and scale shifts in a continuous process distribution.

3. Control chart based on modified Lepage-type statistic

In this Section, we develop a nonparametric control chart based on modified Lepage-type test statistic proposed by Neuhäuser (2000) for simultaneously monitoring the location and the scale parameters of a continuous process. The single plotting statistic for the joint monitoring of location and scale is given L_M in Eq. (3) and chart is called LM chart. To adopt the idea of two sample test for control chart implementation, m independent observations $X=(X_1,X_2,\ldots,X_m)$ from an in-control process are used as reference sample and compared to future sample subgroups of n independent observations $Y=(Y_1,Y_2,\ldots,Y_n)$ an arbitrary test sample.

Proposed charting procedure under SRS scheme

Step1: Collect Phase-I reference sample $X=(X_1,X_2,\ldots,X_m)$ of size m using SRS from an in-control process.

Step2: Let $Y=(Y_1,Y_2,\ldots,Y_n)$ be j^{th} Phase-II (test) sample of size n, using RSS $j=1,2,3,\ldots$

Step 3: Calculate B_j and $(AB)_j$ using (1) and (2) for j^{th} test sample.

Step 4: Compute means and standard deviations of B and AB statistics respectively

Step 5: Calculate the standardized B and AB statistics as

$$T_{1j}=\left(\frac{B-E_o(B)}{\sqrt{Var_0(B)}}\right) \text{and } T_{2j}=\left(\frac{AB-E_o(AB)}{\sqrt{Var_0(AB)}}\right) \text{respectively.}$$

Step 6: Calculate the control chart statistic LM chart as $T_j=T_{1j}^2+T_{2j}^2,\ j=1,2,3,\ldots$

Step 7: Plot T_j against an upper control limit (UCL), $H>0$.

Step 8: If T_j exceed H, the process is out-of-control at the j^{th} test sample. If not, the process is thought to be in-control and testing continues to the next sample.

Proposed charting procedure under RSS scheme

Ranked Set Sampling

McIntyre (1952) introduced the concept of Ranked Set Sampling, which involves ranking samples based on a related variable, instead of directly measuring the variable being investigated. We will now explain the steps involved in selecting a sample using RSS techniques.

1. Select n^2 units with SRS scheme from the target population.
2. Randomly allocate these n^2 units in n groups each of size n.
3. Rank the units in each group in ascending order of magnitude by personal judgement or visual inspection or by using some auxiliary variable.
4. Select smallest value from the first group and second smallest value from the second group.

5. This procedure will continue, and the last sample unit corresponds to the largest value from the n^{th} group.
This gives a ranked set sample of size n .
The charting procedure of the proposed LM chart under RSS scheme is as follows
Step1: Collect a reference sample of size m using RSS from an in-control process $X_{rss} = (X_{rss,1}, X_{rss,2}, .., X_{rss,m})$.
Step2: Collect a j^{th} Phase-II (test) sample of size n using RSS $j = 1, 2, 3, ...$
$Y_{rss} = (Y_{rss,1}, Y_{rss,2}, .., Y_{rss,n})$
Use Step 3 to Step 8 in the procedure of SRS scheme using RSS samples instead of SRS samples.

4. Performance Evaluation and Analysis of LM Chart

Implementation of the proposed LM chart requires the upper control limit H. Typically, in practice, it is determined for specified in-control (ARL_0), say, 500 under SRS and RSS Scheme. The estimation of H is achieved by utilizing a Monte-Carlo simulation method that involves generating a significant number of potential samples. For a given pair of (n, m) values, a search is conducted with different values of H, and that value of H is obtained for which ARL_0 is equal to nominal (target) value. We choose $m = 30, 50, 100$ for the reference sample size and $n = 5, 11, 25$ as the test sample size and target values $ARL_0 = 500$. The results are presented in Table 1.

Table 1: Charting constant H for the proposed chart, for various values of m and n , and for standard (target) value of $ARL_0 = 500$ under SRS and RSS scheme

Reference sample size	Test Sample size	Charting constant (upper control limit): H	
m	n	SRS	RSS
30	5	37.960	5.4212
30	11	37.312	3.2361
30	25	24.820	2.2498
50	5	23.390	5.7895
50	11	20.752	3.3452
50	25	20910	2.3584
100	5	32.800	6.8695
100	11	31.305	3.9292
100	25	28.023	2.3110

To evaluate the efficiency of a control chart, it is common to analyse its run length distribution. In case this distribution shows an asymmetrical shape towards the right side, it becomes necessary to explore different statistics like ARL, standard deviation of run length (SDRL), and specific percentiles including first and third quartiles in order to describe such distribution. We study the performance of the proposed LM chart both under in-control and out-of- control setup under SRS an RSS scheme. For the in-control setup, we simulate both the reference and the test sample from standard normal distribution. We choose $m = 30, 50, 100$ and $n = 5, 11, 25$. For a given pair of (m, n) values, we obtain upper control limits H for nominal (target) ARL_0 of 500 and simulate different characteristics of the in-control run-length distribution.

The results of simulation are shown in Table 2 under SRS scheme and in Table 3 under RSS scheme. It indicates that the target $ARL_0 = 500$ is much larger than the median (Q_2) for all (m, n) combinations. Hence, in-control run-length distribution of the LM chart is highly skewed to the right.

To examine how the proposed LM chart performs in out-of-control scenarios, we analyse its effectiveness using both SRS and RSS schemes with normal and double exponential process distributions. Specifically, we employ the double exponential distribution to evaluate its performance under heavy-tailed conditions. In our analysis, we assume that observations from the process follow a mean of zero and variance of one for both types of distributions being studied.

Table 2: In-control performance characteristics of the LM chart for $ARL_0 = 500$ under SRS scheme simulated values

m	n	H	ARL_0	$SDRL_0$	P_5	Q_1	Q_2	Q_3	P_{95}
30	5	37.960	501.0	500.5	26	146	350	694	1484
30	11	37.312	499.7	499.2	27	145	346	692	1493
30	25	24.820	500.4	499.9	26	144	347	695	1481
50	5	23.390	499.5	499.0	27	143	344	691	1508
50	11	20.752	501.4	500.9	26	144	351	698	1499
50	25	20910	501.3	500.8	26	143	348	692	1502
100	5	32.800	500.6	500.1	26	144	348	696	1506
100	11	31.305	500.1	499.6	26	145	345	694	1499
100	25	28.023	502.9	502.4	26	147	349	695	1508

Table 3: In-control performance characteristics of the LM chart for $ARL_0 = 500$ under RSS scheme simulated values

m	n	H	ARL_0	$SDRL_0$	P_5	Q_1	Q_2	Q_3	P_{95}
30	5	5.4212	501.4	500.9	26	144	348	695	1498
30	11	3.2361	501.5	501.0	26	147	350	691	1497
30	25	2.2498	501.0	500.5	26	144	348	697	1495
50	5	5.7895	500.2	499.7	27	144	350	697	1483
50	11	3.3452	501.2	500.7	27	145	347	691	1498
50	25	2.3584	499.6	499.1	26	144	346	692	1499
100	5	6.8695	504.5	504.0	27	144	349	698	1513
100	11	3.9292	497.5	497.2	26	144	348	687	1473
100	25	2.3110	503.8	503.3	25	145	351	697	1503

4.1 Performance Comparison of LM chart under SRS and RSS scheme for Normal Distribution.

In order to investigate the out-of-control performance comparison of the proposed LM chart under SRS and RSS scheme, we consider the underlying process distribution as normal; samples are taken from $N(\theta,\lambda)$ distribution, with in-control samples coming from $N(0,1)$ distribution. To examine the effects of shifts in process mean and process variance, 25 combinations of (θ,λ) values are considered with $\theta = 0, 0.5, 1.0, 1.5, 2.0$ and $\lambda = 1.0, 1.251.5, 1.75, 2.0$.

Tables 4 and 5 present the performance characteristics of the LM chart when underlying process distribution is normal with combinations of the reference and test sample sizes $m = 50, 100$ and $n = 5$.

Table 4: Performance comparisons of LM chart under SRS and RSS Scheme for the $N(\theta,\lambda)$ distribution with $ARL_0 = 500$

$m = 50,\ n = 5$															
θ	λ	SRS							RSS						
		ARL	$SDRL$	P_5	Q_1	Q_2	Q_3	P_{95}	ARL	$SDRL$	P_5	Q_1	Q_2	Q_3	P_{95}
0.00	1.00	499.5	499.0	27	143	344	691	1508	499.8	499.3	27	145	349	691	1492
0.50	1.00	42.4	41.9	3	13	30	59	126	119.5	119.0	6	35	83	165	354
1.00	1.00	5.9	5.4	1	2	4	8	16	4.0	3.5	1	2	3	5	11
1.50	1.00	1.9	1.3	1	1	1	2	5	1.2	0.5	1	1	1	1	2
2.00	1.00	1.2	0.5	1	1	1	1	2	1.0	0.1	1	1	1	1	1
0.00	1.25	108.1	107.6	6	31	76	150	322	201.5	201.0	11	58	139	277	603
0.50	1.25	24.1	23.6	2	7	17	33	72	60.1	59.6	3	17	42	83	181
1.00	1.25	5.7	5.1	1	2	4	8	16	4.0	3.5	1	1	3	5	11
1.50	1.25	2.2	1.7	1	1	2	3	6	1.3	0.6	1	1	1	1	2
2.00	1.25	1.3	0.7	1	1	1	2	3	1.0	0.1	1	1	1	1	1

0.00	1.50	43.0	42.5	3	13	30	60	127	84.5	84.0	5	25	59	117	253
0.50	1.50	16.6	16.1	1	5	12	23	48	34.0	33.5	2	10	24	47	100
1.00	1.50	5.5	4.9	1	2	4	7	15	3.9	3.3	1	1	3	5	11
1.50	1.50	2.5	1.9	1	1	2	3	6	1.3	0.6	1	1	1	1	3
2.00	1.50	1.5	0.9	1	1	1	2	3	1.0	0.1	1	1	1	1	1
0.00	1.75	22.8	22.3	2	7	16	31	68	43.3	42.8	3	13	30	60	129
0.50	1.75	12.7	12.2	1	4	9	17	37	21.8	21.3	2	7	15	30	64
1.00	1.75	5.3	4.8	1	2	4	7	15	3.7	3.2	1	1	3	5	10
1.50	1.75	2.7	2.2	1	1	2	4	7	1.4	0.7	1	1	1	2	3
2.00	1.75	1.7	1.1	1	1	1	2	4	1.0	0.2	1	1	1	1	1
0.00	2.00	14.5	14.0	1	5	10	20	42	26.0	25.5	2	8	18	36	77
0.50	2.00	9.9	9.4	1	3	7	13	29	15.2	14.7	1	5	11	21	45
1.00	2.00	5.1	4.6	1	2	4	7	14	3.5	3.0	1	1	3	5	9
1.50	2.00	2.9	2.3	1	1	2	4	8	1.4	0.8	1	1	1	2	3
2.00	2.00	1.9	1.3	1	1	1	2	4	1.1	0.2	1	1	1	1	1

Table 5: Performance comparisons of LM chart under SRS and RSS Scheme for the $N(\theta, \lambda)$ distribution with $ARL_0 = 500$

$m=100,\ n=5$															
θ	λ	SRS							RSS						
		ARL	$SDRL$	P_5	Q_1	Q_2	Q_3	P_{95}	ARL	$SDRL$	P_5	Q_1	Q_2	Q_3	P_{95}
0.00	1.00	500.6	500.1	26	144	348	696	1506	502.3	501.8	26	144	347	697	1510
0.50	1.00	66.4	65.9	4	20	46	91	199	202.7	202.2	11	59	140	283	606
1.00	1.00	7.5	7.0	1	3	5	10	21	5.3	4.8	1	2	4	7	15
1.50	1.00	2.1	1.5	1	1	2	3	5	1.3	0.6	1	1	1	1	2
2.00	1.00	1.2	0.5	1	1	1	1	2	1.0	0.1	1	1	1	1	1
0.00	1.25	119.9	119.4	7	35	83	166	355	91.5	91.0	5	27	63	126	274
0.50	1.25	33.3	32.8	2	10	23	46	98	46.0	45.5	3	14	32	64	138
1.00	1.25	6.8	6.3	1	2	5	9	19	4.9	4.4	1	2	4	7	14
1.50	1.25	2.4	1.9	1	1	2	3	6	1.4	0.8	1	1	1	2	3
2.00	1.25	1.4	0.7	1	1	1	2	3	1.0	0.2	1	1	1	1	1
0.00	1.50	48.6	48.1	3	14	34	67	145	21.6	21.1	2	7	15	30	64
0.50	1.50	21.5	21.0	2	7	15	30	63	15.6	15.1	1	5	11	21	46
1.00	1.50	6.4	5.9	1	2	5	9	18	4.1	3.5	1	2	3	5	11
1.50	1.50	2.7	2.2	1	1	2	4	7	1.6	0.9	1	1	1	2	3
2.00	1.50	1.6	1.0	1	1	1	2	4	1.1	0.3	1	1	1	1	2
0.00	1.75	26.3	25.8	2	8	18	36	78	8.9	8.4	1	3	6	12	26
0.50	1.75	15.5	15.0	1	5	11	21	45	7.6	7.1	1	3	5	10	22
1.00	1.75	6.1	5.6	1	2	4	8	17	3.3	2.8	1	1	2	4	9
1.50	1.75	2.9	2.4	1	1	2	4	8	1.6	1.0	1	1	1	2	4
2.00	1.75	1.8	1.2	1	1	1	2	4	1.1	0.4	1	1	1	1	2
0.00	2.00	16.5	16.0	1	5	12	23	49	5.1	4.6	1	2	4	7	14
0.50	2.00	11.8	11.3	1	4	8	16	35	4.7	4.2	1	2	3	6	13
1.00	2.00	5.8	5.2	1	2	4	8	16	2.8	2.2	1	1	2	4	7
1.50	2.00	3.1	2.5	1	1	2	4	8	1.6	1.0	1	1	1	2	4
2.00	2.00	2.0	1.4	1	1	1	2	5	1.2	0.5	1	1	1	1	2

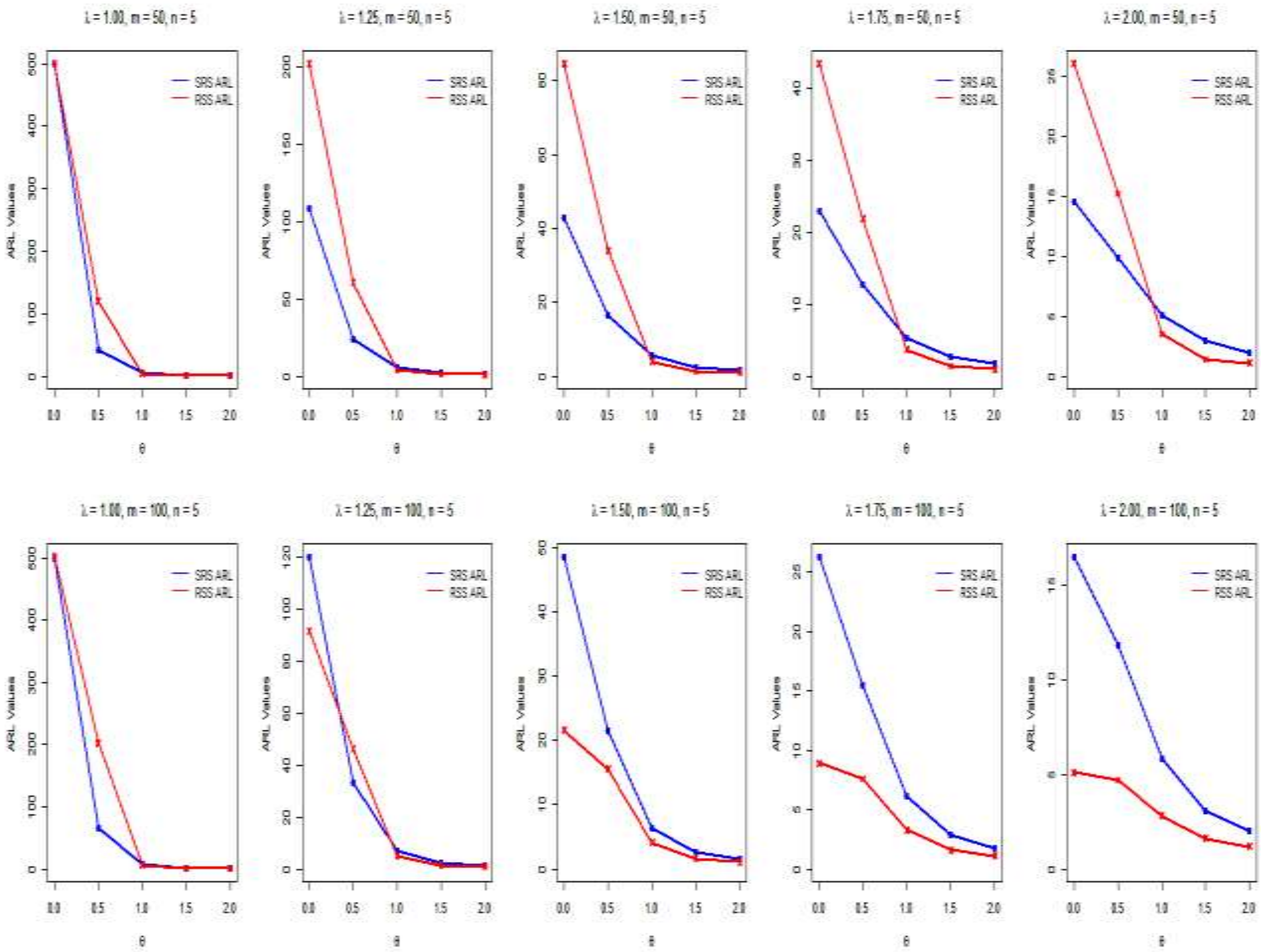

Figure 1 ARL performance of SML chart for normal distribution under SRS and RSS scheme

The results in Table 4, 5 and Figure 1 indicate that the out-of-control run-length distributions are also skewed to right. It is observed that, for a fixed m, n and a given ARL_0 , the out-of-control ARL values as well as the percentiles all decrease sharply with increasing shift in the location and also with the increasing shift in the scale. It indicates that the proposed LM chart
is effective in detecting shifts in location and/or in the scale. The proposed LM chart under SRS and RSS scheme detect shift in the scale more quickly than that in the location. For example, from Table 4, we observe that for 50% increase in location when scale is in-control, the ARL decreases by 91% under SRS scheme and decreases by 76% under RSS scheme, whereas for a 25% increase in a scale when the location is in-control, ARL decreases by 78% under SRS scheme and decreases by 60% under RSS scheme. Finally, when location and scale increases by 50% the ARL decreased by 97% under SRS scheme and decreases 93% under RSS scheme. The pattern is same for SDRL; it decreases for an increase in the shift in both parameters, but decreases more for a shift in scale. For example, from Table 4, for 50% increase in location, the SDRL decreases by 87% under SRS scheme and 76% under RSS scheme but for 25% increase in scale, the SDRL decreases by 78% under SRS scheme and decreases 60% under RSS scheme.
In Table 4, it is evident that the LM chart employing the RSS scheme outperforms the one using the SRS scheme when the distribution is normal and there is a shift in both the location and scale parameter. The LM chart based on RSS scheme displays better performance as the location parameter increases, compared to the one based on SRS scheme. Moreover, the LM chart using RSS scheme shows superior performance even with a minor shift in location and scale parameter, as compared to the one based on SRS scheme.

4.2 Performance Comparison of LM chart under SRS and RSS scheme for double exponential Distribution.

To study the effect of heavy tailed distribution on the performance of the proposed LM chart under SRS and RSS scheme, double exponential distribution is included in the study as heavy ailed process distribution. We conduct simulation study with data from double exponential distribution. The performance characteristics of the run-length were evaluated when the in-control sample is from a $Laplace(0,1)$ distribution that has a mean 0 and a variance 2. And test samples are generated from the double exponential distribution with mean θ and standard deviation λ .

Table 6: Performance comparisons of LM chart under SRS and RSS Scheme for the $Laplace\,(\theta,\lambda)$ distribution with $ARL_0 = 500$

$m = 50,\ n = 5$															
		SRS							RSS						
θ	λ	*ARL*	*SDRL*	P_5	Q_1	Q_2	Q_3	P_{95}	*ARL*	*SDRL*	P_5	Q_1	Q_2	Q_3	P_{95}
0.00	1.00	499.6	499.1	25	143	345	693	1505	501.7	501.2	27	145	349	695	1489
0.50	1.00	18.2	17.7	1	6	13	25	53	100.4	99.9	6	29	70	139	301
1.00	1.00	3.0	2.4	1	1	2	4	8	6.1	5.6	1	2	4	8	17
1.50	1.00	1.5	0.8	1	1	1	2	3	1.7	1.1	1	1	1	2	4
2.00	1.00	1.1	0.4	1	1	1	1	2	1.1	0.3	1	1	1	1	2
0.00	1.25	222.2	221.7	12	64	155	307	663	148.2	147.7	8	43	102	205	444
0.50	1.25	17.0	16.5	1	5	12	23	50	56.0	55.5	3	16	39	77	167
1.00	1.25	3.6	3.0	1	1	3	5	10	6.5	6.0	1	2	5	9	18
1.50	1.25	1.7	1.1	1	1	1	2	4	1.9	1.3	1	1	1	2	4
2.00	1.25	1.3	0.6	1	1	1	1	2	1.2	0.4	1	1	1	1	2
0.00	1.50	128.6	128.1	7	38	90	177	381	44.6	44.1	3	13	31	62	133
0.50	1.50	16.1	15.6	1	5	11	22	47	26.0	25.5	2	8	18	36	76
1.00	1.50	4.2	3.6	1	2	3	6	11	5.8	5.3	1	2	4	8	16
1.50	1.50	2.0	1.5	1	1	2	3	5	2.0	1.4	1	1	1	2	5
2.00	1.50	1.4	0.8	1	1	1	2	3	1.2	0.5	1	1	1	1	2
0.00	1.75	87.1	86.6	5	26	61	121	261	19.3	18.8	2	6	14	27	56
0.50	1.75	15.8	15.3	1	5	11	22	46	13.9	13.4	1	4	10	19	41
1.00	1.75	4.7	4.2	1	2	3	6	13	4.9	4.3	1	2	4	7	14
1.50	1.75	2.3	1.8	1	1	2	3	6	2.0	1.5	1	1	2	3	5
2.00	1.75	1.6	1.0	1	1	1	2	4	1.3	0.6	1	1	1	1	3
0.00	2.0	65.0	64.5	4	19	45	90	193	10.7	10.2	1	3	7	15	31
0.50	2.00	15.4	14.9	1	5	11	21	45	8.5	8.0	1	3	6	12	25
1.00	2.00	5.2	4.7	1	2	4	7	14	4.1	3.5	1	1	3	5	11
1.50	2.00	2.6	2.1	1	1	2	3	7	2.0	1.4	1	1	2	3	5
2.00	2.00	1.8	1.2	1	1	1	2	4	1.3	0.7	1	1	1	2	3

Table 7: Performance comparisons of LM chart under SRS and RSS Scheme for the $Laplace\ (\theta,\lambda)$ distribution with $ARL_0 = 500$

$m = 100,\ n = 5$															
	λ	SRS							RSS						
θ		*ARL*	*SDRL*	P_5	Q_1	Q_2	Q_3	P_{95}	*ARL*	*SDRL*	P_5	Q_1	Q_2	Q_3	P_{95}
0.00	1.00	497.9	497.4	25	142	344	691	1502	498.6	497.9	26	143	344	690	1505
0.50	1.00	194.6	194.1	10	56	134	269	583	281.4	280.9	15	81	196	390	837
1.00	1.00	10.5	10.0	1	3	7	14	30	8.0	7.5	1	3	6	11	23
1.50	1.00	2.2	1.6	1	1	2	3	5	1.7	1.1	1	1	1	2	4
2.00	1.00	1.2	0.5	1	1	1	1	2	1.1	0.4	1	1	1	1	2
0.00	1.25	179.6	179.1	10	52	125	249	536	152.1	151.6	8	44	106	211	452
0.50	1.25	91.5	91.0	5	27	64	126	272	84.5	84.0	5	25	59	116	252
1.00	1.25	9.6	9.1	1	3	7	13	27	7.8	7.3	1	3	6	11	22
1.50	1.25	2.5	2.0	1	1	2	3	6	2.0	1.4	1	1	1	2	5
2.00	1.25	1.4	0.7	1	1	1	2	3	1.2	0.5	1	1	1	1	2
0.00	1.50	91.1	90.6	5	26	64	127	272	45.7	45.2	3	14	32	63	136
0.50	1.50	55.5	55.0	3	16	38	77	164	32.0	31.5	2	10	22	44	96
1.00	1.50	9.3	8.7	1	3	7	13	27	6.6	6.1	1	2	5	9	18
1.50	1.50	2.8	2.3	1	1	2	4	7	2.1	1.5	1	1	2	3	5
2.00	1.50	1.5	0.9	1	1	1	2	3	1.3	0.6	1	1	1	1	2
0.00	1.75	54.4	53.9	3	16	38	75	162	19.4	18.9	1	6	14	27	57
0.50	1.75	37.8	37.3	2	11	26	52	112	15.5	15.0	1	5	11	21	46
1.00	1.75	8.7	8.2	1	3	6	12	25	5.4	4.8	1	2	4	7	15
1.50	1.75	3.1	2.5	1	1	2	4	8	2.1	1.6	1	1	2	3	5
2.00	1.75	1.7	1.1	1	1	1	2	4	1.4	0.7	1	1	1	2	3
0.00	2.00	36.7	36.2	2	11	26	51	109	10.6	10.0	1	3	7	15	31
0.50	2.00	27.9	27.4	2	8	20	39	83	9.2	8.7	1	3	6	12	26
1.00	2.00	8.3	7.8	1	3	6	11	24	4.3	3.8	1	2	3	6	12
1.50	2.00	3.3	2.7	1	1	2	4	9	2.1	1.5	1	1	2	3	5
2.00	2.00	1.8	1.2	1	1	1	2	4	1.4	0.7	1	1	1	2	3

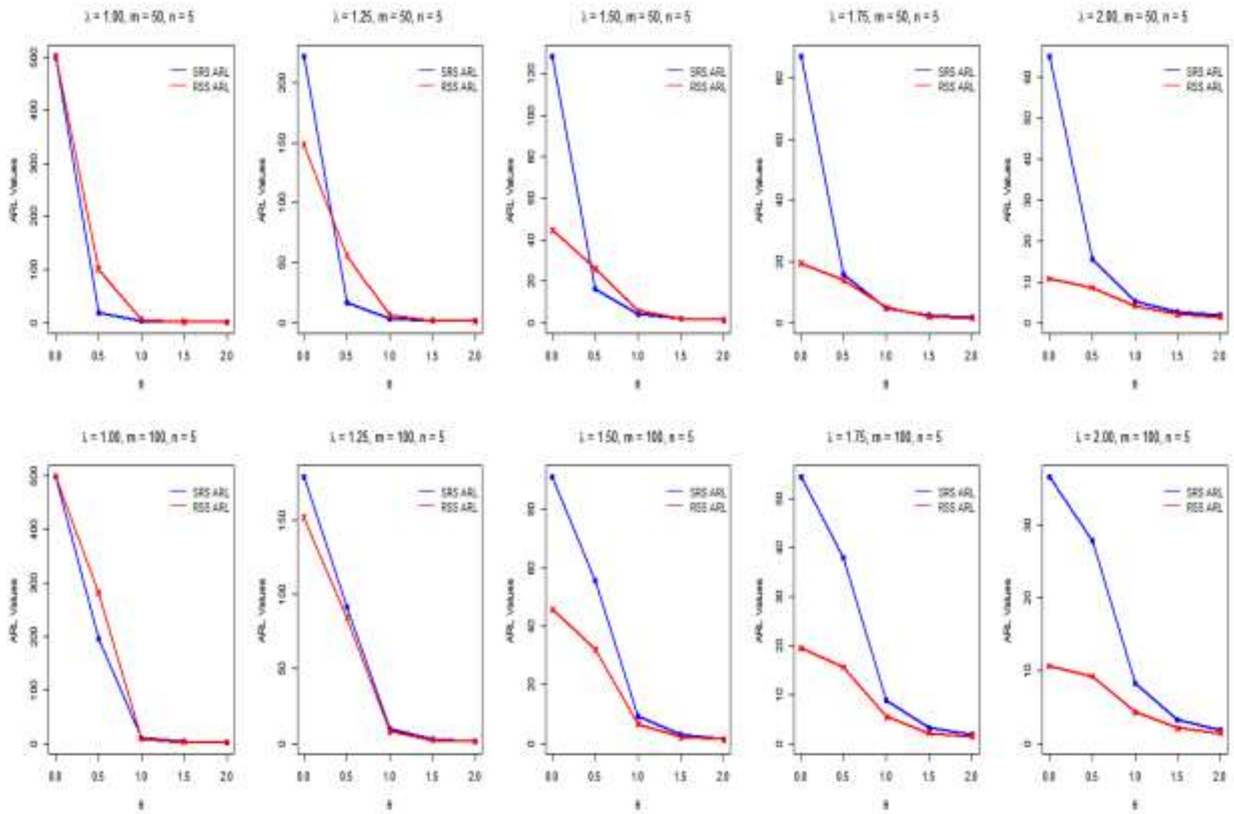

Figure 2 ARL performance of SML chart for double exponential distribution under SRS and RSS scheme

To examine the effect of shifts in location and scale, as in normal case, we studied 25 combinations of (θ, λ) values. Table 6 and Table 7 presents the performance characteristics of proposed LM chart under SRS and RSS scheme when underlying process distribution is double exponential with combinations of reference and test samples of size $m = 50,100$ and $n = 5$.

Tables 6, 7 and Figure 2 demonstrate that for a doubling exponential distribution, the overall pattern of the process remains similar to that of a normal distribution. Nevertheless, when utilizing an SRS scheme to detect shifts in mean and/or variance, the out-of-control ARL values are greater compared to those obtained under normal process distribution. Conversely, with RSS schemes in place, detecting such changes under double exponential distribution is more efficient than using traditional methods as indicated by smaller out-of-control ARL values relative to normal ones. An example of this can be seen in Table 6, where a mean shift and dispersion shift of equal proportions (50%) that is $(\theta = 0.50, \lambda = 1.5)$ are implemented under the SRS scheme. The resulting ARL is 16.1, which is slightly smaller than the ARL of 16.6 observed in the normal case presented in Table 4. On the other hand, when applying these shifts under RSS scheme, the resulting ARL is smaller at 26.0 compared to an ARL of 34.0 for a normal case recorded in previous studies. Furthermore, in the case of double exponential distribution, both percentiles and SDRL exhibit an increase when compared to a normal distribution using SRS scheme. However, these values decrease under RSS scheme.

5. Conclusions

In this article, a non-parametric control chart is introduced. It uses the modified Lepage-type test statistic to monitor both the location and scale parameters of an ongoing continuous process distribution simultaneously. The LM charts performance for both in-control and out-of-control states is examined using SRS and RSS scheme. These tests are conducted on normal distribution, as well as those with heavy tails or double exponential distribution. Various characteristics like mean, median, and percentile run-length distributions are analyzed. The results show that the proposed LM chart maintains its expected ARL under different process distributions when utilizing either SRS or RSS methods. Furthermore, it was observed that the RSS scheme is more efficient than the SRS method for normal and double exponential distributions.

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