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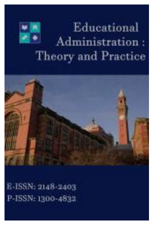

Developing a Hypothetical Learning Trajectory with Problem-Based Learning and a Learning Medium for Middle School

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Abstract

A teacher must consider the learning trajectory that will emerge during the learning process while designing an instructional plan. In mathematics, the degree to which students comprehend the learning context the teacher applies indicates the teacher's ability to construct a learning framework. This study aims to design the Hypothetical Learning Trajectory (HLT) for learning linear equations in two variables using the graphical method, problem-based learning, and the WolframAlpha application (as a learning medium). The design research methodology was used to construct the HLT design. This study begins with interviews with teachers who have taught similar material and students who have received this learning material. The findings from these interviews serve as the foundation for this research. As a result, this study provided an HLT design employing the graphical method for teaching mathematics in middle schools. It was discovered that the order in which students solved problems on the HLT corresponded to their cognitive states. Using the proposed learning trajectory design, students will find it less scary to advance through the current mathematical learning stages. Students might follow each procedural step without teacher pressure using this method. The developed learning strategy is expected to be able to solve the provided mathematical problem. Moreover, this HLT design indicates that the contextual qualities of the problem-based learning model make it best suited for relevant material.

Keywords: Learning Trajectory; Learning Mathematics; Problem-Based Learning; Problem-Solving; Learning Medium

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Introduction

Mathematics is a crucial science for students. By studying mathematics, a student can master other fields of knowledge more effectively. Mathematics is vital to education due to its numerous benefits and applications in daily life (G. S. Pratama & Retnawati, 2018). Mathematics trains the mind to think logically and analytically, which aids in decision-making and problem-solving in ordinary life. Mathematics requires language and mathematical symbols, which can enhance communication and aid in problem-solving (Maulyda et al., 2020). The study of mathematics is essential for forming competent persons who are more creative and independent in tackling complicated challenges. Therefore, a teacher's planning must be thorough for the learning process to proceed as intended and for the objectives of learning mathematics to be attained (Peranginangin et al., 2019).

In mathematics teaching, the extent to which pupils comprehend the learning context the teacher applies indicates the teacher's success in building a learning framework. This demonstrates that learning planning has a strong relationship with the quality of the learning process delivered (Dorovolomo et al., 2010; Hendriana et al., 2019). Students undergo the learning process mentioned earlier to participate according to the learning objectives. In this regard, a teacher must consider the learning trajectory that will emerge during the learning process while designing an instructional plan. Mathematical learning trajectory refers to the sequence of learning phases required to comprehend and master specific mathematical concepts (Spaull & Kotze, 2015). Typically, learning trajectories are created to guide the learning process, aid teachers in lesson planning, and provide students with feedback on their understanding of math concepts. The term "learning trajectory" refers to a series of levels of comprehension required to attain specific mathematics learning objectives (Callejo et al., 2022). Each stage may include more complex concepts, procedures, or understandings, and students must progress sequentially through these stages to achieve a deeper level of comprehension.

The learning trajectory provides the instructor with data for measuring student comprehension of a learning subject. Learning trajectories are designed with the pupils' natural levels of thought in mind (Hendriana et al., 2019). Simon (1995) asserted that developing HLT is an integral element of the mathematical learning cycle, in which students participate in this process while teachers evaluate learning objectives and activities.

In addition to the learning design developed by the teacher, the use of technology as a medium has a significant impact on the success of the learning process in the classroom because students are expected to be able to learn independently when using technology (Pannen, 2015). Various research on the use of applications or software for learning mathematics in the classroom has shown that they can assist pupils in enhancing their problem-solving skills and learning independence (Faroh, 2017; Hendikawati et al., 2019; Kusuma et al., 2020; Natalliasari & Mulyani, 2017; Nuritha & Tsurayya, 2021). In other words, technology has a significant impact on the success of the learning process. Regarding this, the WolframAlpha application, an online service that can answer factual queries by assembling structured replies, represents the technology employed in this study as the media since this application may answer mathematical and scientific problems.

The benefits of employing technology in teaching mathematics include piquing students' interest in learning, making students more independent, and engaging and active (Hwa, 2018; Papadakis et al., 2021). In addition, with technology, time spent learning will be used more effectively, and the learning process will be simplified. NCTM (2000) asserted the same idea, claiming that integrating technology into learning has at least three positive effects on mathematics learning, which can increase learning outcomes in mathematics, increase the effectiveness of teaching mathematics, and influence what and how mathematics should be learned and taught. Thus, technology becomes a requirement for students so that they may engage in a more attractive, effective, and efficient teaching and learning process.

Arends (2008) stated that good learning is when the teacher gives a variety of situations (problems) so that students can experiment, try out various things, ask questions, and find their answers, then compare their findings with those of their peers. NCTM (2000) supported this sentiment, stating that the emphasis on approach in problem-solving is an essential skill for students. The proposed technique involves manipulation, trial and error, case-by-case practice, guessing and checking, knowing the probable patterns that may develop, gathering data in tables, and working with backward patterns. By employing these problem-solving strategies, it is expected that students would be able to make discoveries by investigating to find an appropriate answer; this begins with presenting students with real-world problems in the form of questions or other objects (Ong et al., 2010). Therefore, formal learning to support pupils' math problem-solving skills is problem-based learning. Problem-based learning is also known as learning that focuses primarily on problem-solving (Carriger, 2015). Students' cognitive level-appropriate problem-based mathematics learning is a type of learning that may be adapted to the current environment. Educators are therefore expected to emphasize problem-solving in teaching mathematics to facilitate effective and meaningful learning.

Puspitasari, Yusmin, and Nursangaji (2015) stated that students had conceptual and fundamental difficulties in solving narrative problems involving a system of two-variable linear equations. Based on the data analysis, it is known which difficulties and factors contribute to students' difficulties. These difficulties include transforming story problems into mathematical sentences, executing operations using elimination and substitution methods, performing addition and subtraction, obtaining variable replacement values, and transforming variable replacement values into question sentences.

Several studies have been conducted to overcome difficulties in mastering linear equations in two variables. Aisy, Farida, and Andriani (2020) researched the development of e-modules with the assistance of software to facilitate student comprehension. In addition, Manzilina, Listiawati, and Wijayanti (2020) created video scribes as learning media for this material. Furthermore, several studies have been conducted on the learning trajectory of two-variable linear equations. For instance, Rezky and Jais (2020) created a hypothetical learning trajectory for this subject matter. Meanwhile, Deciku et al. (2022) conducted a similar study using a realistic mathematics education approach. According to the researcher, no research has been conducted on HLT using this material, supported by learning media and problem-based learning.

Based on the described explanations and the results of pre-study interviews, developing an HLT design, specifically for linear equations in two variables material, is necessary to make it easier for teachers to deliver learning material. In addition, it is anticipated that students will be assisted in receiving lessons. Technology as a learning medium has been shown to help students attain better learning outcomes; thus, WolframAlpha was utilized in this study to facilitate the learning process. Therefore, this study aims to construct an HLT design supported by problem-based learning for middle school students studying linear equations in two variables. This study's findings are expected to assist educators in adopting and implementing HLT designs to increase student learning outcomes and make learning more meaningful.

Literature Review

The learning trajectory is the progression of students' cognitive abilities and knowledge during learning activities. The learning trajectory will assist instructors in applying models, teaching material strategies, and assessments that correspond to the cognitive stages of their students (Shepard et al., 2018). Learning trajectory research has been extensively studied in numerous nations. Simon (1995) introduced the term learning trajectory in his article *Reconstructing Mathematics Pedagogy from a Constructivist Perspective*. The article discusses the concept of a mathematics learning trajectory. Using technology and data, Bakker (2003) successfully formulated student learning paths to develop symbols and their meanings in mathematics. In their article "Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory," (Simon & Tzur, 2004) devised learning activities based on elaborating on a hypothetical learning trajectory.

Surya (2018) researched elementary school mathematics learning trajectories. The study results indicate that students' learning trajectory is their path to comprehending learning. The learning trajectory was derived from the verified hypothetical learning trajectory. Meanwhile, Astuti and Wijaya (2021) conducted a similar study about rigid definition material in middle school. In addition, Refianti & Adha (2018) constructed a learning trajectory based on a hypothetical learning trajectory for learning material on the surface area of cubes and blocks. The findings of this study indicate that the use of concrete objects such as cube-shaped boxes and blocks can help students understand the concept of the surface area of cubes and blocks through activities such as identifying cubic and rectangular shapes, drawing nets, and writing definitions of surface areas up to the elevated level, which is to write down the formula for the surface area of cubes and blocks, thus producing a learning trajectory.

Problem-based learning is one of the learning models that can support learning in every discipline, including mathematics (Malasari et al., 2017). Because problem-based learning is implemented to develop students' critical thinking skills further, and in this model, students are encouraged to be more active in problem-solving than a teacher and to concentrate more on investigations or discussions during learning. This learning model is implemented by presenting students with contextual problems designed to encourage them to investigate and solve each problem in collaboration with other group members (Kaendler et al., 2015).

The purpose of mathematics courses is, among other things, to train students to think logically, rationally, critically, meticulously, honestly, and effectively in order for them to be able to adapt to a world that is constantly changing (McPeck, 2016). This extremely high standard cannot be met solely through memorization, routine problem-solving exercises, and the conventional learning process. Therefore, problem-solving is essential to learning mathematics, as it allows students to acquire experience applying their existing knowledge and skills to solve non-routine problems (Chong et al., 2018).

Learning media is a factor that contributes to the success of the school learning process because it can facilitate the transfer of information from instructors to students or vice versa (Abror et al., 2020) . Through digital media and various educational technology, students with varying learning abilities can be supported, and more educational opportunities can be available (Kukulska-Hulme & Viberg, 2018). A lack of instructional media can hinder the learning process. Therefore, a teacher must create a learning medium that can facilitate the learning process and enhance the quality of learning—creating instructional media designed to enhance mathematics learning in schools (Dwijayani, 2019).

Frequently, junior high school students encounter problems because they have not mastered the fundamental mathematics required to comprehend more complex ideas (Abdullah et al., 2015). For instance, the child may struggle with fundamental arithmetic operations like addition, subtraction, multiplication, and division. A linear equation with two variables is more abstract and complex than other mathematical concepts that junior high school students have previously learned (Agrawal & Morin, 2016). Understanding the relationship between two variables and how to represent them in an equation is required for this concept. Understanding two-variable linear equations require analytical thinking skills, such as recognizing patterns, analyzing data, and solving problems (Pratiwi et al., 2021). Students in junior high school who are less adept at analytical reasoning may have difficulty comprehending this material. The learning trajectory benefits for mathematics teachers include the ability to comprehend how students think and, thus, how to assist students in learning mathematics effectively (Wilson et al., 2015). It should be noted in the learning trajectory that each pupil in the class has a distinct learning path. By understanding the learning trajectory, teachers should be able to discern strategies for students' potential difficulties in learning mathematics, allowing them to provide appropriate guidance throughout the process.

Very little research has been conducted on HLT for linear equations with two variables using problem-based learning, explicitly using student-friendly instructional materials. It is anticipated that the developed HLT will make it simpler for teachers to develop curricula and effective lesson plans, as learning trajectories can assist teachers in understanding the concepts that must be taught in stages (Yuliani et al., 2018). In the meantime, with HLT, students can comprehend the material on linear equations with two variables incrementally and systematically, allowing them to develop a strong and thorough understanding of the concept.

Developing a learning trajectory is critical in education (Rich et al., 2017). The learning trajectory can assist educators in planning efficient and effective learning. By identifying specific learning objectives and phases, learning trajectory assists educators in ensuring that students can comprehend concepts in depth and acquire the necessary skills (Wijngaards-de Meij & Merx, 2018). In addition, learning trajectory can assist curriculum makers in creating a structured and unified curriculum. By understanding the phases of learning that students must attain, curriculum makers can develop a curriculum that is appropriate for student development and ensure that students can comprehend concepts in depth and master the necessary skills (Franklin et al., 2017).

Methodology

This study employed a design research methodology. This method adopts the procedures of Gravemeijer and Cobb (2006), namely (1) preparing for the experiment, (2) experimenting in the classroom, and (3) retrospective analysis. However, this study was confined to preparations for the experiment. At this stage, characteristics will be acquired for establishing the initial design of HLT on the linear equations in two variables material using the graphical method and the Wolfram Alpha application. The stages carried out by the researcher in compiling the HLT were: (1) examining the characteristics of students' thinking theoretically in the age range of 12 to 14 years (grade 8); (2) studying history and in-depth study of theory regarding the concept of linear equations for two variables and research on this topic; (3) reviewing the curriculum, syllabus, and mathematics textbooks used by grade 8 students (including examining the methods, approaches, or methods used to convey the concept of linear equations in two variables); (4) reviewing the aspects of learning obstacles that occur during the learning process; (5) examining what didactic situations will be constructed, predicting student responses that might occur.

In this study, the data sources are primary data and secondary data. The primary data consists of results from interviews and observations. In contrast, secondary data consists of the research findings of others published in scholarly journals and publications. The observation was used to collect data for this study, specifically when observing the learning process of eighth-grade students at Dharma Jaya Middle School, Indonesia. There is a need for interviews to acquire data regarding students' difficulties in learning mathematics. Interviews with the mathematics teachers regarding the learning barriers students encounter and the teacher's method of teaching students are required. In addition, the literature review is used as consideration material when conducting research. The researcher examines textbooks and lesson plans used by subject teachers during the learning process to determine whether errors can cause students to encounter learning obstacles.

The study site is Dharma Jaya Middle School in Medan, Indonesia. The research population was divided into two groups: all eighth-grade students and all mathematics teachers. In the meantime, the sample consisted of two eighth-grade mathematics teachers and 27 students. In addition, this development was supported by document analysis and interviews. The guided interview covered the following indicators: students' difficulties, the teacher's role in learning, students' perspectives on how the teacher teaches, the teacher's comprehension of the learning trajectory, and the teacher's experience using learning media. The entire population of teachers was chosen as subjects. Meanwhile, four eighth-grade students were selected at random from the population.

Results

Pre-Study Analysis

A literature review, needs analysis, curriculum analysis, and concept analysis was conducted at this stage. An analysis of the learning flow following the problem-based learning model and the application of technology was examined. In the needs analysis activity, the limitations and strengths of the learning design implemented thus far in the schools where the research was conducted were analyzed, focusing on linear equations in two variables. After that, the appropriate instructional design for teaching linear equations in two-variable was determined. The problem-based learning model was ultimately selected as the method for constructing the learning trajectory in this study.

After conducting a needs analysis, a curriculum analysis was performed, examining the curriculum's content to see if it was sufficient to meet the program's goals. The next step is to review the curriculum and lesson plan for subjects associated with the material. In the final stage of pre-study analysis, concept analysis was conducted, i.e., an analysis of the fundamental concepts that must exist in learning linear equations in two variables to achieve the primary learning objectives: to enhance problem-solving skills and student learning independence.

Interview Study

The outcomes of interviews with teachers and students reveal several discoveries regarding mathematics instruction thus far conducted in the classroom (Table 1). Based on these findings, it is essential to implement a learning design with a learning trajectory so that students can learn without feeling forced or pressured.

Table 1. Interview Results

Respondent Responses	**Findings**
Student 1: "Every time I study mathematics, I feel anxious because of its difficulty."	Students are always worried about math lessons.
Student 2: "Every time I study mathematics, the teacher guides me in answering the questions given."	Students could not uncover the theoretical concept of a system of linear equations in two variables on their own.
Student 3: "I lacked the prerequisite knowledge for learning a system of linear equations in two variables."	Students are unfamiliar with the prerequisite material that must be acquired before studying a linear equation system in two variables.
Student 4: "I like teachers who demonstrate in depth how to answer questions and provide comprehensive material."	Students prefer teachers who thoroughly explain the material.
Teacher 1: "I am unfamiliar with the concept of a learning trajectory and have never applied it to my teaching."	The teacher has never designed a student learning trajectory for studying the material of a system of linear equations in two variables.
Teacher 2: "I have never used a third-party application. I have only used what is provided."	When teaching linear equations in two variables, the teacher never incorporates technology.

Study Design

Table 2, 3, and 4 describe the activities performed by students using the WolframAlpha application based on the problem-based learning model. Students learned the concept of a system of linear equations in two variables and how to calculate solutions using graphics during the early learning activity stage. All provided concepts were derived from daily occurrences or problems. Meanwhile, core learning activities were conducted following the model of problem-based learning. In addition, an HLT design scheme derived from the preceding activities is shown in Figure 5.

Table 2. Early Learning Activities

Stage	**Teacher Activities**	**Student Activities**
1	Before initiating the lesson, the teacher reviewed the concept of linear equations in two variables and the solutions.	Students listened carefully to the teacher's explanation. Students tried to recall by answering the teacher's questions.
2	The teacher motivated students by emphasizing that if they grasped this material, they could solve story problems involving systems of linear equations in everyday life.	Students inquired and responded to the teacher's explanation.

Table 3. Core Learning Activities

Syntax	Teacher Activities	Student Activities
1. Orienting students on the problem	The teacher instructed students on the project tasks, which included buying and selling illustrations needed to answer questions in the Student Worksheets. The teacher asked students to solve problems in the Student Worksheets.	Students listened to the teacher's instructions on the illustrations of buying and selling that will be used to answer questions in the Student Worksheets.
2. Organizing students to study	The teacher organized students into diverse study groups. Each group contained three to four students. The teacher distributed the Student Worksheets to each group and asked the group leader to lead the discussion.	Forming groups according to the teacher's instructions. Discussion of the Student Worksheets in Groups: Students planned completion steps by collaborating, discussing, and consulting with their peers in each group.
3. Guiding group investigations	The teacher instructed three students to perform buying and selling illustration activities using the math problems shown in Figure 1 in front of the class. (Note: To help students grasp the concept of a system of linear equations in two variables, the student book provides a solution method using graphics)	Three students came to the front of the class to demonstrate the buying and selling of stationery, one as a seller and two as a buyer. Students identified buying and selling in the market as one of the everyday difficulties associated with linear equations in two variables.
	The teacher instructed students to create mathematical equations based on the given scenario.	Students created equations for the given problem.
	The teacher-guided students to solve the assigned problem using the graphical method.	Students solved the problem using the graphical method.
	The teacher instructed students to use the taught WolframAlpha application to explore other solutions or compare their solutions.	Students accessed the WolframAlpha application on a computer or mobile device.
4. Developing and presenting works	The teacher requested that each group submit problem-solving results on the Student Worksheets. The teacher instructed each group to prepare presentation materials about resolving problems. The teacher managed the presentation process and encouraged student participation. The teacher asks a representative of one of the groups to explain the problem-solving results, and other group members are requested to provide feedback.	Students submitted assignments on the Student Worksheets to the teacher. Students developed presentation materials. Students listened to the teacher's instructions in making a presentation. Students presented their work to the class (one student represented the group), and other groups provided feedback.
5. Analyzing and evaluating the process of problem-solving	The teacher instructed each group to double-check the complete process in the Student Worksheets. The teacher assisted students in evaluating and reflecting (cooperation, communication, and asking questions)	Students examined the completed problem-solving process Students analyzed and reflected on the results of their problem-solving abilities.

Table 4. Closing Activities

Teacher Activities	Student Activities
The teacher helped students summarize the definition of a system of linear equations in two variables.	Students summarized what they had learned.
The teacher concluded the learning activity by encouraging students to continue their studies and assigning homework.	Students were given homework to do at home.

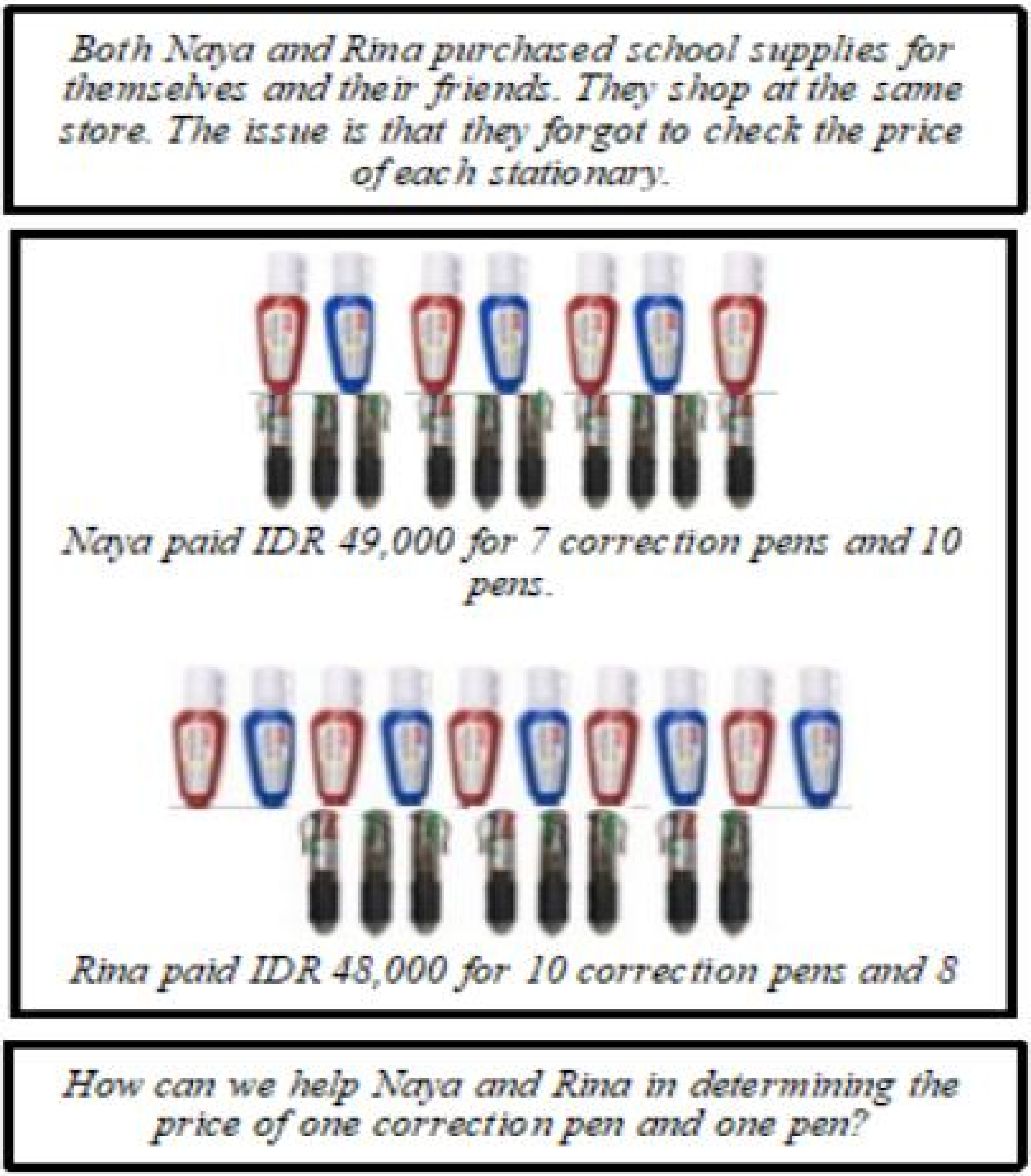

Figure 1. Example of Mathematical Problem

From the previous problem, we can express:
The price of one correction pen = x
The price of one pen = y

The mathematical equations of the problem are
Naya: $7x + 10y = 49{,}000$
Rina: $10x + 8y = 48{,}000$

Determine the line coordinates of the two equations:
Naya case:
given $y=0$, *so* $7x + 10y = 49{,}000 \rightarrow 7x = 49{,}000 \rightarrow x = 7{,}000$
given $x=0$, *so* $7x + 10y = 49{,}000 \rightarrow 10y = 49{,}000 \rightarrow y = 4{,}900$
The coordinates are determined as $(x, y) \rightarrow (7{,}000, 0)$ *and* $(0, 4{,}900)$
Rina case:
given $y=0$, *so* $10x + 8y = 48{,}000 \rightarrow 10x = 48{,}000 \rightarrow x = 4{,}800$
given $x=0$, *so* $10x + 8y = 49{,}000 \rightarrow 8y = 48{,}000 \rightarrow y = 6{,}000$
The coordinates are determined as $(x, y) \rightarrow (4{,}800, 0)$ *and* $(0, 6{,}000)$

Figure 2. Step-by-step in Solving Mathematical Problems Given

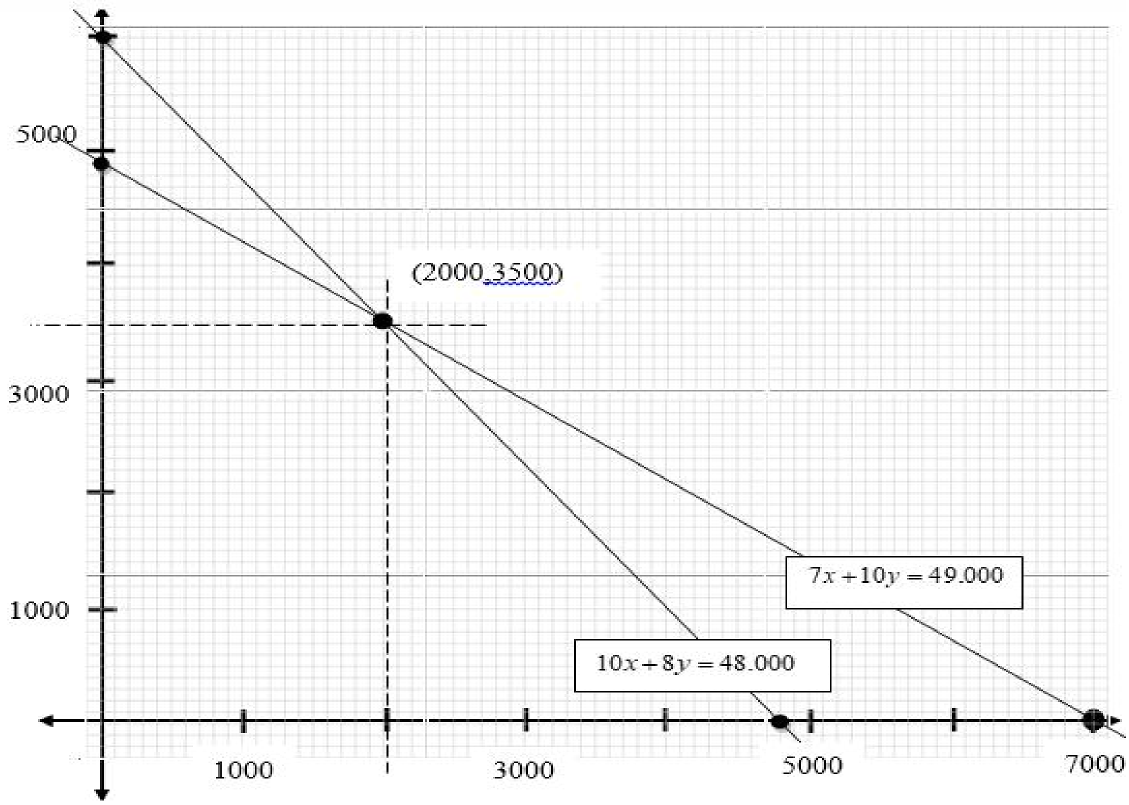

Figure 3. Graph Generated from Solution in Figure 2

Figure 1 is an example of a problem illustrated by students in front of the class. Figure 2 is the initial step to solve the problem in Figure 1. Then, based on Figure 2 and Figure 3, students were guided to plot a graph. Meanwhile, Figure 4 displays the solution using WolframAlpha as a comparison for the solution using graphics (Figure 5).

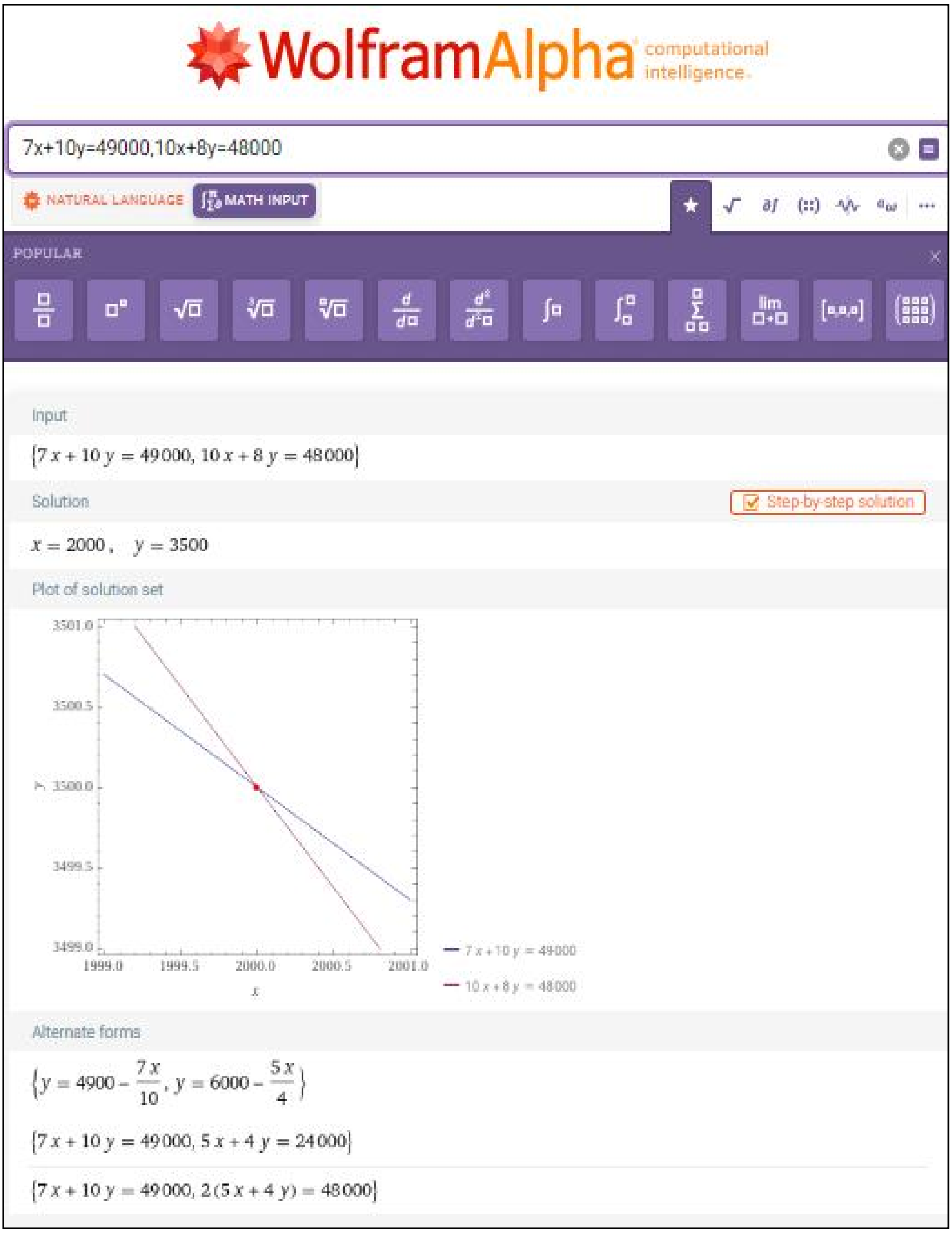

Figure 4. Example of Mathematical Solution using WolframAlpha Application

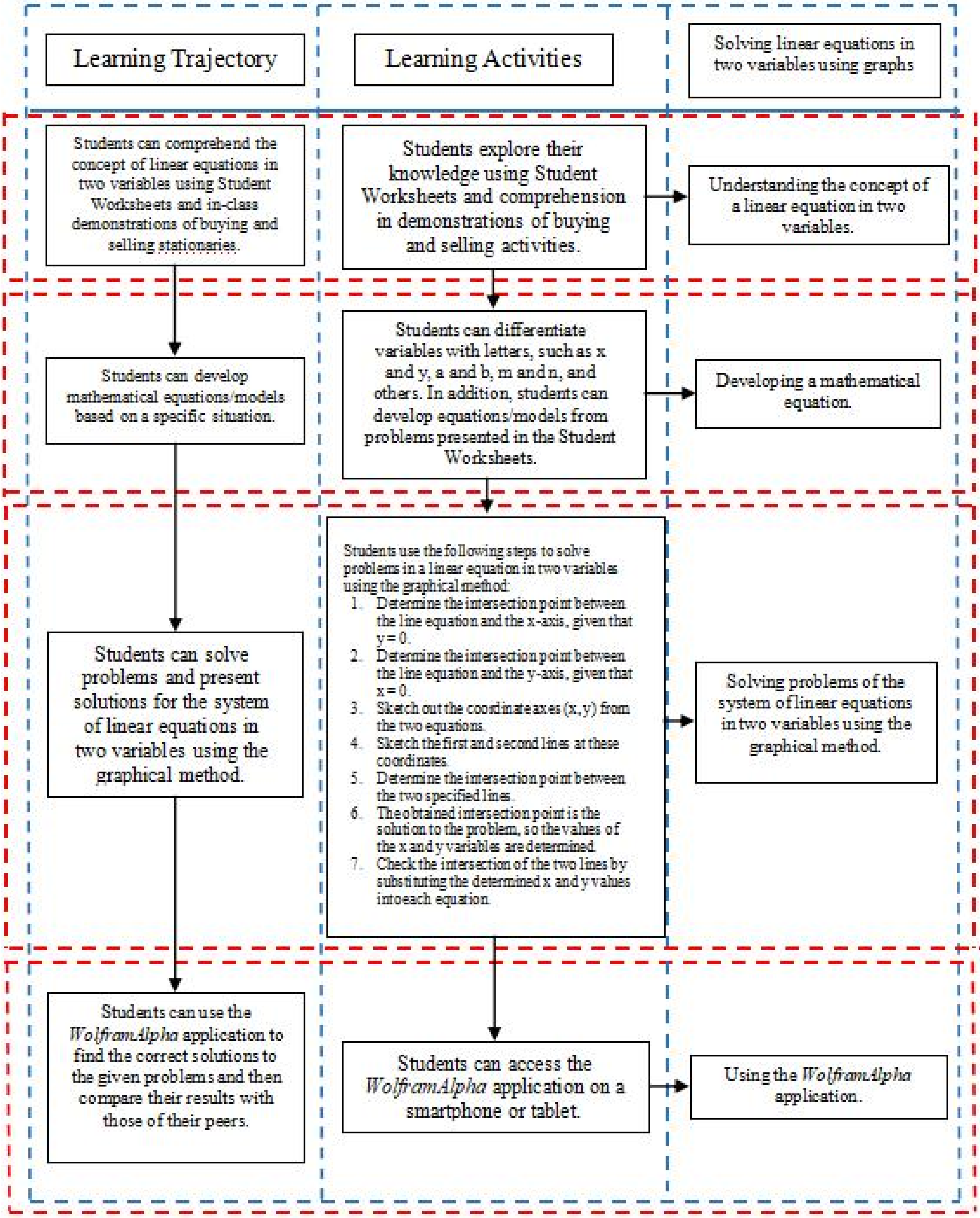

Figure 5. Design of HLT for Learning Linear Equations in Two Variables using Graphical Methods

Discussion

The choice of problem-based learning as the platform for constructing student learning trajectories is based on the rarity of its use by researchers in designing HLT. Typically, researchers employ the Realistic Mathematics Education (RME) method while constructing HLT for mathematics learning (Annisa & Mauleto, 2020; Armanto, 2002; Edo & Tasik, 2019; Febriani & Sidik, 2020; Indriani et al., 2018; Lede & Kii, 2018; Liu, 2019; Meirida et al., 2021; Rangkuti & Siregar, 2019; Yulia et al., 2020). In addition, the researchers are interested in students' problem-solving skills in this current study. Learning that can enhance students' problem-solving skills is acquired through problem-based learning (Newmann & Wehlage, 1993; Pertiwi et al., 2019; M. A.

R. Pratama et al., 2019; Subekti & Jazuli, 2020).

According to Permendikbud No. 58 (2014), one of the purposes of learning mathematics is for pupils to be able to solve problems using critical thinking and logical thinking. Meanwhile, the learning that has been accomplished thus far does not support government rules in achieving the objectives of mathematical education. As a result, problem-solving skills must be fostered through student education. Problem-based learning presents students with actual, everyday situations. Eventually, it is possible to bridge formal mathematical principles with real-world mathematical concepts through problem-based learning.

In this study, the problem-based learning model was implemented using a five-stage, step-based learning procedure: 1) Orienting students on the problems, 2) Organizing students to study, 3) Guiding group investigations, 4) Developing and presenting works, and 5) Analyzing and evaluating the process of problem-solving (Arends, 2008). At each level of the learning process, the teacher's and potential student activities are planned. These student activities serve as the foundation for developing learning flow designs. The highlight of student activities in designing HLT is the content for solving linear equations in two variables using the graphical method in the third stage of the problem-based learning model. At this point, students began to discuss the Student Worksheets they were working on. In problem-solving, the student focuses on their methods to get the desired results.

This study uses the problem-based learning model for its reasons, specifically to examine how well students can solve mathematical problems. Finding non-routine problems is highly related to problem-solving ability. Problem-based learning helps increase problem-solving skills by applying relevant learning (Newmann & Wehlage, 1993; Pertiwi et al., 2019; M. A. R. Pratama et al., 2019; Subekti & Jazuli, 2020). Since problem-based learning emphasizes problem-solving in the learning process, it is ideally suited for problem-based learning.

Because the model contains contextual characteristics, the design of the HLT using the problem-based learning model is consistent with the material of the system of linear equations in two variables, as indicated by the preceding description. Amalia et al. (2017), Khoirudin and Rizkianto (2018), and Nupus et al. (2022) have conducted research relating to HLT and problem-based learning models. As a further study, this current study discusses the design of the learning trajectory in mathematics learning and its impact on mathematical comprehension. The created learning trajectory distinguishes this study from previous research significantly, notably in establishing the learning flow of linear equations in two variables with the WolframAlpha application and the application's problem-solving efficiency. In addition, prior research did not link HLT design to technology or learning medium.

The use of technology in mathematics education is necessary to improve the teaching and learning process. In this study, the use of applications gives benefits such as the ability to stimulate students' interest in learning, make students more independent, and make students more active and enthusiastic. In addition, with technology, time spent learning will be used more effectively, and the learning process will be simplified (Hwang et al., 2015). Thus, technology becomes necessary for students, making the teaching and learning process more enjoyable, effective, and efficient.

During interviews with students regarding classroom learning, it was discovered that students tend to feel bored when learning mathematics and that some students fear learning mathematics because it is a subject that is difficult to comprehend. In addition, students like a teacher who thoroughly explains each step involved in constructing sample questions and assigns questions identical to the examples but with different numbers so that students only follow the examples of already-existing questions. This indicates that students are never presented with non-routine problems, resulting in a lack of mathematical knowledge (Johar & Lubis, 2018; Öztürk et al., 2020).

Moreover, during the interview with teachers, it was discovered that they could not comprehend the term Hypothetical Learning Trajectory since they had never developed a lesson program for mathematics based on this theory. Teachers typically employ the lecture method (conventional). Suppose teachers are not equipped with teaching knowledge or do not update their expertise, in this case, every teaching-learning process implemented will fail to meet the objectives of the national educational system. Therefore, training them on the most recent concerns and developments in mathematics education is necessary so that teachers, especially in rural locations, are not left out of information (Venkat & Spaull, 2015).

Based on the study's findings, it was discovered that the stages used by students in solving problems in the HLT corresponded to the flow of students' mindsets. The HLT design derived in this study is a valuable resource for teachers instructing linear equations in two variables. This design offers the instructor an overview of pupils' steps to comprehend the material's concepts. In addition, teachers will find it easier to apply this design to their students by selecting a method that matches their students' characteristics. Furthermore, the proposed learning trajectory design will make it less intimidating for students to progress through each of the existing mathematical learning stages. Therefore, students might follow each step using this approach without teacher pressure. In this approach, the teacher's role in learning is limited to that of facilitator and scaffolding.

Conclusion

Based on the findings, it was determined that the HLT design through a problem-based learning model on linear equations in two variables using the WolframAlpha application could be used by teachers as a reference for designing mathematical learning. The constructed learning design may solve the specified mathematical problem. In addition, this HLT design demonstrates that the contextual characteristics of the problem-based learning model make it ideally suited for relevant material. However, teachers may also select different learning strategies or models appropriate for the subject. Based on the results, the researcher identified several limitations of this study, including the lack of student achievement that could support the research findings. Student achievement is crucial for determining the efficacy of HLT implementation. In addition, this research necessitates the results of a questionnaire that reveals students' perspectives on the HLT-based learning process. Therefore, additional research is required to determine the impact of HLT on student learning outcomes.

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