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Research Article

The Connected Certified Domination Number Of Join Of Two Graphs

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ARTICLE INFO	ABSTRACT

A subset S from the vertex set V(G) of a graph G is known a dominating set. This means that every vertex in S must be adjacent to at least one vertex in S. Now, when we refer to a dominating set S as a certified dominating set of G, it has a special rule. Each vertex in S has either zero or at least two neighbors that are outside of S. Furthermore, a certified dominating set S can be called a connected certified dominating set if the subgraph created by S remains connected within G. The minimum size of the connected certified dominating set is what we term the connected certified domination number, denoted as $\gamma_{cer}(G)$. In this study, we find the bounds for $\gamma_{cer}(G)$ and determine its exact value for several classes of graphs.

Keywords: Certified Domination, Connected Dominating set, Connected Certified Domination Number.

1. Introduction

The graph G, which is expressed as G=(V,E), signifies a finite undirected & connected graph. Import, this type of graph contains neither nor multiple edges. We use the terms "order" and "size" to describe G, represented by n and m respectively. For details on graph terminology, one can refer to West [4].
Let $v \in V$, the open neighbourhood and the closed neighbourhood of v are denoted by $N(v)$ and $N[v] = N(v) \cup \{v\}$, respectively. If $S \subseteq V$, then $N(S) = \cup_{v\in S} N(v)$ and $N[S] = N(S) \cup S$. The minimum degree of the vertices in G is represented by δ(G), whereas the maximum degree is denoted by Δ(G). We write K_n, P_n and C_n for a complete graph, a path graph, and a cycle graph of order n, respectively.
A set S⊆V(G) is called a dominating set. This is true if every vertex v∈V-S is either part of S or is right next to a vertex in S. Now, a γ-set is simply a dominating set with the minimum number of elements. The domination number, denoted as γ(G), represents the smallest size of these γ-sets. A concise overview of dominating sets and related ideas can be found in Haynes et al. [5]. Numerous variations of domination have been introduced by combining two different dominating parameters [5,6,7].
The focus here is on one specific variant: connected certified domination in graphs. To clarify, a set S⊆V qualifies as a certified dominating set if every vertex that is not in S is adjacent to at least one vertex in S. Additionally, it must have either zero or at least two neighbors in V-S. The certified domination number, represented by $\gamma_{cer}(G)$, indicates the smallest size of a certified dominating set within G. This concept has remarkable applications in various real-life scenarios. It drove us to explore the connected certified domination number for the join of two graphs.
According to [1], the definition of connected certified domination in graphs is articulated as follows:

Definition 1.1
A certified dominating set, referred to as S, is recognized as a connected certified dominating set if the subgraph that it induces is indeed connected. The connected certified domination number, denoted $\gamma_{cer}(G)$ represents the smallest size of any connected certified dominating set for G.

Definition 1.2
We designate the join of two graphs with the symbol $G_1 + G_2$. This graph is formed from two separate copies of G_1 and G_2 by linking every vertex in G_1 to every vertex in G_2.[7]

2. Main Results

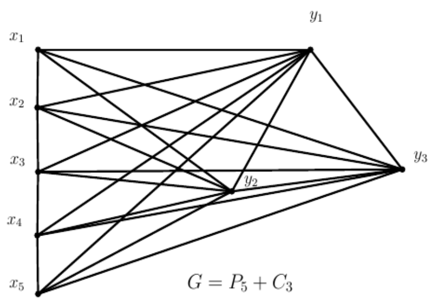
Theorem 2.1
For the two connected graphs G and H, $1 \leq \gamma_{cer}^{c}(G + H) \leq 2$.

Proof.
Let G and H be two connected graphs. It is obvious that every minimum connected certified dominating set of $G + H$ contains minimum one vertex. Therefore, $\gamma_{cer}^{c}(G + H) \geq 1$. Next to prove $\gamma_{cer}^{c}(G + H) \leq 2$.
Suppose $|V(G)| = 1$. Take $S = \{x\} = V(G)$. By definition in $G + H$, that x is adjacent to every vertex in $V(G + H) - \{x\}$. So if $|V(H)| = 1$, then $\gamma_{cer}^{c}(G + H) = 2$. Otherwise $V(G)$ is a connected certified dominating set in $G + H$ and hence $\gamma_{cer}^{c}(G + H) \leq 2$.
Similarly we can verify that $|V(H)| = 1$.
Now assume $|V(G)| \geq 2$. Since H is connected that $|V(H)| \geq 1$. We can select any vertex x from V(G) which has at least two neighbors in V(G)-{x}. Therefore, if we take some vertex x in G along with any vertex y in H, this pair dominates all other vertices in G+H. Moreover that x and y must be adjacent. So $\gamma_{cer}^{c}(G + H) \leq 2$. Similarly we ccan verified for that $|V(H)| \geq 1$. Thus proved.

Theorem 2.2
Let H be any connected on $n \geq 1$ vertices. Then, $\gamma_{cer}^{c}(H + C_3) = 1$.
Proof.
Consider $G = H + C_3$. That is, G is the join of H and C_3. So the vertex set $V(G) = \{x_1, x_2, x_3, y_i; 1 \leq i \leq n\}$ and the edge set $E(G) = \{x_1x_{2,}x_1x_3, x_2x_3, x_1y_i, x_2y_i, x_3y_i; 1 \leq i \leq n\} \cup E(H)$. Therefore $|V(G)| = n + 3$ and $|E(G)| = 3n + 3 + E(H)$.

Now we select a set $S = \{v; v = \Delta(G)\}$ in G. Clearly that S dominates $V(G)$ and has minimum two adjacent vertices in both C_3 and H. So that $\gamma_{cer}^{c}(G) \leq 1$. By Theorem 2.1, we conclude that $\gamma_{cer}^{c}(G) = 1$.

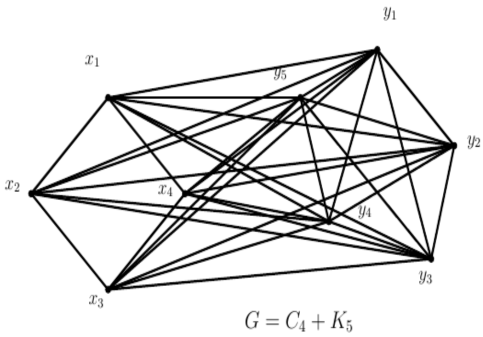
Illustration 2.3
Consider the graph G as join of P_5 and C_3. That is, $G = P_5 + C_3$ in Figure below.



Here $S_1 = \{y_1\}, S_2 = \{y_2\}$, and $S_3 = \{y_3\}$ are the only minimum connected certified dominating sets of G and hence $\gamma_{cer}^{c}(G) = 1$.

Theorem 2.4
If H is a connected graph with $m \geq 1$ vertices. Then, $\gamma_{cer}^{c}(H + K_n) = 1$.
Proof.
Consider $G = H + K_n$, K_n is a complete graph with n vertices, while H is a connected graph with m vertices. The vertex set of G can be represented as V(G) = $\{x_i, y_j; 1 \leq i \leq n, 1 \leq j \leq m\}$ and the edge set of G as $E(G) = \{x_ix_{i+1}; 1 \leq i \leq n - 1\} \cup \{x_ix_j; 1 \leq i \leq n, 1 \leq j \leq m\} \cup E(H)$. Then $|V(G)| = m + n$ and $|E(G)| = nm + n + \frac{n(n+1)}{2}$.
Now we select a set $S = \{x; x = \Delta(G)\}$. S is the only vertex in G that dominates all vertices and has at least two neighbors in $V(G).-S$.. Moreover that $< S >$ is connected. Therefore that S is a connected certified dominating set of G and so $\gamma_{cer}^{c}(G) \leq |S| = 1$. Thus, by Theorem 2.1, we conclude that $\gamma_{cer}^{c}(G) = 1$.

Illustration 2.5
Consider the graph G as join of C_5 and K_5. That is, $G = C_4 + K_5$ given in Figure below.



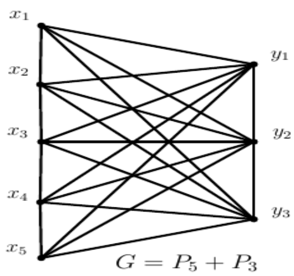
$G = C_4 + K_5$

Here $S_1 = \{y_1\}$, $S_2 = \{y_2\}, S_3 = \{y_3\}, S_4 = \{y_4\}$ and $S_5 = \{y_5\}$ are the minimum connected certified dominating sets of G and hence, $\gamma_{cer}^{c}(G) = 1$.

Theorem 2.6
Let H be any connected graph on $n \geq 1$ vertices. Then $\gamma_{cer}^{c}(H + P_3) = 1$.
Proof.
Consider $G = H + P_3$, The path graph of order 3 is represented by P_3, while the connected graph of order n is represented by H. Then the graph G has vertex set $V(G) = \{x_1, x_2, x_3 y_i; 1 \leq i \leq n\}$ and the edge set of G as $E(G) = \{x_1x_2, x_1x_3, x_2x_3, x_1y_i, x_2y_i, x_3y_i; 1 \leq i \leq n\} \cup E(H)$. Clearly, $|V(G)| = n + 3$ and $E(G) = 3n + 3 + |E(H)|$.

Now, we select a set $S = \{x; x = \Delta(G)\}$. Then $N[S] = V(G)$ and S has minimum two neighbours in $V(G) - S$. Moreover, by the definition of join of graphs that, $< S >$ is connected. Thus, S is a certified dominating set of G that is connected. and so $\gamma_{cer}^{c}(G) \leq |S| = 1$. Theorem 2.1 states that $\gamma_{cer}^{c}(G) = 1$.

Illustration 2.7
Consider the graph G as join of P_5 and P_3. That is $G = P_5 + P_3$ shown in Figure below.



$G = P_5 + P_3$

Here , $\gamma_{cer}^{c}(G) = 1. S = \{y_2\}$ is the unique connected certified dominating set of minimum cardinality.

Theorem 2.8
Let G and H be any connected graph of order n and m, respectively. Then, $\gamma_{cer}^{c}(G + H) = 2$ if and only if neither $\Delta(H) = m - 1$ and $\Delta(G) = n - 1$.

Proof.
Let G and H be connected graphs of order n and m respectively. First assume, $\gamma_{cer}^{c}(G + H) = 2$. Then, $G + H$ has no vertex of degree $m + n - 1$. It follows that G and H has no vertex of degree $n - 1$ and $m - 1$, respectively.

Conversely, assume that $\Delta(G) \neq n - 1$ and $\Delta(H) \neq m - 1$. Then G and H has no vertex of degree $n - 1$ and $m - 1$, respectively. So, $\gamma_{cer}^{c}(G) \geq 2$ and $\gamma_{cer}^{c}(H) \geq 2$. This shows that $\gamma_{cer}^{c}(G + H) \geq 2$. Theorem 2.1 suggests that $\gamma_{cer}^{c}(G + H) = 2$.

Conclusion

We have studied the connected certified domination number of the join of two graphs obtained through various graph operations.

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