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Research Article

Semi Simple Ternary Ideals In Ternary Semi Rings

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ARTICLE INFO **ABSTRACT**

We introduce the notion of a generalized semi-ideal in a ternary semiring. Various examples to establish a relationship between ideals, bi-ideals, quasi-ideals and generalized semi-ideals are furnished. A criterion for a commutative ternary semiring without any divisors of zero to a ternary division semiring is given.

Key words and phrases: Ternary semiring, generalized semi-ideals and ideals in ternary semirings, ternary division semiring

1. Introduction

Ternary rings and their structures were investigated by Lister [4] in 1971. In fact, Lister characterized those additive subgroups of rings which are closed under the triple product. In 2003, T. K. Dutta and S. Kar [3] introduced the notion of a ternary semiring as a generalization of a ternary ring. A ternary semiring arises naturally as follows. Consider the subset Z^- of all negative integers of Z. Then $Z^-$is an additive semigroup which is closed under the triple product. Z^- is a ternary semiring. Note that Z^- does not form a semiring. In [3] T. K. Dutta and S. Kar introduced the notions of left/right/lateral ideals of ternary semirings and also characterized regular ternary semirings. In 2005, S. Kar [1] introduced the notions of quasi-ideals and bi-ideals in a ternary semiring. The notion of a generalized semi-ideal in a ring has been introduced and studied by T. K. Dutta in [2]. In this paper we introduce the notion of a generalized semi- ideal in a ternary semiring and study them. Also, we establish a relationship between generalized semi-ideals, ideals, bi-ideals, etc. in a ternary semiring to study some properties of a generalized semi-ideals in ternary semirings.

2. Preliminaries

For preliminaries we refer to [1] and [3].

Definition 2.1. An additive commutative semigroup S, together with a ternary multiplication denoted by [ ] is said to be a ternary semiring if $[abc]de] = [a[bcd]e] = [ab[cde]]$,

i) $[(a + b)cd] = [acd] + [bcd]$,
ii) $[a(b + c)d] = [abd] + [acd]$,
iii) $[ab(c + d)] = [abc] + [abd]$ for all $a, b, c, d, e \in S$.

Throughout, S will denote a ternary semiring unless otherwise stated.

Definition 2.2. If there exists an element 0 S such that $0 + x = x$ and $[0xy] = [xy0] = [x0y] = 0$ for all x, y S, then 0 is called the zero element of
S. In this case we say that S is a ternary semiring with zero. ∈ ∈

Definition 2.3. S is called a commutative ternary semiring if $[abc] = [bac] = [bca]$, for all $a, b, c \in S$.

Definition 2.4. An additive subsemigroup T of S is called a ternary subsemir- ing of S if $[t_1t_2t_3] \in T$ for all t_1, t_2, $t_3 \in T$.

Definition 2.5. An element a in S is called regular if there exists an element $x \in S$ such that $[axa] = a$. S is called regular if all of its elements are regular.

Definition 2.6. S is said to be zero-divisor free (ZDF) if for a, b, c S, $[abc] = 0$ implies that $a = 0$ or $b = 0$ or $c = 0$. ∈

Definition 2.7. S with $|S| \geq 2$ is called a ternary division semiring if for any non-zero element a of S , there exists a non-zero element $b \in S$ such that $[abx] = [bax] = [xab] = [xba] = x$, for all $x \in S$.

Definition 2.8. A left (right/lateral) ideal I of S is an additive subsemigroup of S such that $[s_1s_2i]$ I

($[is_1s_2]$ $I/[s_1is_2]$ I) for all i I, for all s_1, s_2 S. If I is a left, a right and a lateral ideal of S, then I is called an ideal of S.

Definition 2.9. An additive subsemigroup Q of S is called a quasi-ideal of S if $[QSS]\,([SQS] + [SSQSS])\,[SSQ] \subseteq Q$.

Definition 2.10. A ternary subsemiring B of S is called a bi-ideal of S if $[BSBSB] \subseteq B$.

3. Generalized semi-ideals in ternary semirings

Generalized semi-ideals in semirings are introduced and studied by T .K. Dutta in [1]. As a generalization, we define generalized semi-ideals in ternary semirings.

Definition 3.1. A non-empty subset A of S satisfying the condition $a + b \in A$, for all $a, b \in A$ is called

i) generalized left semi-ideal of S if $[[xxx]xa] \in A$ for all $a \in A$ for all $x \in S$,

ii) generalized right semi-ideal of S if $[axx]xx] \in A$ for all $a \in A$, for all $x \in S$,

iii) generalized lateral semi-ideal of S if $[xxa]xx] \in A$ for all $a \in A$, for all $x \in S$,

iv) generalized semi-ideal of S if it is a generalized left semi-ideal, a generalized right semi-ideal and a generalized lateral semi-ideal of S.

Example 3.2. Consider a ternary semiring Z of all integers. The subset A of Z containing all non-negative integers and the set B of all non-positive integers are generalized semi-ideals of Z.

Remark 3.3. The concepts of generalized semi-ideal and ternary subsemiring are independent in S. This means that is every ternary subsemiring of S need not be a generalized semi-ideal of S and every generalized semi-ideal of S need not be a ternary subsemiring of S. For this, consider the following examples.

Example 3.4. Let $S = M_2(Z_0^-)$ be the ternary semiring of the set of all 2×2 square matrices over Z_0 , the set of all non-positive integers.

Let $T = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} / a \in Z_0^- \right\}$. T is a ternary subsemiring of S, but it is not a generalized semi-ideal of S.

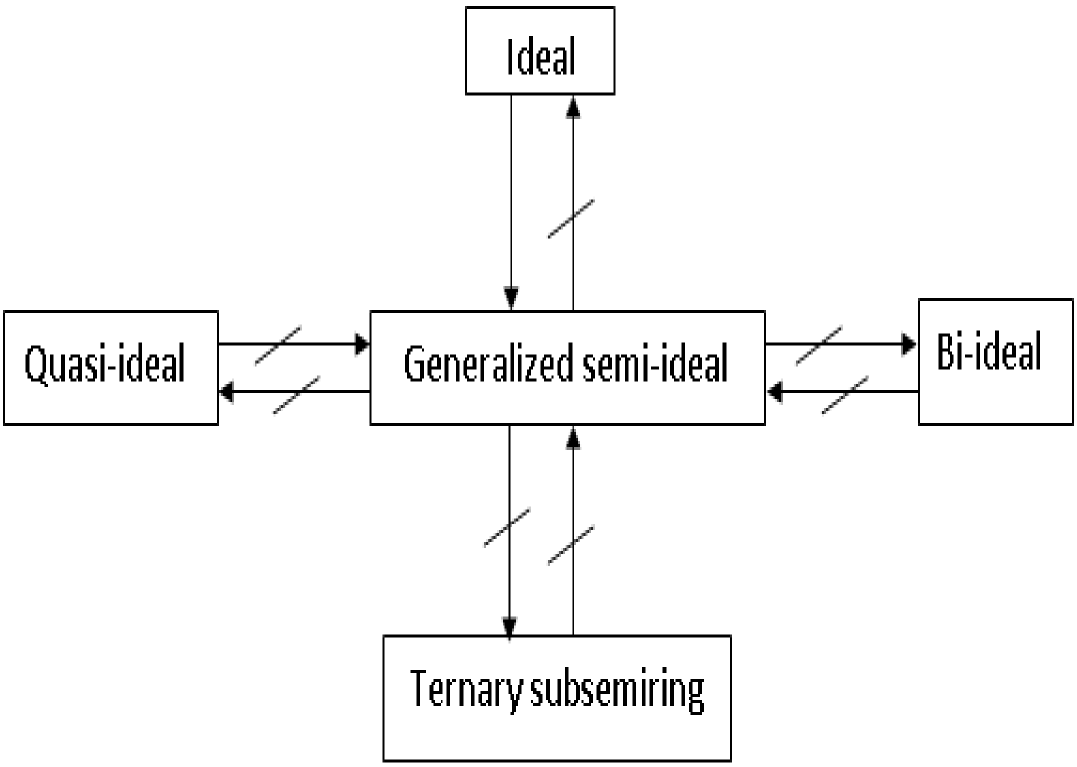
Example 3.5. Let $S = \{\ldots, -2i, -i, 0, i, 2i, \ldots\}$ be a ternary semiring with respect to addition and complex triple multiplication. Let $A = \{0, i, 2i, \ldots\}$. A is a generalized semi-ideal of S, but not a ternary subsemiring of S.

Every ideal of S is a generalized semi-ideal of S but the converse need not be true.

Example 3.6. Every quasi-ideal need not be a generalized semi-ideal and every generalized semi-ideal need not be a quasi-ideal of S. In Example 3.4), T is a quasi-ideal of S, but it is not a generalized semi-ideal of S. In Example 3.5, A is a generalized semi-ideal of S, but not a quasi-ideal of S.

Every quasi-ideal is a bi-ideal in S [2]. Hence, the bi-ideals and generalized semi-ideals in S are independent concepts.

The flow-chart of the relationship between the ideals, bi-ideals, quasi-ideals, ternary subsemiring and generalized semi-ideals in a ternary semiring is given below.



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