Educational Administration: Theory and Practice
2024, 30(2) 1496-1499
ISSN: 2148-2403
https://kuey.net/

Research Article

An Approach To Solve Sequencing Problem With Picture Fuzzy Numbers

Harpreet Kaur[1*]

[1*]Post Graduate Department of Mathematics, GSSDGS Khalsa College, Patiala, Punjab (India)" harpreet31kaur@gmail.com

Citation: Harpreet Kaur (2024) An Approach To Solve Sequencing Problem With Picture Fuzzy Numbers, *Educational Administration: Theory and Practice*, *30(2) 1496-1499*
Doi: 10.53555/kuey.v30i2.7764

ARTICLE INFO	ABSTRACT
	In this paper, a new method to solve Job Sequencing problem is given and by taking processing times as picture fuzzy numbers (PFNs) proposed method is applied to find total elapsed time. Job sequencing technique is used to find optimal sequence in order to minimize the total elapsed time of the sequence. For solving job sequencing problem in picture fuzzy environment basic definitions of picture fuzzy numbers are given. Firstly, an algorithm is designed to solve the problem and is applied on numerical example to clarify the concept. Finally, the conclusion is given.

Key Words: Picture fuzzy set, picture fuzzy number, absolute score of PFNs, actual score of PFNs.

1. Introduction

PFN [4,5] is a new tool to overcome the problem of vagueness and uncertainty in different fields. These fuzzy numbers have advantage over intuitionistic fuzzy numbers[6,7,12] that here both neutral and refusal membership degrees are introduced to manage uncertainty. PFSs are extension of Atanassov's [1,2,3] Intuitionistic fuzzy sets and are adequate in situations when more options are involved like 'yes', 'abstain', 'no', and 'refusal'. Due to the capability of handling more opinions, PFS has become more suitable to deal with imprecise and ambiguous information and many researchers [15,16] have used these in real-life problems. Singh [14] presented a geometrical interpretation of picture fuzzy sets and proposed correlation coefficients for picture fuzzy sets to apply it to clustering analysis under fuzzy environment. Guiwu [9] established a multiple attribute decision making method which is based on picture fuzzy cross entropy in which attribute values for alternatives are picture fuzzy numbers. Garg [8] presented some series of the aggregation operators for the picture fuzzy sets and based on these operators a decision-making approach was established. Some researchers compared the PFNs and proposed ranking method based on positive ideal solution, positive and negative goal differences, and score and accuracy degrees of the PFNs. Shit et al. [13] introduced a multi-attribute decision-making problem using harmonic mean aggregation operators with trapezoidal fuzzy number under picture fuzzy information. Hasan et al. [10] defined the arithmetic operations of generalized trapezoidal picture fuzzy numbers by vertex method and explored their properties. Kaur [11] solved job sequencing problem by taking processing times as intuitionistic fuzzy numbers in which we consider only two factors membership function and non-membership function. Due to the capability of handling more opinions, in this article we propose new algorithm to solve job sequencing problem in which processing times are taken as picture fuzzy numbers.

2. PRELIMINARIES

In this section, definition of Picture Fuzzy number(PFN) and related concepts are given which are used to solve numerical examples.

2.1 Crisp Set

A crisp set or a classical set A is defined as a collection of distinct and distinguishable objects. The objects are called elements of A.

2.2 Characteristic Function

A crisp set A, defined on the universal set X, can also be represented by where $A = \{(x,)\mu_A(x); x\epsilon X\}$ is called characteristic function defined by

$$\mu_A(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A \end{cases}$$

2.3 Fuzzy Set

The characteristic function of a crisp set $A \subseteq X$ assigns a value either 0or 1to each member in X. This function can be generalized to a function μ_A such that the value assigned to the element of the universal set X fall within a specified range [0,1] i.e $\mu_A: X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A. The function μ_A is called the membership function and the set $A = \{(x,)\mu_A(x); x\epsilon X\}$ defined by μ_A for each $x\epsilon X$ is called a fuzzy set.

2.4 Fuzzy Number

A fuzzy set A,defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_A: R \to [0,1]$ is continuous.
2. $\mu_A = 0$for all x in $(-\infty, c] \cup [d, \infty)$.
3. Is strictly increasing on $[a, c]$ and strictly decreasing on $[b, d]$.
4. $\mu_A = 1$for all x in $[a, b]$.

2.5 Intuitionistic fuzzy set

For universal set $X = \{x_1, x_2, \dots \dots, x_n\}$, an intuitionistic fuzzy set is defined as $A = \{\langle x, \mu_A(x), \theta_A(x)\rangle : x\epsilon X\}$ in which functions $\mu_A(x): X \to [0,1]$ and $\theta_A(x): X \to [0,1]$ called membership function and non-membership function respectively and for every $x\epsilon X, 0 \le \mu_A(x) + \theta_A(x) \le 1$ always holds.

2.6 Intuitionistic fuzzy Number

An intuitionistic fuzzy number is an intuitionistic fuzzy set which satisfies following conditions.

i) It is fuzzy normal.

ii) Membership function is convex i.e.

$\mu_A(\lambda a + (1-\lambda)b) \ge min(\mu_A(a), \mu_A(b))$ for all $a, b\epsilon R, \lambda\epsilon[0,1]$

iii) Non-Membership function is concave i.e.

$\theta_A(\lambda a + (1-\lambda)b) \le max(\theta_A(a), \theta_A(b))$ for all $a, b\epsilon R, \lambda\epsilon[0,1]$

2.7 Picture Fuzzy Set

A picture fuzzy set (PFS) A on the universe X is an object in the form of

$$A = \{(x, \mu_A(x), \theta_A(x), \nu_A(x)) | x\epsilon X\}$$

where $\mu_A(x)\epsilon[0,1]$ be the degree of positive membership of x in A, $\theta_A(x)$ and $\nu_A(x)\epsilon[0,1]$ are respectively called the degrees of neutral and negative membership of x in A. These three parameters $(\mu_A(x), \theta_A(x), \nu_A(x))$ of the picture fuzzy set A satisfy the condition $0 \le \mu_A(x) + \theta_A(x) + \nu_A(x) \le 1, \forall x\epsilon X$. Furthermore, (1-$\mu_A(x), -\theta_A(x) - \nu_A(x)$) is called degree of refusal.

2.8 Picture Fuzzy Number

A picture fuzzy set (PFS) A on the universe X is an object in the form of

$$A = \{(x, \mu_A(x), \theta_A(x), \nu_A(x)) | x\epsilon X\}$$

and for simplicity we denote it as $A = (\mu, \theta, \nu)$ and is called picture fuzzy number.

2.9 Ranking of Picture Fuzzy Numbers

Let $f_i = (\mu_i, \theta_i, \nu_i), i = 1,2, \dots, n$ be the set of picture fuzzy numbers, where μ_i, θ_i and ν_i be the positive, neutral and negative membership degree respectively.

1. Find $f^+ = (max\mu_i, max\theta_i, max\nu_i) = (\mu^+, \theta^+, \nu^+)$
2. Calculate positive goal difference (PGD)μ^* and negative goal difference (NGD) ν^* for all PFNs as ${\mu_i}^* = \mu^+ - \mu_i$ and ${\nu_i}^* = \nu_i - \nu^+$
3. Find Absolute score (p_i) as $p_i = (1 - {\mu_i}^*) - {\nu_i}^*$
4. Calculate average neutral degree as $\bar{\theta} = \frac{\sum_{i=1}^{n} \theta_i}{n}$
5. Find actual score of the PFNs $f_i = (\mu_i, \theta_i, \nu_i), i = 1,2, \dots, n$ as $S_i = \frac{p_i}{1-(\bar{\theta}-\theta_i)}$

3. Proposed method to solve Job Sequencing problem

We have many types of sequencing problems. In every problem objective is to minimize the total elapsed time and waiting time for the jobs. Here let us consider the problem consisting of n-jobs processed through 2-machines. In this method, processing time of machines is taken as Picture fuzzy numbers and thus sequencing problem can be written as follows:

Jobs→ Machines↓	J_1	J_2	…….	J_n
M_1	t_{11}	t_{12}	………	t_{1n}
M_2	t_{21}	t_{22}	…….	t_{2n}

Here t_{ij} is Picture Fuzzy number which denotes time duration taken by i^{th} job on j^{th} machine.
Algorithm for solving problem:
1. Start
2. Find actual score of all processing times t_{ij}.
3. Add processing times of two machines corresponding to each job.
4. From these numbers, find the smallest sum and if it corresponds to first machine then place that job in the first available position in the sequence.
5. If this is for second machine, then write that job in the last available position in the sequence. In case of tie : if it is for same machine then select lowest subscript job for machine 1 and largest subscript for machine 2.
6. Do not consider jobs that have assigned position.
7. Repeat these steps until all the jobs are included in the sequence.
8. Calculate overall elapsed time and idle time for both machines.
9. End.

6.4 Numerical Example

Example 6.4.1 *There are 5 jobs each of which must go through the two machines* M_1 *and* M_2 *in the order* M_1M_2 *with following processing times. Find optimal sequence of jobs that minimizes the total elapsed time required to complete all jobs.*

obs→ Machines↓	J1	J2	J3	J4	J5
M_1	(0.55,0.34,0.11)	(0.60,0.25,0.10)	(0.61,0.19,0.18)	(0.57,0.21,0.17)	(0.65,0.15,0.15)
M_2	(0.25,0.45,0.25)	(0.58,0.13,0.19)	(0.56,0.24,0.20)	(0.48,0.26,0.21)	(0.72,0.10,0.09)

Solution:

Step 1: Find positive ideal solution

$$f^+ = (max\mu_i, max\theta_i, maxv_i) = (\mu^+, \theta^+, v^+) = (0.72,0.45,0.25)$$

Step 2: Calculate positive goal difference (PGD)μ^* and negative goal difference (NGD) v^* for all PFNs as ${\mu_i}^* = \mu^+ - \mu_i$ and ${v_i}^* = v_i - v^+$

Jobs→ Machines↓	J1		J2		J3		J4		J5	
	PGD	NGD	PGD	NGD	PGD	NGD	PGD	NGD	PGD	NGD
M_1	0.17	0.14	0.12	0.15	0.11	0.07	0.15	0.08	0.07	0.10
M_2	0.47	0	0.14	0.06	0.16	0.05	0.24	0.04	0	0.16

Step 3: Find Absolute score (p_i) as $p_i = (1 - {\mu_i}^*) - {v_i}^*$

Jobs→ Machines↓	J1	J2	J3	J4	J5
M_1	0.69	0.73	0.82	0.77	0.83
M_2	0.53	0.80	0.79	0.72	0.84

Step 3: Calculate average neutral degree as $\bar{\theta} = \frac{\sum_{i=1}^{n} \theta_i}{n} = 0.227$

Step 4: Find actual score of the PFNs $f_i = (\mu_i, \theta_i, v_i), i = 1,2, \dots, n$ as $S_i = \frac{p_i}{1-(\bar{\theta}-\theta_i)}$

Jobs→ Machines↓	J1	J2	J3	J4	J5
M_1	0.62	0.71	0.85	0.78	0.90
M_2	0.43	0.89	0.77	0.70	0.96

Step5: With the help of proposed algorithm find optimal sequence of jobs which comes out to be

J_2	J_5	J_3	J_4	J_1

Step6: Find total elapsed time

Job Sequence	M_1		M_2	
	Time in	Time out	Time in	Time out
J_2	0	0.71	0.71	1.50
J_5	0.71	1.61	1.61	2.57
J_3	1.61	2.46	2.57	3.34
J_4	2.46	3.24	3.34	4.04
J_1	3.24	3.86	4.04	4.47

Thus the minimum total elapsed time is 4.47 hours.

7. Conclusion:

In this Job sequencing problem is solved with picture fuzzy numbers by using proposed method to find optimal sequence of jobs and minimum total elapsed time. Due to the capability of handling more opinions, PFNs have become more suitable to deal with imprecise and ambiguous information

References:

1. Atanassov K., (1986) Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20, 87-96.
2. Atanassov K. and Gargov G.,(1989) Interval-Valued Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 31, 343-349.
3. Atanassov K., (1989) More on Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 33, 37-46.
4. Cuong B.C. and Kreinovich V., (2014) Picture Fuzzy Sets, Journal of Computer Science and Cybernetics, 30, 409-416.
5. Cuong B.C. and Kreinovich V., (2013) Picture Fuzzy Sets- a New Concept for Computational Intelligence Problems, Departmental Technical Reports (CS), 809.
6. Das S., Kar S., Pal T. (2017) Robust Decision Making using Intuitionistic Fuzzy Numbers, Granular Computing, 2, 41-54.
7. Das S., Kar M.B., Kar S., Pal T. (2018) An Approach for Decision Making using Intuitionistic Trapezoidal Fuzzy Soft Set, Annals of Fuzzy Mathematics and Informatics, 16(1), 85-102.
8. Garg H. (2017) Some Picture Fuzzy Aggregation Operators and their Applications to Multicriteria Decision Making, Arabian Journal for Science and Engineering, 42(12), 5275-5290.
9. Guiwu W. (2016) Picture Fuzzy Cross-entropy for Multiple Attribute Decision Making Problems, Journal of Business Economics and Management, 17, 491-502.
10. Hasan M.K., Sultana A., Mitra N. K., (2023) Arithmetic Operations of Generalized Trapezoidal Picture Fuzzy Numbers by Vertex Method, American Journal of Computational Mathematics, 13, 99-121.
11. Kaur H. (2021), An Algorithm to solve Fuzzy Job Sequencing Using Pythagorean Numbers, High Technology Letters, 27(7), 626-631.
12. Li D.F. (2008) Extension of the LINMAP for Multiattribute Decision Making under Atanassov's Intuitionistic Fuzzy Environment, Fuzzy Optimization and Decision Making, 7, 17-34.
13. Shit C., Ghorai G., Xin Q., Gulzar M. (2022) Harmonic Aggregation Operator with Trapezoidal Picture Fuzzy Numbers and its Application in a Multiple-Attribute Decision-Making Problem, Summetry, 14, 135.
14. Singh P. (2015) Correlation Coefficients for Picture Fuzzy Sets, Journal of Intelligent and Fuzzy Systems, 28(2), 591-604.
15. Wang C., Zhou X., Tu H. and Tao S. (2017), Some aggregation operators based on picture fuzzy sets and their application in multiple attribute decision making, Italian Journal of Pure and Applied Mathematics, 37, 477-492.
16. Zhang X. and Xu Z. (2012), A new method for ranking intuitionistic fuzzy values and its application in multi-attribute decision making, Fuzzy Optimization and Decision Making, 11, 135-146.