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Research Article

A Study On Absolute Mean Cordial Labeling Of Various Graphs

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ABSTRACT

In this paper we investigate absolute mean cordial labeling for some graphs. We have proved that complete biapartite graph $K_{m,n}$, grid graph $P_m \times P_n$, step grid graph St_n, double step grid graph DSt_n, wheel W_n, sunlet graph S_n, gear graph G_n, swastik graph SW_n, sunflower graph SF_n, flower graph F_n and extended friendship graph $F_{4,n}$ are absolute mean cordial graphs.

Keywords: Labeling, Cordial Labeling, Absolute Mean Cordial Labeling.

AMS Subject Classification(2020): 05C78 .

1 Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G),E(G))$. the concept of cordial labeling was introduced by Cahit [5]. laege number of papers are found with variety of applicationsin cordial theory, radar communication, cryptography etc. For an extensive survey on graph labeling and bibliographic references we refer to Gallian [3]. Most of the graph labeling techniques trace their origin to graceful labeling introduced independently by Rosa [2] and Golomb [4]. Kaneria and Kanani introduced absolute mean cordial labeling. we have used all terminologies and notations from Harary [1]. We will give brief summary of definitions and other information which are useful for the present investigations.

Definition 1.1. *A function $f : V(G) \longrightarrow \{0,1\}$ is called binary vertex labeling of a graph G and $f(v)$ is called label of vertex of G under f. for an edge $e = (uv)$, the induced function $f^* : E(G) \longrightarrow \{0,1\}$ defined as $f^*(e) = \lfloor f(u) - f(v) \rfloor$. let $v_f(0), v_f(1)$ be number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0), e_f(1)$ be number of edges of G having edge labels 0 and 1 eespectively under f^*. a binary vertex labeling f of graph is called cordial labeling if $|v_f(0) - v_f(1) \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. a graph is called cordial graph if it admits a cordial labeling.*

Definition 1.2. *A function f is called an absolute mean cordial labeling of a graph $G = (V(G),E(G))$, if $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm 2\lfloor \frac{q}{2} \rfloor\}$ is one-one and the induced function $f^* : E(G) \longrightarrow \{0,1\}$ defined as,*

$$f^*(e) = \begin{cases} 1 & ; if \ \frac{|f(u)-f(v)|}{2} \leq \lceil \frac{q}{2} \rceil \\ 0 & ; otherwise \end{cases}$$

and the satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ is onto for every edge $e = (u,v) \in E(G)$. a graph is called an absolute mean cordial if it admits an absolute mean cordial labeling.

Definition 1.3. *A function f is called an absolute mean graceful of a graph G with q edges, if the vertex labeling function $f : V(G) \rightarrow \{0, \pm 1, \pm 2, \pm 3, ..., \pm q\}$ is one-one and the edge labeling function $f^* : E(G) \rightarrow \{1,2,3,...,q\}$ defined as $f^*(uv) = \lceil \frac{|f(u)-f(v)|}{2} \rceil$ is bijective, for every edge $uv \in E(G)$.*
If a graph G admits absolute mean graceful labeling, then it is called absolute mean graceful graph.

2 Main Results

Theorem 2.1. *Every complete bipartite graph $K_{m,n}$ is absolute mean cordial graph.*

Proof. let $G = K_{m,n}$ be complete bipartite graph. let vertex set $M = \{u_1,u_2,...,u_m\}$ of $m-$ part and $N = \{v_1,v_2,...,v_n\}$ of $n-$ part of G.

i.e. $V(G) = \{u_1, u_2, ..., u_m, v_1, v_2, ..., v_n\}$ and $E(G) = \{u_i v_j / 1 \leq i \leq m, 1 \leq j \leq n\}$. to obtain vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$, we take following cases.

Case-1: $n \equiv 0(mod2)$

$$f(u_i) = \begin{cases} q + 2 - 2i & ; \text{if } i = 1, 2, ..., m \end{cases}$$

$$f(v_j) = \begin{cases} 2j - q - 2 & ; \text{if } j = 1, 2, ..., \frac{n}{2} \\ 2j - n & ; \text{if } j = \frac{n+2}{2}, \frac{n+4}{2}, ..., n \end{cases}$$

Case-2: $n \equiv 1(mod2)$

subcase-1: $m \equiv 0(mod2)$

$$f(u_i) = \begin{cases} q + 2 - 2i & ; \text{if } i = 1, 2, ..., m \end{cases}$$

$$f(v_j) = \begin{cases} 2j - q - 2 & ; \text{if } j = 1, 2, ..., \frac{n-1}{2} \\ -m & ; \text{if } j = \frac{n+1}{2} \\ 2j - n - 1 & ; \text{if } j = \frac{n+3}{2}, \frac{n+5}{2}, ..., n \end{cases}$$

subcase-2: $m \equiv 1(mod2)$

$$f(u_i) = \begin{cases} q + 2 - 2i & ; \text{if } i = 1, 2, ..., m \end{cases}$$

$$f(v_j) = \begin{cases} 2j - q - 2 & ; \text{if } j = 1, 2, ..., \frac{n-1}{2} \\ 1 - m & ; \text{if } j = \frac{n+1}{2} \\ 2j - n - 1 & ; \text{if } j = \frac{n+3}{2}, \frac{n+5}{2}, ..., n \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. hence, f is an absolute mean cordial labeling for G. therefore, complete bipartite graph is an absolute mean cordial graph.

Illustration 2.2. *Absolute mean cordial labeling for complete bipartite graph $K_{4,3}$ with $p = 7$ and $q = 12$ is shown in following Figure 1.*

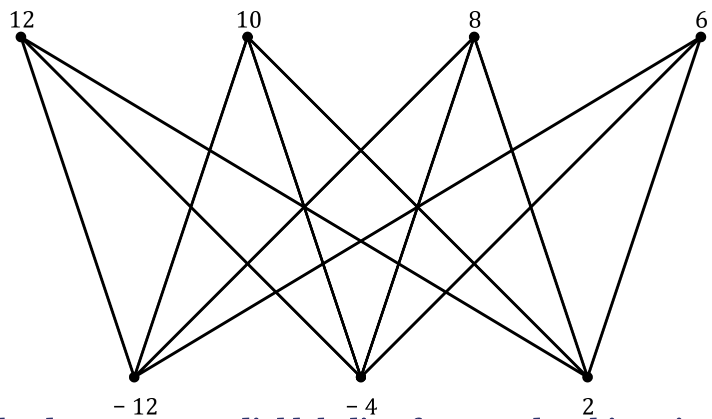

Figure 1: Absolute mean cordial labeling for complete bipartite graph $K_{4,3}$.

Theorem 2.3. *Every grid graph $P_m \times P_n$ is absolute mean cordial graph.*

Proof. let $G = P_m \times P_n$ be grid graph with vertex set

$V(G) = \{u1,1, u1,2, ..., u1,m, u2,1, u2,2, ..., u2,m, ..., un,1, un,2, ..., un,m\}$

clearly $|V(G)| = mn, |E(G)| = 2mn - (m + n)$. to obtain vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$, we take following cases.

Case-1: $n \equiv 0(mod2)$

$$f(u_{i,j}) = \begin{cases} (-1)^{i+j}[q + 2(1 - j - \lceil \frac{i-1}{2} \rceil m)] & ;\text{if } i = 1, 2, ..., \frac{n}{2} \ \& \ j = 1, 2, ..., m \\ (-1)^{i+j}2[1 - j + \lceil \frac{n-i+2}{2} \rceil m] & ;\text{if } i = \frac{n+2}{2}, \frac{n+4}{2}, ..., n \ \& \ j = 1, 2, ..., m \end{cases}$$

Case-2: $n \equiv 1(mod2)$

subcase-1: $m \equiv 0(mod2)$

$$f(u_{i,j}) = \begin{cases} (-1)^{i+j}[q + 2(1 - j - \lceil \frac{i-1}{2} \rceil m)] & ;\text{if } i = 1, 2, ..., \frac{n-1}{2} \ \& \ j = 1, 2, ..., m \\ (-1)^{i+j}[q + 2(1 - j - \lceil \frac{i-1}{2} \rceil m)] & ;\text{if } i = \frac{n+1}{2} \ \& \ j = 1, 2, ..., \frac{m}{2} \\ (-1)^{i+j}2[1 - j + \lceil \frac{n-i+2}{2} \rceil m] & ;\text{if } i = \frac{n+1}{2} \ \& \ j = \frac{m+2}{2}, \frac{m+4}{2}, ..., m \\ (-1)^{i+j}2[1 - j + \lceil \frac{n-i+2}{2} \rceil m] & ;\text{if } i = \frac{n+3}{2}, \frac{n+5}{2}, ..., n \ \& \ j = 1, 2, ..., m \end{cases}$$

subcase-2: $m \equiv 1(mod2)$

$$f(u_{i,j}) = \begin{cases} (-1)^{i+j}[q + 2(1 - j - \lceil \frac{i-1}{2} \rceil m)] & ;\text{if } i = 1, 2, ..., \frac{n-1}{2} \ \& \ j = 1, 2, ..., m \\ (-1)^{i+j}[q + 2(1 - j - \lceil \frac{i-1}{2} \rceil m)] & ;\text{if } i = \frac{n+1}{2} \ \& \ j = 1, 2, ..., \frac{m-1}{2} \\ (-1)^{i+j}\frac{q}{2} & ;\text{if } i = \frac{n+1}{2} \ \& \ j = \frac{m+1}{2} \\ (-1)^{i+j}2[1 - j + \lceil \frac{n-i+2}{2} \rceil m] & ;\text{if } i = \frac{n+1}{2} \ \& \ j = \frac{m+3}{2}, \frac{m+5}{2}, ..., m \\ (-1)^{i+j}2[1 - j + \lceil \frac{n-i+2}{2} \rceil m] & ;\text{if } i = \frac{n+3}{2}, \frac{n+5}{2}, ..., n \ \& \ j = 1, 2, ..., m \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.
hence, f is an absolute mean cordial labeling for G. Therefore, grid graph is an absolute mean cordial graph.

Illustration 2.4. *Absolute mean cordial labeling for grid graph $P_6 \times P_4$ with $p = 24$ and $q = 38$ is shown in following Figure 2.*

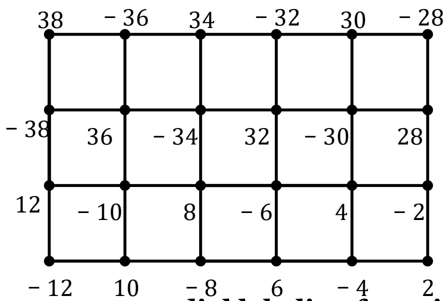

Figure 2: Absolute mean cordial labeling for grid graph $P_6 \times P_4$.

Theorem 2.5. *Every step grid graph St_n is absolute mean cordial graph.*

Proof. let $G = St_n$ be step grid graph with vertex set
$V(G) = \{u_{1,1}, u_{1,2}, ..., u_{1,n}, u_{2,1}, u_{2,2}, ..., u_{2,n-1}, u_{3,1}, u_{3,2}, ..., u_{3,n-2}, ..., u_{n,1}\}$
clearly $|V(G)| = \frac{n^2+3n-2}{2}$, $|E(G)| = n^2 + n - 2$. the vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$ defined as follows.

$$f(u_{i,j}) = \begin{cases} (-1)^{i+j}(2n - 2j + 2) & ;\text{if } i = 1, 2 \ \& \ j = 2, 4, ..., n \\ (-1)^{i+j}[i(n-1) + n - 2j - 2\sum(i-2) + 3] & ;\text{if } i = 3, 5, 7, ... \ \& \ j = i+1, i+2, ..., n-i+2 \\ (-1)^{i+j}[i(n-1) - 2j - 2\sum(i-3) + 4] & ;\text{if } i = 4, 6, 8, ... \ \& \ j = i+1, i+2, ..., n-i+2 \\ (-1)^{i+j}(q - 2n + 2i) & ;\text{if } j = 1, \ \& \ i = 1, 2, ..., n \\ (-1)^{i+j}(q - 2n + 2i) & ;\text{if } j = 2, \ \& \ i = 3, 4, ..., n \\ (-1)^{i+j}[q - (j+1)n + 2i + 2\sum(j-2)] & ;\text{if } j = 3, 5, 7, ... \ \& \ i = j+1, j+2, ..., n-j+2 \\ (-1)^{i+j}[q - jn + 2i + 2\sum(j-3)] & ;\text{if } j = 4, 6, 8, ... \ \& \ i = j+1, j+2, ..., n-j+2 \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. hence, f is an absolute mean cordial labeling for G. therefore, step grid graph is an absolute mean cordial graph.

Illustration 2.6. *Absolute mean cordial labeling for step grid graph St_5 with p = 18 and q = 28 is shown in following Figure 3.*

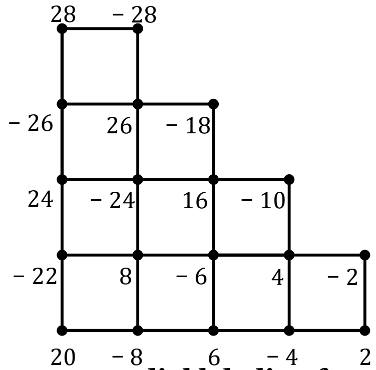

Figure 3: Absolute mean cordial labeling for step grid graph St_5.

Theorem 2.7. *Every double step grid graph DSt_n is absolute mean cordial graph.*

Proof. let $G = DSt_n$ be step grid graph with vertex set
$V(G)=\{u_{1},1,u_{2},1,...,u_{n},1,u_{1},2,u_{2},2,...,u_{n}-1,2,u_{1},3,u_{2},3,...,u_{n}-2,3,...,u_{1},n-1,$
$v1,1,v2,1,...,vn,1,v1,2,v2,2,...,vn-1,2,v1,3,v2,3,...,vn-2,3,...,v1,n-1, \}$
clearly $|V(G)| = n^2 + n - 2, |E(G)| = 2n^2 - n - 2$. the vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$ defined as follows.

$$f(u_{i,j}) = \begin{cases} (-1)^{i+j}(2i + 2\lfloor \frac{j-1}{2} \rfloor n) & ; \text{if } j = 1, 2, ..., 4 \ \& \ i = 1, 2, ..., n - j + 1 \\ (-1)^{i+j}[n(j-1) + 2i - 4\sum(\frac{j-3}{2})] & ; \text{if } j = 5, 7, 9, ... \ \& \ i = 1, 2, ..., n - j + 1 \\ (-1)^{i+j}[n(j-2) + 2i - 4\sum(\frac{j-4}{2})] & ; \text{if } j = 6, 8, 10, ... \ \& \ i = 1, 2, ..., n - j + 1 \end{cases}$$

$$f(v_{i,j}) = \begin{cases} (-1)^{i+j+1}(q - 2n + 2i) & ; \text{if } j = 1, 2 \ \& \ i = 1, 2, ..., n - j + 1 \\ (-1)^{i+j+1}[q - (j+1)n + 2i + 4\sum(j-2)] & ; \text{if } j = 3, 5, 7, ... \ \& \ i = 1, 2, ..., n - j + 1 \\ (-1)^{i+j+1}[q - jn + 2i + 4\sum(j-3)] & ; \text{if } j = 4, 6, 8, ... \ \& \ i = 1, 2, ..., n - j + 1 \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.
hence, f is an absolute mean cordial labeling for G. therefore, double step grid graph is an absolute mean cordial graph.

Illustration 2.8. *Absolute mean cordial labeling for double step grid graph DSt_4 with p = 18 and q = 26 is shown in following Figure 4.*

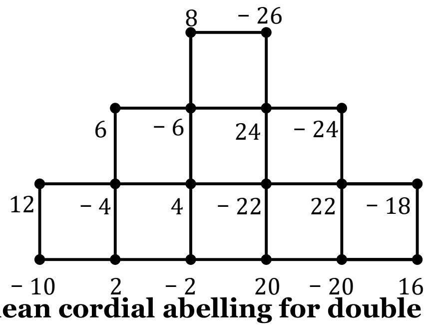

Figure 4: Absolute mean cordial abelling for double step grid graph DSt_4.

Theorem 2.9. *Every wheel W_n is absolute mean cordial graph.*
Proof. let $G = W_n$ be a wheel. let $v_1, v_2, ..., v_n$ be rim vertices and v_0 be apex vertex. i.e. $V(G) = \{v_1, v_2, ..., v_n\}$ and $E(G) = \{v_0v_i / 1 \leq i \leq n\} \bigcup \{v_iv_{i+1} / 1 \leq i < n\} \bigcup \{v_1v_n\}$. the vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$ defined as follows.

$$f(v_i) = \begin{cases} q - 2i + 2 & ; \text{if } i = 1, 2, ..., n \\ -2q & ; \text{if } i = 0 \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. hence, f is an absolute mean cordial labeling for G. therefore, wheel is an absolute mean cordial graph.

Illustration 2.10. *Absolute mean cordial 3098abelling for wheel W_8 with $p = 9$ and $q = 16$ is shown in following Figure 5.*

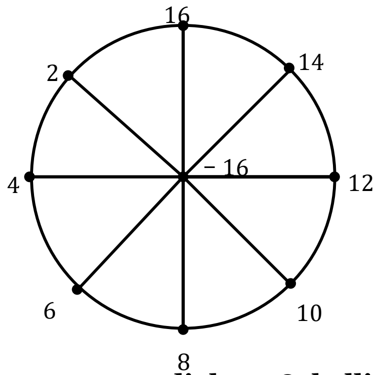

Figure 5: Absolute mean cordial 3098abelling for wheel W_8.

Theorem 2.11. *Every sunlet graph S_n is absolute mean cordial graph.*

Proof. let $G = S_n$ be a sunlet graph obtained by adding n pendent vertices at each vertices in cycle C_n. let $v_1,v_2,...,v_n$ be vertices of cycle C_n. now we shall add n pendent vertices at vertices in cycle C_n to obtain graph G. let v_{i+1} be pendent vertex to vertex $v_i, i \in \{1,2,...,n\}$.
i.e. $V(G) = \{v_1,v_2,...,v_{2n}\}$ and $E(G) = \{v_iv_{i+1}/1 \le i < n\}^S\{v_1v_n\}^S\{v_iv_{n+i}/1 \le i \le n\}$. the vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$ defined as follows.

$$f(v_i) = \begin{cases} q - 2i + 2 & ; \text{if } i = 1, 2, ..., n \\ 2n - 2i & ; \text{if } i = n+1, n+2, ..., 2n \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \le 1$. hence, f is an absolute mean cordial labeling for G. therefore, sunlet graph is an absolute mean cordial graph.

Illustration 2.12. *Absolute mean cordial labeling for sunlet graph S_7 with $p = 14$ and $q = 14$ is shown in following Figure 6.*

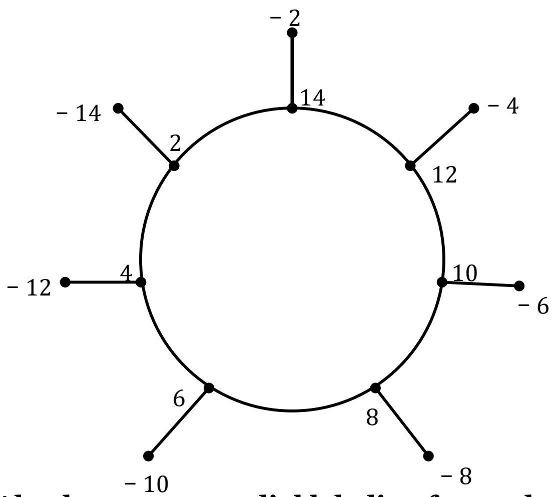

Figure 6: Absolute mean cordial labeling for sunlet graph S_7.

Theorem 2.13. *Every gear graph G_n is absolute mean cordial graph.*

Proof. let $G = G_n$ be a gear graph. let cycle C_{2n} with $V(C_{2n}) = \{v_1,v_2,...,v_{2n}\}$ and $E(G) = \{v_iv_{i+1}/1 \le i < 2n\}^S\{v_1v_{2n}\}$. let $G = G_n$ be gear graph obtained by adding one vertex v_0 in centre of C_{2n} such a way that vertices $v_1,v_3,...,v_{2n-1}$ are connected with vertex v_0.
i.e. $V(G) = \{v_1,v_2,...,v_{2n}\}$ and $E(G) = \{v_iv_{i+1}/1 \le i < 2n\}^S\{v_1v_{2n}\}^S\{v_0v_i/i = 1,3,...,2n - 1\}$. to obtain vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$, we take following cases.

Case-1: $n \not\equiv 3(mod4)$

$$f(v_i) = \begin{cases} (-1)^i(q+2-2i) & ; \text{if } i = 1,2,...,2n \\ -q & ; \text{if } i = 0 \end{cases}$$

Case-2: $n \equiv 3(mod4)$

$$f(v_i) = \begin{cases} (-1)^i(q+2-2i) & ; \text{if } i = 1,2,...,2n-1 \\ (-1)^{i+1}(q+4-2i) & ; \text{if } i = 2n \\ -q & ; \text{if } i = 0 \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. hence, f is an absolute mean cordial labeling for G. therefore, gear graph is an absolute mean cordial graph.

Illustration 2.14. *Absolute mean cordial labeling for gear graph G_8 with p = 17 and q = 24 is shown in following Figure 7.*

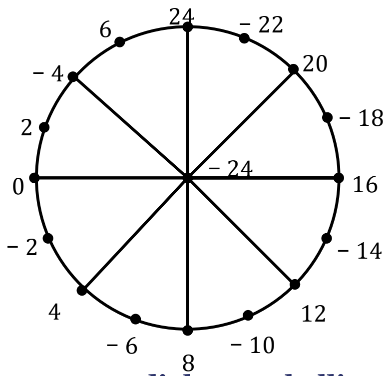

Figure 7: Absolute mean cordial 3099abelling for gear graph G_8.

Theorem 2.15. *The swastik graph $SW_n, n \in N - \{1\}$ is absolute mean cordial graph.*

Proof. let $G = SW_n$ be a swastik graph of size n, where $n \in N - \{1\}$. we mention each vertices of SW_n like $v_{i,j}(i = 1,2,3,4; j = 1,2,...,4n)$. we see that $|V(G)| = 16n - 4$ and $|E(G)| = q = 16n$. the vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$ defined as follows.

$$f(v_{1,j}) = \begin{cases} (-1)^j(8n+2j) & ; \text{if } j = 1,2,...,4n \end{cases}$$

$$f(v_{2,j}) = \begin{cases} (-1)^{j+1}(16n-2j+4) & ; \text{if } j = 2,3,...,4n \\ 16n & ; \text{if } j = 1 \end{cases}$$

$$f(v_{3,j}) = \begin{cases} (-1)^j(8n+4j) & ; \text{if } j = 1 \\ (-1)^j(2j-2) & ; \text{if } j = 2,3,...,4n \end{cases}$$

$$f(v_{4,j}) = \begin{cases} (-1)^{j+1}(2n-2j-2) & ; \text{if } j = 1,2,...,4n-1 \\ (-1)^{j+1}(2n+2) & ; \text{if } j = 4n \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.
hence, f is an absolute mean cordial labeling for G. therefore, swastik graph is an absolute mean cordial graph.

Illustration 2.16. *Absolute mean cordial labeling for swastik graph SW_2 with p = 28 and q = 32 is shown in following Figure 8.*

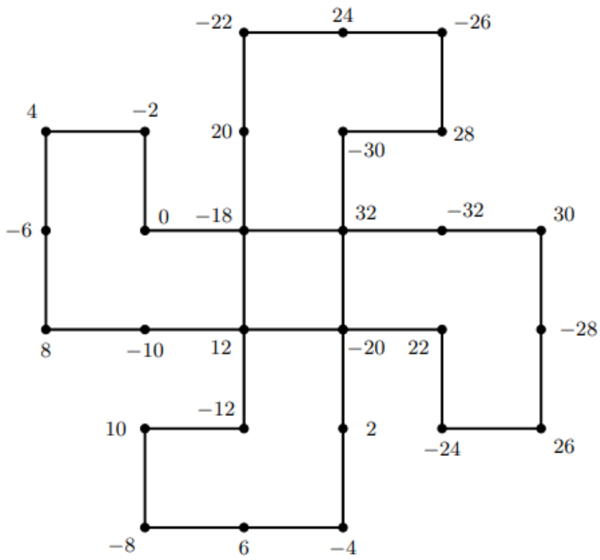

Figure 8: Absolute mean cordial labeling for swastik graph SW_2.

Theorem 2.17. *Every sunflower graph SF_n is absolute mean cordial graph.*

Proof. let $G = SF_n$ be a sunflower graph obtained from wheel W_n. let v_0 be apex vertex and $v_1,v_2,...,v_n$ be the n rim vertices of wheel W_n.
the sunflower graph SF_n is obtained from wheel by adding n vertices $u_1,u_2,...,u_n$, where u_i is connected with v_i and $v_{i+1}, i = 1,2,...,n-1$ and u_n is coonected with v_1 and v_n.
i.e. $V(G) = \{v_1,v_2,...,v_n,u_1,u_2,...,u_n\}$ and
$E(G) = \{v_0v_i/1 \le i \le n\}$ $^S\{v_iv_{i+1}/1 \le i < n\}$ $^S\{v_1v_n\}$ $^S\{u_iv_i/1 \le i \le n\}$ $^S\{u_nv_1\}$ S$\{uivi+1/1 \le i < n\}$ S$\{unv1\}$.
to obtain vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm\lfloor \frac{q}{2} \rfloor\}$, we take following cases.

Case-1: $n \equiv 0(mod2)$

$$f(v_i) = \begin{cases} q+2-2i & ; \text{if } i = 1,2,...,n \\ -q & ; \text{if } i = 0 \end{cases}$$

$$f(u_i) = \begin{cases} 2i-q & ; \text{if } i = 1,2,...,\frac{n}{2} \\ 2n-2i & ; \text{if } i = \frac{n+2}{2}, \frac{n+4}{2}, ..., n \end{cases}$$

Case-2: $n \equiv 1(mod2)$

$$f(v_i) = \begin{cases} q+2-2i & ; \text{if } i = 1,2,...,n \\ -q & ; \text{if } i = 0 \end{cases}$$

$$f(u_i) = \begin{cases} 2i-q & ; \text{if } i = 1,2,...,\frac{n-1}{2} \\ 2n-2i-2 & ; \text{if } i = \frac{n+1}{2}, \frac{n+3}{2}, ..., n \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \le 1$.
hence, f is an absolute mean cordial labeling for G. therefore, sunflower graph is an absolute mean cordial graph.
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Illustration 2.18. *Absolute mean cordial 3100abelling for sunflower graph SF_5 with $p = 11$ and $q = 20$ is shown in following Figure 9.*

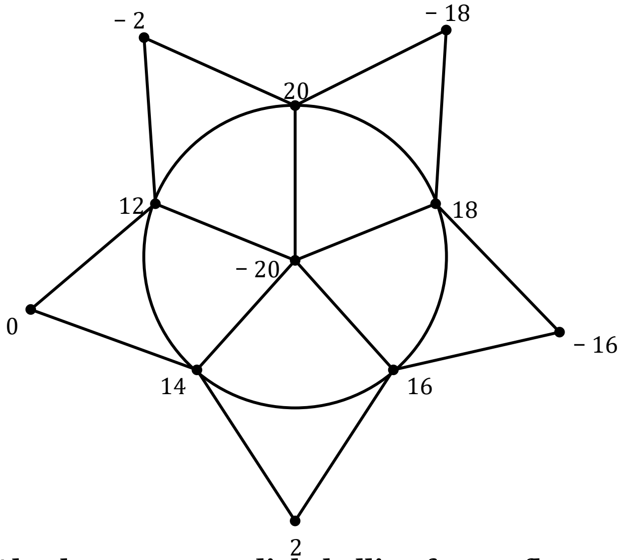

Figure 9: Absolute mean cordial abelling for sunflower graph SF_5.

Theorem 2.19. *Every flower graph F_n is absolute mean cordial graph*

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Proof. let $G = F_n$ be a flower graph obtained from helm H_n. let v_0 be the apex vertex and $v_1,v_2,...,v_n$ be the n vertices of cycle and $u_1,u_2,...,u_n$ be the pendent vertices of helm H_n. the flower graph is obtained from helm H_n by joining eachpendent vertices to the aopex vertex of helm. i.e. $V(G) = \{v_1,v_2,...,v_n,u_1,u_2,...,u_n\}$ and $E(G) = \{v_0v_i/1 \le i \le n\}^{S}\{v_0u_i/1 \le i \le n\}^{S}\{v_1v_n\}^{S}\{v_iv_{i+1}/1 \le i < n\}$. the vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm \lfloor \frac{q}{2} \rfloor\}$ defined as follows.

$$f(v_i) = \begin{cases} q - 2i + 2 & ;\text{if } i = 1,2,...,n \\ -q & ;\text{if } i = 0 \end{cases}$$
$$f(u_i) = \begin{cases} 2n - 2i + 2 & ;\text{if } i = 1,2,...,n \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \le 1$.
hence, f is an absolute mean cordial labeling for G. therefore, flower graph is an absolute mean cordial graph.

Illustration 2.20. *Absolute mean cordial labeling for flower graph F_3 with $p = 7$ and $q = 12$ is shown in following Figure 10.*

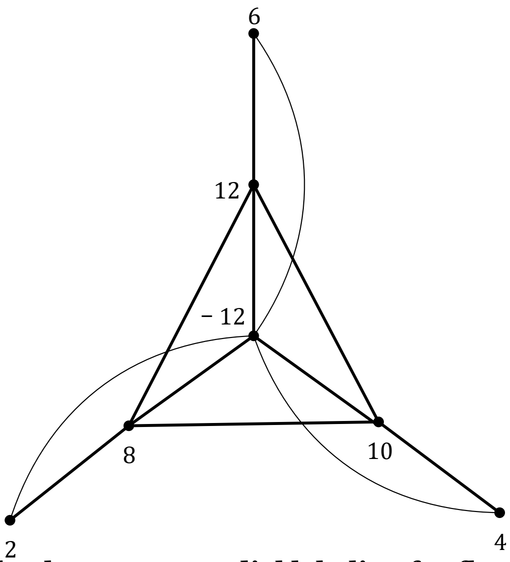

Figure 10: Absolute mean cordial labeling for flower graph F_3.

Theorem 2.21. *Every extended frienship graph $F_{4,n}$ is absolute mean cordial graph.*
Proof. let $G = F_{4,n}$ be a extended friendship graph obtained by considering n 4-cycles with common vertex. let v_0 be centre vertex and let $\{v_1,v_2,v_3,..,v_{3n}\}$ be vertices such that $\{v_i/i \in \{1,3,...,3n\}\}$ are connected to v_0. also v_i

and v_{i+2} are connected to $v_{i+1}, i \in \{1,4,7,...,3n-2\}$ to obtain graph G. the vertex labeling functioin $f : V(G) \longrightarrow \{0, \pm 2, \pm 4, ..., \pm\lfloor \frac{q}{2} \rfloor\}$ defined as follows.

$$f(v_i) = \begin{cases} q - i + 1 & ;\text{if } i = 1, 3, ..., 3n \\ 2 - 2i & ;\text{if } i = 2, 4, ..., 3n - 1 \\ -q & ;\text{if } i = 0 \end{cases}$$

the labeling function f defined as above is one-one, as there is no repeated vertex labels and induced funtion f^* : $E(G) \longrightarrow \{0,1\}$ is onto. also f satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.
hence, f is an absolute mean cordial labeling for G. therefore, extended friendship graph is an absolute mean cordial graph.

Illustration 2.22. *Absolute mean cordial labeling for extended friendship graph $F_{4,5}$ with $p = 15$ and $q = 20$ is shown in following Figure 11.*

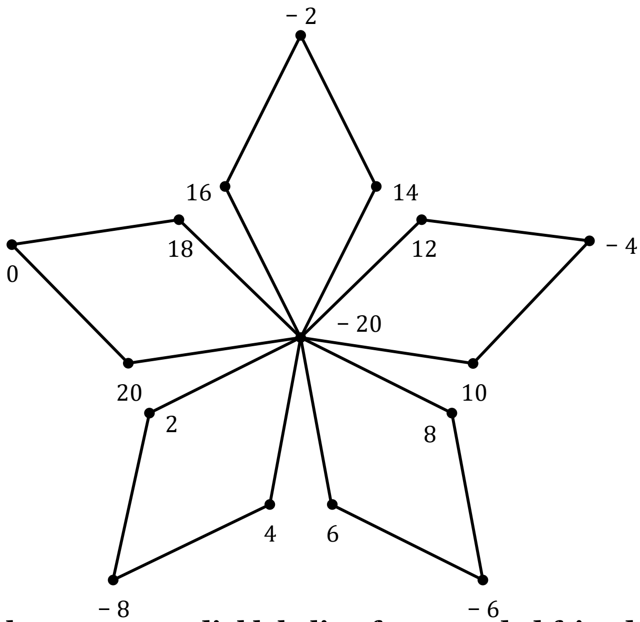

Figure 11: Absolute mean cordial labeling for extended friendship graph $F_{4,5}$.

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