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Research Article

Graceful Labels Of Some Graphs Joined By An Arbitrary Path

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ARTICLE INFO | **ABSTRACT**

We proved the gracefulness of graphs formed by connecting different graphs. And found the result (1) The graph G is obtained by connecting a quadrilateral and a quadrilateral with one chord through a path of arbitrary length is graceful. (2) The graph G obtained through joined a quadrilateral with one chord and a barycentric subdivision of C_n ($n \equiv 0,2 \pmod 4$) by a path of arbitrary length is graceful. (3)The graph G obtained through joined a quadrilateral with one chord $G_1$and a quadrilateral snake G_2 by a path of arbitrary length is graceful.

Keywords: Graceful labeling, barycentric subdivision of cycle C_n, quadrilateral with one chord, quadrilateral snake QS_n.

Ams classification no: 05C78

1 Introduction

The concept of graceful labels was proposed by Rosa [8] in 1967, and the numbers in the figure are defined by S.W. Golomb [4]. Many researchers have studied the gracefulness of graphs, please refer to Gallian's survey [3]. A large number of papers were found to have various applied in coding theory, radar communication, cryptography, etc. For in-depth details on graph labeling applications, see Bloom and Golomb [2]. We accept all the symbols and terms proposed by Harary [5]. We recall some of the definitions used in this article.
If $f : V \rightarrow \{0,1,\ldots,q\}$ is injective, and the induce function $f^* : E \rightarrow \{1,\ldots,q\}$ is defined as $f^*(e) = |f(u) - f(v)|$. For each edge $e = (u,v) \in E(G)$ is bijective. Graph G is called graceful graphics (if it allows graceful labels).
A graph is connected if every pair of points are joined by a path. [5]
A chord of a cycle is an edge joining two non-adjacent vertices of cycle C_n. [9]
The quadrilateral snake Q_n is obtained from the path P_n by replacing every edge of a path by cycle C_4.[1]
Let G = (V, E) be a graph. If each edge of a graph G is subdivided, the resulting graph is called the barycentric subdivision of graph G. In other words, the barycentric subdivision is a graph obtained by inserting 2 degree vertices. Enter each edge of the original graphic. The subdivision of the barycentric of any graph G is represented by the S(G). Very easily observe $|VS(G)| = |V(G)| + |E(G)|$ and $|ES(G)| = 2|E(G)|$. [10]
In this paper, the gracefulness of the graph formed by connecting different graphs is discussed. And found the following result
1.The graph G is obtained by connecting a quadrilateral and a quadrilateral with one chord through a path of arbitrary length is graceful.
2. The graph G obtained through joined a quadrilateral with one chord and a barycentric subdivision of C_n ($n \equiv 0,2 \pmod 4$) by a path of arbitrary length is graceful.
3. The graph G obtained through joined a quadrilateral with one chord G_1 and a quadrilateral snake G_2 by a path of arbitrary length is graceful.

For a detailed investigation of graph labeling, we refer to Gallian[3].

2 Main Results:

2.1 Theorem

The graph G is obtained by connecting a quadrilateral and a quadrilateral with one chord through a path of arbitrary length is graceful.

***Proof*:**
Let G = (V, E) be a graph obtained by connecting a chord quadrilateral and a quadrilateral with a path P_k of length k-1. Let $\{u_1,u_2,u_3,u_4\}$ be vertices of a quadrilateral G_1, $\{w_1,w_2,w_3,w_4\}$ be vertices of a quadrilateral with one chord G_2 and $\{v_1,v_2,.....,v_k\}$ are the vertices of the path P_k, where $v_1 = u_4$ and $v_k = w_1$. We consider the following two cases.

***Case-1*:** Length of P_k is odd.
Define $f : v \to \{0,1,....q\}$, where q (number of edges of graph G) = 9 + k.

$f(u_1) = 9 + k$ $\quad$ $f(w_1) = f(v_{2n})$

$f(u_2) = 0$ $\quad$ $f(w_2) = f(v_{2n-1}) + 1$

$f(u_3) = f(u_1) - 1$ $\quad$ $f(w_3) = f(w_1) - 2$

$f(u_4) = 2$ $\quad$ $f(w_4) = f(w_2) + 1$

$f(v_1) = f(u_4)$ $\quad$ $f(v_2) = f(u_3) - 1$

$f(v_3) = f(v_1) + 1$ $\quad$ $f(v_4) = f(v_2) - 1$

$f(v_5) = f(v_3) + 1$ $\quad$ $f(v_6) = f(v_4) - 1$

$f(v_7) = f(v_5) + 1$ $\quad$ $f(v_8) = f(v_6) - 1$

. $\quad$.

. $\quad$.

$f(v_{2n+1}) = f(v_{2n-1}) + 1$ $\quad$ $f(v_{2n+2}) = f(v_{2n}) - 1.$

$(\forall\, n = 1, 2, ...)$ $\quad$ (n = graphic position, k-1 = path length P_k)

2.2Illustration

The graceful labeling of the diagram J acquired by interfacing a quadrilateral and a quadrilateral with one chord through the way P_6 displayed in FIG.1.

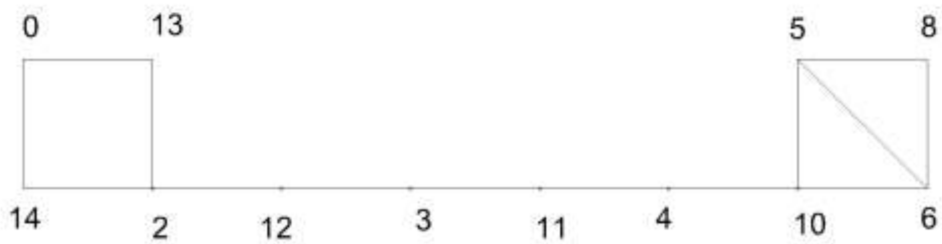

Figure 1: An graceful labeling of a diagram J, which is framed by interfacing a quadrilateral and a quadrilateral with one chord of way P_6, where p = 12 (the quantity of vertices for chart G) and q = 14 (the quantity of edges for diagram G).

***Case-2*:** Length of P_k is even.
Define $f : v \to \{0,1,....q\}$, where q (number of edges of graph G) = 9 + k.

$f(u_1) = 9 + k$ $\quad$ $f(w_1) = f(v_{2n-1})+1$

$f(u_2) = 0$ $\quad$ $f(w_2) = f(v_{2n}) - 1$

$f(u_3) = f(u_1) - 1$ $\quad$ $f(w_3) = f(w_1) + 2$

$f(u_4) = 2$ $f(w_4) = f(w_2) - 1$

$f(v_1) = f(u_4)$ $f(v_2) = f(u_3) - 1$
$f(v_3) = f(v_1)+1$ $f(v_4) = f(v_2) - 1$

$f(v_5) = f(v_3)+1$ $f(v_6) = f(v_4) - 1$

$f(v_7) = f(v_5)+1$ $f(v_8) = f(v_6) - 1$

. .

. .

$f(v2n+1) = f(v2n-1) + 1$ $f(v2n+2) = f(v2n) - 1$.
($\forall\, n = 1,2,....,\forall\, k = 1,2,....$). (n = graphic position, k-1 = path length P_k)

In both cases, we can verify that f is a graceful label of graph G.

2.3 Theorem

The graph G obtained through joined a quadrilateral with one chord and a barycentric subdivision of cycle C_n ($n \equiv 0,2 \pmod 4$) by a path of arbitrary length is graceful.

***Proof*:**
***Case-1*:** First, for C_n, we take n = 4, that is C_4
Let G = (V, E) be the graph obtained through joined two graphs, a quadrilateral with one chord and a barycentric subdivision of cycle C_n ($n \equiv 0,2 \pmod 4$) by a path P_k of length k-1. Let $\{u_1,u_2,u_3,u_4\}$ be vertices of a quadrilateral with one chord G_1, $\{w_1,w_2,w_3,w_4\}$ be vertices of C_4 and $\{x_1,x_2,x_3,x_4\}$ are inserted vertices due to barycentric subdivision , i.e. $\{w_1,x_1,w_2,x_2,w_3,x_3,w_4,x_4\}$ be vertices of a barycentric subdivision of cycle G_2 and $\{v_1,v_2,.....,v_k\}$ be vertices of the path P_k with $v_1 = u_4$ and $v_k = w_1$. We consider the following cases.

(*A*): Length of P_k is odd.
Define $f : v \to \{0,1,....q\}$, where q (number of edges of graph G) = 13 + k.

$f(u_1) = 13 + k$ $f(w_1) = f(v_{2n})$
$f(u_2) = 0$ $f(w_2) = f(w_1) - 1$
$f(u_3) = f(u_1) - 2$ $f(w_3) = f(w_2) - 1$
$f(u_4) = 1$ $f(w_4) = f(w_3) - 1$

$f(v_1) = f(u_4)$ $f(v_2) = f(u_3) - 1$
$f(v_3) = f(v_1) + 1$ $f(v_4) = f(v_2) - 1$
$f(v_5) = f(v_3) + 1$ $f(v_6) = f(v_4) - 1$
$f(v_7) = f(v_5) + 1$ $f(v_8) = f(v_6) - 1$
. .
. .
$f(v_{2n+1}) = f(v_{2n-1}) + 1$ $f(v_{2n+2}) = f(v_{2n}) - 1$

$f(x_1)= f(v_{2n-1})+1$ $f(x_2) = f(x_1) + 1$
$f(x_3) = f(x_2) + 2$ $f(x_4) = f(x_3) + 1$

($\forall\, n = 1, 2, ..., \forall\, k = 1, 2, ...$).
(n = graphic position, k-1 = path length P_k)

2.4 Illustration

The graceful labeling of the graph G obtained by connecting a quadrilateral with one chord and a barycentric subdivision of cycle C_4 through the path P_4 shown in FIG. 2.

Figure 2: **Graceful labeling of a graph G, which is formed by connecting a quadrilateral with one chord and a barycentric subdivision of cycle C_4 through the path P_4, where p = 14 (the number of vertices for graph G) and q = 16 (the number of edges for graph G).**

***(B)*: Length of P_k is even.**
Define $f : v \rightarrow \{0,1,....q\}$, where q (number of edges of graph G) = 13 + k.

$$\begin{array}{ll} f(u_1) = 13 + k & f(w_1) = f(v_{2n-1}) + 1 \\ f(u_2) = 0 & f(w_2) = f(w_1)+1 \\ f(u_3) = f(u_1) - 2 & f(w_3) = f(w_2+1 \\ f(u_4) = 1 & f(w_4) = f(w_3)+1 \end{array}$$

$$\begin{array}{ll} f(v_1) = f(u_4) & f(v_2) = f(u_3) - 1 \\ f(v_3) = f(v_1)+1 & f(v_4) = f(v_2) - 1 \\ f(v_5) = f(v_3)+1 & f(v_6) = f(v_4) - 1 \\ f(v_7) = f(v_5)+1 & f(v_8) = f(v_6) - 1 \\ . & . \\ . & . \\ f(v_{2n+1}) = f(v_{2n-1})+ 1 & f(v_{2n+2}) = f(v_{2n})- 1 \end{array}$$

$$\begin{array}{ll} f(x_1) = f(v_{2n}) - 1 & f(x_2) = f(x_1) - 1 \\ f(x_3) = f(x_2) - 2 & f(x_4) = f(x_3) - 1 \end{array}$$

$$(\forall\ n = 1, 2, ..., \forall\ k = 1, 2, ...)$$

(n = graphic position, k-1 = path length P_k)
In both cases, we can verify that f is a graceful label of graph G.

***Case-2*:** Now we take n = 6 for C_n, that is C_6
Let G = (V, E) be the graph obtained through joined two graphs, a quadrilateral with one chord and a barycentric subdivision of cycle C_6 by the path P_k of length k-1. Let $\{u_1,u_2,u_3,u_4\}$ be vertices of quadrilateral with one chord G_1, $\{w_1,w_2,w_3,w_4,w_5,w_6\}$ be vertices of C_6 and $\{x_1,x_2,x_3,x_4,x_5,x_6\}$ are inserted vertices due to barycentric subdivision, i.e. $\{w_1,x_1,w_2,x_2,w_3,x_3,w_4,x_4,w_5,x_5,w_6,x_6\}$ be vertices of a barycentric subdivision of cycle G_2 and $\{v_1,v_2,.....,v_k\}$ be vertices of the path P_k with $v_1 = u_4$ and $v_k = w_1$. We consider the following cases.

***(A)*: Length of P_k is odd.**
Define $f : v \rightarrow \{0,1,....q\}$, where q (number of edges of graph G) = 17 + k.

$$f(u_1) = 17 + k \qquad f(u_2) = 0$$
$$f(u_3) = f(u_1) - 2 \qquad f(u_4) = 1$$

$$f(w_1) = f(v_{2n}) \qquad f(x_1) = f(v_{2n-1})+1$$
$$f(w_2) = f(w_1) - 1 \qquad f(x_2) = f(x_1) + 1$$
$$f(w_3) = f(w_2) - 1 \qquad f(x_3) = f(x_2) + 1$$

$$f(w_4) = f(w_3) - 1 \qquad f(x_4) = f(x_3) + 2$$
$$f(w_5) = f(w_4) - 1 \qquad f(x_5) = f(x_4) + 1$$
$$f(w_6) = f(w_5) - 1 \qquad f(x_6) = f(x_5) + 1$$

$$f(v_1) = f(u_4) \qquad f(v_2) = f(u_3) - 1$$
$$f(v_3) = f(v_1) + 1 \qquad f(v_4) = f(v_2) - 1$$
$$f(v_5) = f(v_3) + 1 \qquad f(v_6) = f(v_4) - 1$$
$$f(v_7) = f(v_5) + 1 \qquad f(v_8) = f(v_6) - 1$$
$$. \qquad .$$
$$. \qquad .$$
$$f(v_{2n+1}) = f(v_{2n-1}) + 1 \qquad f(v_{2n+2}) = f(v_{2n}) - 1$$

$(\forall\ n = 1, 2, ..., \forall\ k = 1, 2, ...)$.
(n = graphic position, k-1 = path length P_k)

2.5Illustration

The graceful labeling of the graph G obtained by connecting a quadrilateral with one chord and a barycentric subdivision of C_6 through the path P_4 shown in FIG. 3

Figure 3: Graceful labeling of the graph G, which is formed by connecting a quadrilateral with one chord and a barycentric subdivision of C_6 of path P_4, where p = 18 (number of vertices for graph G) and q = 20 (number of edges for graph G).

(B): Length of P_k is even.

Define $f : v \rightarrow \{0,1,....q\}$, where q (number of edges of graph G) = 17 + k.

$$f(u_1) = 17 + k \qquad f(u_2) = 0$$
$$f(u_3) = f(u_1) - 2 \qquad f(u_4) = 1$$

$$f(w_1) = f(v_{2n-1})+1 \qquad f(x_1) = f(v_{2n}) - 1$$
$$f(w_2) = f(w_1) + 1 \qquad f(x_2) = f(x_1)-1$$
$$f(w_3) = f(w_2) + 1 \qquad f(x_3) = f(x_2)-1$$
$$f(w_4) = f(w_3) + 1 \qquad f(x_4) = f(x_3)-2$$
$$f(w_5) = f(w_4) + 1 \qquad f(x_5) = f(x_4)-1$$

$$f(w_6) = f(w_5) + 1 \qquad f(x_6) = f(x_5)-1$$

$$f(v_1) = f(u_4) \qquad f(v_2) = f(u_3)-1$$
$$f(v_3) = f(v_1)+1 \qquad f(v_4) = f(v_2)-1$$
$$f(v_5) = f(v_3)+1 \qquad f(v_6) = f(v_4)-1$$
$$f(v_7) = f(v_5)+1 \qquad f(v_8) = f(v_6)-1$$

. .

. .

$$f(v_{2n+1}) = f(v_{2n-1}) + 1$$
$$f(v_{2n+2}) = f(v_{2n}) - 1$$

$$(\forall\ n = 1, 2, ..., \forall\ k = 1, 2, ...)$$
(n = graphic position, k-1 = path length P_k)

Case-3: In General for C_n

Let G = (V, E) be the graph obtained through joined two graphs, a quadrilateral with one chord and a barycentric subdivision of cycle C_n ($n \equiv 0,2 \pmod 4$) by a path P_k of length k-1. Let $\{u_1,u_2,u_3,u_4\}$ be vertices of a quadrilateral with one chord G_1, $\{w_1,w_2,w_3,...,w_{2i+2}\}$ be vertices of cycle C_{2i+2} and $\{x_1,x_2,x_3,...,x_{2i+2}\}$ are inserted vertices due to barycentric subdivision i.e. $\{w_1,x_1,w_2,x_2,w_3,x_3,...,w_{2i+2},x_{2i+2}\}$ be vertices of a barycentric subdivision of cycle G_2 and $\{v_1,v_2,.....,v_k\}$ be the vertices of the path P_k with $v_1 = u_4$ and $v_k = w_1$. We consider the following cases.

(A): Length of P_k is odd.

Define $f : v \rightarrow \{0,1,....q\}$, where q (number of edges of graph G) = (4j + 9) + k.

$$f(u_1) = (4j + 9) + k. \qquad f(u_2) = 0$$
$$f(u_3) = f(u_1) - 2 \qquad f(u_4) = 1$$

$$f(w_1) = f(v_{2n}) \qquad f(x_1) = f(v_{2n-1})+1$$
$$f(w_2) = f(w_1) - 1 \qquad f(x_2) = f(x_1)+1$$
$$f(w_3) = f(w_2) - 1 \qquad f(x_3) = f(x_2)+1$$

. .

. $\quad f(x_{n+2}) = f(x_{n+1}) + 2$

. .

$$f(w_n) = f(w_{n-1}) - 1 \qquad f(x_{2n+2}) = f(x_{2n+1}) + 1$$

$$f(v_1) = f(u_4) \qquad f(v_2) = f(u_3) - 1$$
$$f(v_3) = f(v_1) + 1 \qquad f(v_4) = f(v_2) - 1$$
$$f(v_5) = f(v_3) + 1 \qquad f(v_6) = f(v_4) - 1$$
$$f(v_7) = f(v_5) + 1 \qquad f(v_8) = f(v_6) - 1$$

. .

. .

$$f(v_{2n+1}) = f(v_{2n-1})+1 \qquad f(v_{2n+2}) = f(v_{2n}) - 1.$$

$$(\forall\ n = 1, 2, ..., \forall\ k = 1, 2, ..., \forall\ j = 1, 2, ...)$$

(n = graphic position, k-1 = path length P_k, j = labeling according to C_n ($n \equiv 0,2 \pmod 4$))

***(B)*: Length of P_k is even.**

Define f : $v \rightarrow \{0,1,....q\}$, where q (number of edges of graph G) = (4j + 9) + k.

$$f(u_1) = (4j + 9) + k \qquad f(u_2) = 0$$
$$f(u_3) = f(u_1) - 2 \qquad f(u_4) = 1$$

$$f(w_1) = f(v_{2n-1})+1 \quad f(x_1) = f(v_{2n}) - 1$$
$$f(w_2) = f(w_1)+1 \qquad f(x_2) = f(x_1) - 1$$
$$f(w_3) = f(w_2)+1 \qquad f(x_3) = f(x_2) - 1$$
$$. \qquad .$$
$$. \qquad .$$
$$. \qquad f(x_{n+2}) = f(x_{n+1}) - 2$$
$$. \qquad .$$
$$. \qquad .$$
$$f(w_n) = f(w_{n-1})+1 \quad f(x_{2n+2}) = f(x_{2n+1})+1$$

$$f(v_1) = f(u_4) \qquad f(v_2) = f(u_3) - 1$$
$$f(v_3) = f(v_1) + 1 \qquad f(v_4) = f(v_2) - 1$$
$$f(v_5) = f(v_3) + 1 \qquad f(v_6) = f(v_4) - 1$$
$$f(v_7) = f(v_5) + 1 \qquad f(v_8) = f(v_6) - 1$$
$$. \qquad .$$
$$f(v_{2n+1}) = f(v_{2n-1}) + 1 \quad f(v_{2n+2}) = f(v_{2n}) - 1.$$

($\forall$ n = 1,2,....,$\forall$ k = 1,2,...,$\forall$ j = 1,2,...)
(n = graphic position, k-1 = path length P_k, j = labeling according to C_n ($n \equiv 0,2 \pmod 4$))

In both cases, we can verify that f is a graceful label of graph G.

2.6 Theorem

The graph G obtained through joined a quadrilateral with one chord $G_1$and a quadrilateral snake G_2 by a path of arbitrary length is graceful.

***Proof* :**

Let G = (V, E) be the graph obtained through joined two graphs, a quadrilateral with one chord G_1 and a quadrilateral snake G_2 by a path P_k of length k-1. Let $\{u_1,u_2,u_3,u_4\}$ be vertices of a quadrilateral with one chord G_1, $\{w_1,w_2,w_3,...,w_k\}$ be vertices of a quadrilateral snake G_2 and $\{v_1,v_2,.....,v_k\}$ be vertices of the path P_k with $v_1 = u_4$ and $v_k = w_1$. We consider the following cases.

***Case-1*:** Length of P_k is odd

Define f : $v \rightarrow \{0,1,....q\}$, where

$$q = 4n + \left\{ 2j + 4, for\ k = 2j - 1 \right.$$

($\forall\ n = 1, 2, ..., \forall\ j = 1, 2, ..., \forall\ k = 1, 2, ...$)
(j = labeling in the graph, k-1 = path length P_k, q = number of edges of a graph G, n = no. of snakes in graph G)

f(u_1) = q   f(u_2) = 0
f(u_3) = f(u_1) - 2   f(u_4) = 1

f(v_1) = f(u_4)   f(v_2) = f(u_3) - 1
f(v_3) = f(v_1) + 1   f(v_4) = f(v_2) - 1
.   .
.   .
f(v_{2j+1}) = f(v_{2j-1}) + 1   f(v_{2j+2}) = f(v_{2j}) - 1
.   .
.   .
f(w_1) = f(v_{2j})   for k = 2j - 1
f(w_2) = f(v_{2j-1}) + 1   for k = 2j - 1
.   .
.   .
f(w_{6j-3}) = f(w_{6j-5}) - 2
f(w_{6j-2}) = f(w_{6j-4}) + 1
f(w_{6j-1}) = f(w_{6j-3}) - 1
f(w_{6j}) = f(w_{6j-2}) + 2
f(w_{6j+1}) = f(w_{6j-1}) - 1
f(w_{6j+2}) = f(w_{6j}) + 1

2.7 Illustration

The graceful labeling of the graph G obtained by connecting a quadrilateral with one chord and a quadrilateral snake through the path P_4 shown in FIG. 4

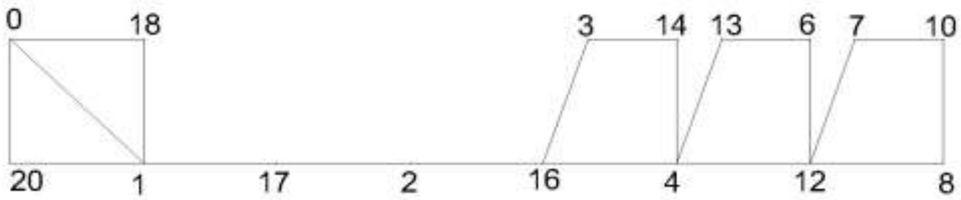

Figure 4: Graceful labeling of the graph G, which is formed by connecting a quadrilateral with one chord and a quadrilateral snake of path P_4, where p = 16(no. of vertices for graph G) and q = 20(no. of edges for graph G).

Case-2: Length of P_k is even. Define $f : v \rightarrow \{0,1,....q\}$, where

$$q = 4n + \begin{cases} 2j + 5, for\ k = 2j \end{cases}$$

$(\forall\ n = 1, 2, ..., \forall\ j = 1, 2, ..., \forall\ k = 1, 2, ...)$
(j = labeling in the graph, k-1 = path length P_k, q = number of edges of a graph G, n = no. of snakes in graph G)

f(u_1) = q f(u_2) = 0
f(u_3) = f(u_1) - 2 f(u_4) = 1

f(v_1) = f(u_4) f(v_2) = f(u_3) - 1
f(v_3) = f(v_1) + 1 f(v_4) = f(v_2) - 1
. .
. .
f(v_{2j+1}) = f(v_{2j-1}) + 1 f(v_{2j+2}) = f(v_{2j}) - 1
. .
. .
f(w_1) = f(v_{2j+1}) for k = 2j
f(w_2) = f(v_{2j}) - 1 for k = 2j
. .
. .
f(w_{6j-3}) = f(w_{6j-5}) + 2
f(w_{6j-2}) = f(w_{6j-4}) - 1
f(w_{6j-1}) = f(w_{6j-3}) + 1
f(w_{6j}) = f(w_{6j-2}) - 2
f(w_{6j+1}) = f(w_{6j-1}) + 1
f(w_{6j+2}) = f(w_{6j}) - 1

In both cases, we can verify that f is a graceful label of graph G.

2.8 Concluding Remark

The current work has contributed some new results. We discussed the gracefulness of the graph obtained by connecting (1) a quadrilateral with one chord and a quadrilateral (2) a quadrilateral with one chord and a barycentric subdivision of cycle C_n (3) a quadrilateral with one chord and a quadrilateral snake, through a path of arbitrary length. The mark pattern is displayed through illustrations to better understand the derived results.

2.9 Acknowledgement

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