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Research Article

Absolute Mean Cordial Labelling

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ARTICLE INFO	ABSTRACT

The focus of this article is to demonstrate a new graph labeling which is called absolute mean cordial labeling. We proved every triangular snake T_r, $\forall\, r \geq 3$, cycle C_s, $s \geq 4$, s 6= 5, friendship graph F_t, helm graph H_m, alternate triangular snake AT_r, $\forall\, r \geq 3$, double triangular snake DT_r, $\forall\, r \geq 3$, double alternate triangular snake $DA(T_r)$, $\forall\, r \geq 3$ are absolute mean cordial graphs.

AMS classification no: 05C78

Keywords: Graphs, triangular snake, absolute mean cordial labeling.

1. Introduction

The idea of cordial labeling was presented by Cahit [2] in 1987 and for numbering in graph was characterized by S.W.Golomb [5]. Various analysts have considered cordialness of graphs, refer Gallian survey [4]. A conventional number of papers are found with grouping of utilizations in coding theory, radar communication, cryptography etc. A significance bits of knowledge with respect to utilizations of graph labeling is found in Bloom and Golomb [5]. We recognize all documentations and expressing from Harary [6]. We audit some definitions which are utilize in this paper.
A function $f : V \rightarrow \{0,1\}$ is called binary vertex labeling of a graph G and $f(v)$ is called label of the vertex v of G under f. For an edge $e = (uv)$, the induced function $f^* : E \rightarrow \{0,1\}$ defined as $f^*(e) = |f(u) - f(v)|$. Let $vf(0)$, $vf(1)$ be number of vertices of G having labels 0 and 1 respectively under f and let $ef(0)$, $ef(1)$ be number of edges of G having labels 0 and 1 respectively under f^*. A binary vertex labeling f of a graph it is called cordial labeling if $|\, vf(0) - vf(1) \,| \leq 1$ and $|\, ef(0) - ef(1) \,| \leq 1$. A graph which admits cordial labeling is called *cordial graph*.
A function f is called an absolute mean cordial labeling of a graph $G = (V,E)$, if f :
$V(G) \rightarrow \{0, \pm 2, \pm 4, ..., \pm 2\left\lfloor \frac{q}{2} \right\rfloor\}$ is injective and the induced function $f^* : E(G) \rightarrow \{0,1\}$
defined as

$$f^*(e) = \begin{cases} 1, & \text{if} \quad \left\lceil \frac{|\, f(u) - f(v) \,|}{2} \right\rceil \leq \left\lceil \frac{q}{2} \right\rceil \\ 0, & \quad \text{otherwise} \end{cases}$$

and satisfies the condition $|\, ef(0) - ef(1) \,| \leq 1$ is surjective for every edge $e = (u,v) \in E(G)$. A graph is called absolute mean cordial, if it admits absolute mean cordial labeling.
A cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices (at least 3, if the graph is simple) connected in a closed chain. The cycle graph with n vertices is called C_n. The number of vertices in C_n equals the number of edges, and every vertex has degree 2; that is, every vertex has exactly two edges incident with it.
The triangular snake graph T_n is obtained from the path v_i by replacing every edge of P_n by C_3.
An alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i. That is every alternate edge of path is replaced by C_3.
Friendship graph F_n is one-point union of n copies of cycles C_3
The helm graph H_n is the graph obtained from an n-wheel graph by adjoining a pendant edge at each node of the cycle.
The double triangular snake DT_n consists of two triangular snakes which have a common path.

A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path.That is, double alternate triangular snake is obtained from a path $v_1, v_2, ..., v_n$ by joining v_i and v_{i+1} (alternately) to new vertices u_i and w_i.

In this paper, we displayed absolute mean cordial labeling of triangular snake T_r, $\forall\ r \geq 3$, cycle C_s, $s \geq 4$,s 6= 5 friendship graph F_t, helm graph H_m, alternate triangular snake AT_r, $\forall\ r \geq 3$, double triangular snake DT_r, $\forall\ r \geq 3$, double alternate triangular snake $DA(T_r)$, $\forall\ r \geq 3$. For detail outline of graph labeling, we imply Gallian [4].

2. Main Results:

2.1. Theorem. Every triangular snake T_r, $\forall\ r \geq 3$ is absolute mean cordial graph.

***Proof*:** Let vertices of $T_r = \{v_1, v_2, ..., v_r\}$ and edges of $T_r = \{e_1, e_2, ..., e_{r-1}\}$.
To set up triangular snake T_r from path P_r, connect v_j and v_{j+1} to new vertex u_j by edges $e_{2j}{}'_{-1} = v_j u_j$ and $'e_{2j} = v_{j+1} u_j$ $\forall\ j = 1, 2, ..., r-1$.
Then $|\ V(T_r)\ | = 2r - 1$, $|\ E(T_r)\ | = 3r - 3$.

We determine vertex labeling $f : V(T_r) \to \{0, \pm 2, \pm 4, ..., \pm 2\left\lfloor \frac{q}{2} \right\rfloor\}$ as follows.

$$f(u_j) = \begin{cases} 2\left\lfloor \frac{q}{2} \right\rfloor, & j = 1 \\ (-1)^{j+1}\ [\ |\ f(u_{j-1})\ | - 2\ ], & j = 2, 3, ..., r-1. \end{cases}$$

For choice of $f(v_j)$ there are two cases:

***Case-1*:** When r is an odd $r \geq 3$.

$$f(v_j) = \begin{cases} (-1)^r\ 2, & j = 1 \\ (-1)^{r+j+1}\ [\ |\ f(v_{j-1})\ | + 2\ ], & j = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor \&\ j = \left\lfloor \frac{r}{2} \right\rfloor + 2,\ j = \left\lfloor \frac{r}{2} \right\rfloor + 3, ..., r. \\ (-1)^{r+j+1}\ [\ 2\left\lfloor \frac{q+2}{2} \right\rfloor - 2\left\lceil \frac{r}{2} \right\rceil\ ] & j = \left\lfloor \frac{r}{2} \right\rfloor + 1 \end{cases}$$

***Case-2*:** When r is an even $r \geq 4$.

$$f(v_j) = \begin{cases} (-1)^r\ 2, & j = 1 \\ (-1)^{r+j+1}\ [\ |\ f(v_{j-1})\ | + 2\ ], & j = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor - 1\ \&\ j = \left\lfloor \frac{r}{2} \right\rfloor + 1,\ j = \left\lfloor \frac{r}{2} \right\rfloor + 2, ..., r. \\ (-1)^{r+j+1}\ [\ 2\left\lceil \frac{q-2}{2} \right\rceil - 2\left\lceil \frac{r}{2} \right\rceil\ ] & j = \left\lfloor \frac{r}{2} \right\rfloor \end{cases}$$

2.2. Illustration. A triangular snake T_7 is absolute mean cordial graph.

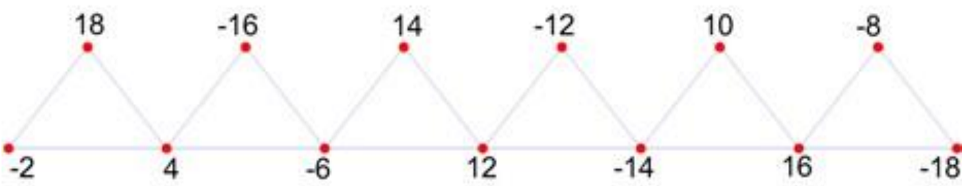

figure 1: A triangular snake T_7 with p = 13 and q = 18 is absolute mean cordial graph.

Obviously the condition $|\ e_f(0) - e_f(1)\ | \leq 1$ is satisfies.
Therefore triangular snake T_r, $\forall\ r \geq 3$ is absolute mean cordial graph.

2.3. Theorem. Every cycle C_s, $s \geq 4$,s 6= 5 is absolute mean cordial graph. *Proof***:** Let C_s be cycle graph with vertex set $V(C_s) = \{v_1, v_2, ..., v_s\}$ and q be number of edges. Clearly p = q = s.

Define vertex labeling function $f : V(C_s) \to \{0, \pm 2, \pm 4, ..., \pm 2\left\lfloor \frac{q}{2} \right\rfloor\}$ as follows.

***Case-1*:**

***Subcase-1*:** s = 7, 11, 15,......

$$f(v_j) = \begin{cases} 0, & j = 1 \\ 2, & j = 2 \\ -f(v_{j-1}), & j = 3, 5, 7, ..., (\frac{s-1}{2}) \\ j, & j = 4, 6, 8, ..., (\frac{s+1}{2}) \\ -(s-1), & j = (\frac{s+3}{2}) \\ (s-1), & j = (\frac{s+5}{2}) \\ (-1)^j \; [\; |\; f(v_{j-1}) \;| - 2 \;] & j = (\frac{s+7}{2})(\frac{s+11}{2}), ..., s \\ [\; |\; f(v_{j-2}) \;| - 2 \;] & j = (\frac{s+9}{2})(\frac{s+13}{2}), ..., (s-1) \end{cases}$$

***Subcase-2*:** s = 9, 13, 17,......

$$f(v_j) = \begin{cases} 0, & j = 1 \\ 2, & j = 2 \\ -f(v_{j-1}), & j = 3, 5, 7, ..., s. \\ j, & j = 4, 6, 8, ..., (\frac{s-1}{2}) \\ (s-1), & j = (\frac{s+3}{2}) \\ [\; |\; f(v_{j-2}) \;| - 2 \;] & j = (\frac{s+7}{2})(\frac{s+11}{2}), ..., (s-1) \\ -f(v_{j-1}) & j = (\frac{s+5}{2}) \\ (-1)^j [\; |\; f(v_{j-2}) \;| - 2 \;] & j = (\frac{s+9}{2}), ..., s. \end{cases}$$

***Case-2*:** s is an even.

$$f(v_j) = \begin{cases} 0, & j = 1 \\ (-2), & j = 2 \\ 4, & j = 3 \\ -(f(v_{j-2}) + 2), & j = 4, 6, 8, ..., s \\ (f(v_{j-2}) + 2), & j = 5, 7, 9, ..., s-1 \end{cases}$$

2.4. Illustration. A cycle C_{11} is absolute mean cordial graph.

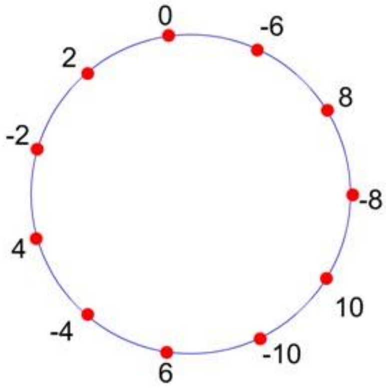

figure 2: **A cycle C_{11} with p = 11 and q = 11 is absolute mean cordial graph.**

Obviously the condition $| e_f(0) - e_f(1) | \leq 1$ is satisfies.
Therefore cycle C_s, $s \geq 4, s$ 6= 5 is absolute mean cordial graph.

2.5. Theorem. Every friendship graph F_t, $\forall\, t \geq 2$ is absolute mean cordial graph. *Proof*: Friendship graph, F_t, is derived by joining t copies of cycle graph C_3 with common vertex. Let v = $\{v_1, v_2, ..., v_{2t+1}\}$ be the vertex set of F_t and let the vertex at the centre be labelled by v_1. F_t is a planar undirected graph with p = 2t + 1 and q = 3t.
Define $f : V(F_t) \to \{0, \pm 2, \pm 4, ..., \pm 2\left\lfloor \frac{q}{2} \right\rfloor\}$ as follows.

***Case-1*:** t is an odd.

$$f(v_j) = \begin{cases} q-1, & j = 1 \\ -f(v_{j-1}), & j = 2 \\ 2, & j = 3 \\ (f(v_{j-2}) - 2), & j = 4, 6, 8, ..., (t+1) \\ -f(v_{j-1}), & j = 5, 7, 9, ..., (2t+1) \\ \left\lfloor \frac{q}{2} \right\rfloor, & j = (t+3) \quad for \quad t = 3, 7, 11, ... \\ \left\lceil \frac{q}{2} \right\rceil, & j = (t+3) \quad for \quad t = 5, 9, 13, ... \\ [\, f(v_{j-2}) - 2\,] & j = (t+5), ..., (2t). \end{cases}$$

***Case-2*:**
***Subcase-1*:** t = 2

$$f(v_j) = \begin{cases} q, & j = 1 \\ -f(v_{j-1}), & j = 2 \\ 2, & j = 3 \\ 0, & j = 4 \\ -2, & j = 5 \end{cases}$$

***Subcase-2*: t** is an even $t \geq 4$

$$f(v_j) = \begin{cases} q, & j = 1 \\ -f(v_{j-1}), & j = 2 \\ 2, & j = 3 \\ [\,|\,f(v_{j-2})\,| - 2\,] & j = 4, 6, 8, ..., t. \\ -f(v_{j-1}), & j = 5, 7, 9, ..., (2t+1). \\ \frac{q}{2} & j = (t+2),\ for\ t = 4, 8, 12, ... \\ \left\lfloor \frac{q-1}{2} \right\rfloor, & j = (t+2)\ for\ t = 6, 10, 14.... \\ (f(v_{j-2}) - 2), & j = t+4, ..., 2t. \end{cases}$$

2.6. Illustration. A friendship graph F_5 is absolute mean cordial graph.

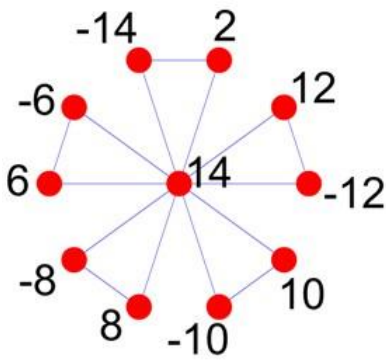

figure 3: **A friendship graph F_5 with p = 11 and q = 15 is absolute mean cordial graph.**

Obviously the condition $|\,e_f(0) - e_f(1)\,| \leq 1$ is satisfies.
Therefore friendship graph $F_t, \forall\, t \geq 2$ is absolute mean cordial graph.

2.7. Theorem. Every helm graph H_m, $\forall\, m \geq 3$ is absolute mean cordial graph. *Proof*: By the definition of helm graph, H_m is obtained from a wheel by attaching a pendant edge at each vertex of the m-cycle. Let $V(H_m) = V_1{}^S V_2{}^S V_3$. Where V_1 is the central vertex, $V_2 = \{V_j \,/\, 2 \leq j \leq m+1\}$ be the vertices on the m-cycle and $V_3 = \{V_j \,/\, m+2 \leq j \leq 2m+1\}$ be the pendant vertices incident with m cycle such that V_{m+j} is adjacent with V_j, $2 \leq j \leq m+1$. It has p = 2m + 1 vertices and q = 3m edges.

Define $f : V(H_m) \rightarrow \{0, \pm 2, \pm 4, ..., \pm 2\left\lfloor \frac{q}{2} \right\rfloor\}$ as follows.

***Case-1*:** m is an odd.
***Subcase-1*:** m = 3

$$f(v_j) = \begin{cases} q-1, & j = 1 \\ -f(v_{j-1}), & j = 2 \\ (-1)^{j-1}[\,|\,f(v_{j-1})\,| - 2\,], & j = 3, 4 \\ 4, & j = 5 \\ -6, & j = 6 \\ 2, & j = 7. \end{cases}$$

***Subcase-2*:** m = 5

$$f(v_j)=\begin{cases} q-1, & j=1\\ -f(v_{j-1}), & j=2\\ [\,|\,f(v_{j-1})\,|-2\,], & j=3,\dots,m+1\\ -f(v_{j-3}), & j=m+2\\ -[\,|\,f(v_{j-1})\,|+2\,], & j=m+3\\ [\,|\,f(v_{j-1})\,|-6\,], & j=m+4\\ (-1)^{j+1}[\,|\,f(v_{j-1})\,|-2\,], & j=m+5\ to\ 2m+1. \end{cases}$$

***Subcase-3*:** m = 7, 11, 15, 19,....

$$f(v_j)=\begin{cases} q-1, & j=1\\ -f(v_{j-1}), & j=2\\ (-1)^{j+1}\,[\,|\,f(v_{j-1})\,|-2\,] & j=3,4,\dots,m+1.\\ (-1)^{j}\,[\,|\,f(v_{j-1})\,|+2\,] & j=(m+3),\dots,(\frac{7m-1}{4}).\\ -2, & j=m+2\\ (-1)^{j+1}\,[\,|\,f(v_{j-1})\,|+(m-3)\,] & j=(\frac{7m+3}{4})\\ (-1)^{j+1}\,[\,|\,f(v_{j-1})\,|+2\,] & j=(\frac{7m+7}{4}),\dots,(2m+1). \end{cases}$$

***Subcase-4*:** m = 9, 13, 17, 21,....

$$f(v_j)=\begin{cases} q-1 & j=1\\ -f(v_{j-1}) & j=2\\ (-1)^{j+1}\,[\,|\,f(v_{j-1})\,|-2\,] & j=3,4,\dots,m+1\\ 2 & j=m+2\\ (-1)^{j+1}\,[\,|\,f(v_{j-1})\,|+2\,] & j=(m+3),\dots,(\frac{7m+1}{4})\\ (-1)^{j+1}\,[\,|\,f(v_{j-1})\,|+(m-3)\,] & j=(\frac{7m+5}{4})\\ (-1)^{j+1}\,[\,|\,f(v_{j-1})\,|+2\,] & j=(\frac{7m+9}{4}),\dots,(2m+1). \end{cases}$$

***Case-2*:** m is an even.
***Subcase-1*:** m = 4, 8, 12,...

$$f(v_j)=\begin{cases} q & j=1\\ -f(v_{j-1}) & j=2\\ (-1)^{j+1}\,[\,|\,f(v_{j-1})\,|-2\,] & j=3,4,\dots,m+1\\ 0 & j=m+2\\ -2 & j=m+3\\ (-1)^{j}\,[\,|\,f(v_{j-1})\,|+2\,] & j=(m+4),\dots,(\frac{7m+8}{4})\\ (-1)^{j}\,[\,|\,f(v_{j-1})\,|+(m-2)\,] & j=(\frac{7m+12}{4})\\ (-1)^{j}\,[\,|\,f(v_{j-1})\,|+2\,] & j=(\frac{7m+16}{4}),\dots,(2m+1). \end{cases}$$

***Subcase-2*:** m = 6, 10, 14,...

$$f(v_j) = \begin{cases} q & j = 1 \\ -f(v_{j-1}) & j = 2 \\ (-1)^{j+1} [\ |\ f(v_{j-1})\ | - 2\ ] & j = 3, 4, ..., m+1 \\ 0 & j = m+2 \\ 2 & j = m+3 \\ (-1)^{j+1} [\ |\ f(v_{j-1})\ | + 2\ ] & j = (m+4), ..., (\frac{7m+6}{4}) \\ (-1)^{j} [\ |\ f(v_{j-1})\ | + (m-2)\ ] & j = (\frac{7m+10}{4}) \\ (-1)^{j} [\ |\ f(v_{j-1})\ | + 2\ ] & j = (\frac{7m+14}{4}), ..., (2m+1) \end{cases}.$$

2.8. Illustration. A helm graph H_9 is absolute mean cordial graph.

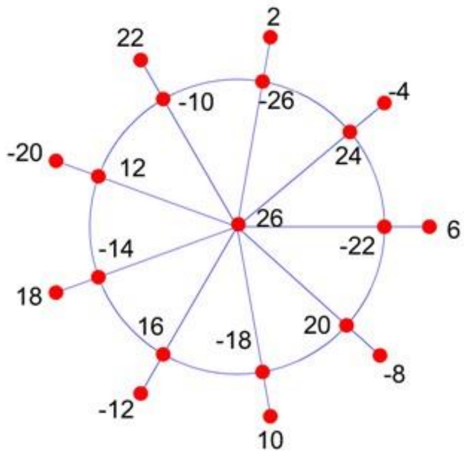

figure 4: A helm graph H_9 is with p = 19 and q = 27 is absolute mean cordial graph.

Obviously the condition | $e_f(0) - e_f(1)$ | ≤ 1 is satisfies.
Therefore helm graph H_m, ∀ $m \geq 3$ is absolute mean cordial graph.

2.9. Theorem. Every alternate triangular snake AT_r, ∀ $r \geq 3$ is absolute mean cordial graph.
*Proof***:** Let $V(P_r) = \{v_1, v_2, \ldots, v_r\}$ and $E(P_r) = \{e_1, e_2, \ldots, e_{r-1}\}$.
To build up $A(T_r)$ we coupling v_i and v_{i+1} (alternately) to new vertex u_i where $1 \leq i \leq r-1$ for even r and $1 \leq i \leq r-2$ for odd r.

so, $V(A(T_r)) = \{v_i, u_j \,/\, 1 \leq i \leq n,\ 1 \leq j \leq \left\lfloor \frac{r}{2} \right\rfloor\}$

$$|\ V(A(T_r))\ | = \begin{cases} \frac{3r}{2}, & if\ even\ r \geq 3 \\ \frac{3r-1}{2}, & if\ odd\ r \geq 3 \end{cases}$$

$$|\ E(A(T_r))\ | = \begin{cases} 2r-1 & if\ even\ r \geq 3 \\ 2r-2, & if\ odd\ r \geq 3 \end{cases}$$

We define vertex labeling

Define $f : V(A(T_r)) \to \{0, \pm 2, \pm 4, ..., \pm 2\left\lfloor \frac{q}{2} \right\rfloor\}$ as follows.

***Case-1*:** When r is an odd $r \geq 3$.

$$f(v_i) = \begin{cases} q & i = 1 \\ 2 & i = 2 \\ (-1)^i \, [\, | \, f(v_{i-1}) \, | + 2 \,], & i = 3, 4, \ldots, r. \end{cases}$$

$$f(u_i) = \begin{cases} -2 & i = 1 \\ -(q-2) & i = 2 \\ (-1)^{i+1} \, [\, | \, f(v_{i-1}) \, | - 2 \,], & i = 3, 4, \ldots, \left\lfloor \frac{r}{2} \right\rfloor. \end{cases}$$

***Case-2*:** When r is an even $r \geq 3$.

$$f(v_i) = \begin{cases} 2 & i = 1 \\ (-1)^{i+1} \, [\, | \, f(v_{i-1}) \, | + 2 \,], & i = 2, 3, \ldots, r-1 \\ -f(v_{i-1}) & i = r. \end{cases}$$

$$f(u_i) = \begin{cases} -2 & i = 1 \\ (q-3) & i = 2 \\ (-1)^i \, [\, | \, f(v_{i-1}) \, | - 2 \,], & i = 3, 4, \ldots, \frac{r}{2}. \end{cases}$$

2.10. Illustration. An alternate triangular snake AT_7 is absolute mean cordial graph.

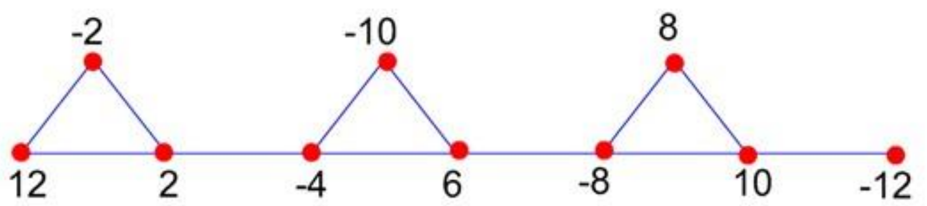

figure 5: An alternate triangular snake AT_7 with p = 10 and q = 12 is absolute mean.

Obviously the condition $| \, e_f(0) - e_f(1) \, | \leq 1$ is satisfies.
Therefore alternate triangular snake AT_r, $\forall \, r \geq 3$ is absolute mean cordial graph.

2.11. Theorem. Every double triangular snake DT_r, $\forall \, r \geq 3$ is absolute mean cordial graph.
***Proof*:** Let $V(P_r) = \{v_1, v_2, \ldots, v_r\}$ and $E(P_r) = \{e_1, e_2, \ldots, e_{r-1}\}$.
For construct $D(T_r)$, join v_j and v_{j+1} to the new vertices u_j, w_j by edges $e_{2j-1} = u_j v_j$, $e2j = ujvj+1$, $e2j' - 1 = wjvj$ and $' e2j = wjvj+1$ j = 1, 2,..., r-1.
And $| \, V(D(T_r)) \, | = 3r - 2$, $| \, E(D(T_r)) \, | = 5r - 5$. We define vertex labelling

$f : V(D(T_r)) \to \{0, \pm 2, \pm 4, \ldots, \pm 2 \left\lfloor \frac{q}{2} \right\rfloor\}$ as follows.

***Case-1*:** When r is an odd $r \geq 3$.

***Subcase-1*:** r = 3

$$f(v_j) = \begin{cases} q & j = 1 \\ (-1)^{j+1} \, [\, | \, f(v_{j-1}) \, | - 2 \,], & j = 2, 3 \end{cases}$$

$$f(w_j) = \begin{cases} (f(v_3) - 2) & j = 1 \\ -f(w_{j-1}) & j = 2 \end{cases}$$

$$f(u_j) = \begin{cases} -2 & j = 1 \\ -f(v_3) & j = 2 \end{cases}$$

***Subcase-2*:** r is an odd $r \geq 5$

$$f(v_j) = \begin{cases} q & j = 1 \\ (-1)^{j+1}\ [\ |\ f(v_{j-1})\ | - 2\ ], & j = 2,3,..,r \end{cases}$$

$$f(w_j) = \begin{cases} (f(v_r) - 2) & j = 1 \\ -f(w_{j-1}) & j = 2,4,6,\dots,r-1 \\ (f(w_{j-2}) - 2) & j = 3,5,7,\dots,r-2 \end{cases}$$

$$f(u_j) = \begin{cases} -2 & j = 1 \\ (-1)^j\ [\ |\ f(u_{j-1})\ | - 2\ ], & j = 2,3,\dots\left\lceil\frac{r}{2}\right\rceil \\ -f(u_{j-1}) & j = \left\lceil\frac{r}{2}\right\rceil + 1 \\ (-1)^j\ [\ |\ f(u_{j-1})\ | + 2\ ], & j = \left\lceil\frac{r}{2}\right\rceil + 2,\dots,r-1. \end{cases}$$

***Case-2*:** When r is an even $r \geq 4$

$$f(v_j) = \begin{cases} q - 1 & j = 1 \\ (-1)^{j+1}\ [\ |\ f(v_{j-1})\ | - 2\ ], & j = 2,3,..,r \end{cases}$$

$$f(w_j) = \begin{cases} -f(v_r) & j = 1 \\ (-1)^{j+1}\ [\ |\ f(w_{j-1})\ | + 2\ ], & j = 2,3,..,r-1 \end{cases}$$

$$f(u_j) = \begin{cases} 0 & \\ (-1)^j\ [\ |\ f(u_{j-1})\ | + 2\ ], & j = r-1\, j = r-2,\dots 1. \end{cases}$$

2.12. Illustration. A double triangular snake DT_5 is absolute mean cordial graph.

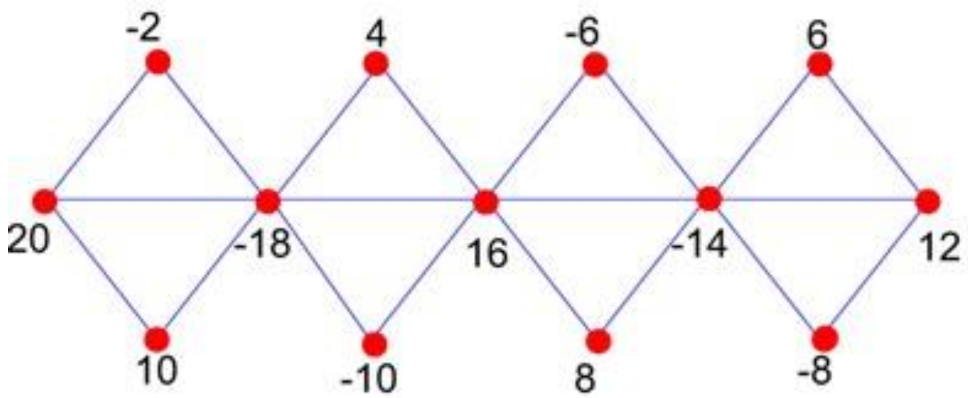

figure 6: A double triangular snake DT_5 with p = 13 and q = 20 is absolute mean cordial graph.

Obviously the condition $|\ e_f(0) - e_f(1)\ | \leq 1$ is satisfies.
Therefore double triangular snake DT_r, $\forall\ r \geq 3$ is absolute mean cordial graph.

2.13. Theorem. Every double alternate triangular snake DAT_r, $\forall\ r \geq 3$ is absolute mean cordial graph.

***Proof*:** Let $V(P_r) = \{v_1, v_2, \dots, v_r\}$ and $E(P_r) = \{e_1, e_2, \dots, e_{r-1}\}$.
To construct $DA(T_r)$, we coupling v_j and v_{j+1} (alternately) to the new vertices u_j, w_j respectively. Let G be a double alternate triangular snake $DA(T_r)$ then

$V(G) = \{v_j, u_k, w_k \,/\, 1 \leq j \leq r,\ 1 \leq k \leq \left\lfloor\frac{r}{2}\right\rfloor\}$

We consider that

$$V(DA(T_r)) = \begin{cases} 2r & when\ r\ is\ an\ even \\ 2r-1 & when\ r\ is\ an\ odd \end{cases}$$

$$E(DA(T_r)) = \begin{cases} 3r-1 & when\ r\ is\ an\ even \\ 3r-3 & when\ r\ is\ an\ odd \end{cases}$$

We define vertex labeling
$f : V(DA(T_r)) \to \{0, \pm 2, \pm 4, ..., \pm 2\left\lfloor \frac{q}{2} \right\rfloor\}$ as follows.

***Case-1*:** When r is an odd $r \geq 3$

$$f(v_j) = \begin{cases} q & j = 1 \\ (-1)^{j+1}\ [\ |\ f(v_{j-1})\ | - 2\ ], & j = 2, 3, .., r \end{cases}$$

$$f(w_j) = \begin{cases} -\left\lceil \frac{r}{2} \right\rceil & j = 1\ \ when\ r = 3, 11, 19, ... \\ (-1)^j\ [\ |\ f(w_{j-1})\ | + 2\ ], & j = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor \\ \left\lceil \frac{r}{2} \right\rceil & j = 1\ \ when\ r = 7, 15, 23, ... \\ (-1)^{j+1}\ [\ |\ f(w_{j-1})\ | + 2\ ], & j = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor \\ -\left\lceil \frac{r+2}{2} \right\rceil & j = 1\ \ when\ r = 5, 13, 21, ... \\ (-1)^j\ [\ |\ f(w_{j-1})\ | + 2\ ], & j = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor \\ \left\lceil \frac{r+2}{2} \right\rceil & j = 1\ \ when\ r = 9, 17, 25, ... \\ (-1)^{j+1}\ [\ |\ f(w_{j-1})\ | + 2\ ], & j = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor \end{cases}$$

$$f(u_j) = \begin{cases} r + \left\lfloor \frac{r}{2} \right\rfloor & j = 1\ \ when\ r = 3, 11, 19, ... \\ (-1)^{j+1}\ [\ |\ f(u_{j-1})\ | + 2\ ], & j = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor \\ -(r + \left\lceil \frac{r}{2} \right\rceil) & j = 1\ \ when\ r = 5, 13, 21, ... \\ (-1)^j\ [\ |\ f(u_{j-1})\ | + 2\ ] & = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor \\ -(r + \left\lfloor \frac{r}{2} \right\rfloor) & j = 1\ \ when\ r = 7, 15, 23, ... \\ (r + \left\lceil \frac{r}{2} \right\rceil) & j = 1\ \ when\ r = 9, 17, 25, ... \\ (-1)^{j+1}\ [\ |\ f(u_{j-1})\ | + 2\ ], & j = 2, 3, ... \left\lfloor \frac{r}{2} \right\rfloor \end{cases}$$

***Case-2*:** When r is an even $r \geq 4$

$$f(v_j) = \begin{cases} q-1 & j = 1 \\ [\ |\ f(v_{j-2})\ | - 2\ ] & j = 3, 5, .., (r-1) \\ -f(v_{j-1}) & j = 2 \\ -[\ |\ f(v_{j-2})\ | - 2\ ], & j = 4, 6, ...r \end{cases}$$

$$f(w_j) = \begin{cases} (p-2) & j = 1 \\ [\ |\ f(w_{j-2})\ | - 2\ ] & j = 3, 5, ..(\frac{r}{2} - 1)\ when\ r = 4, 8, 12, ... \\ [\ |\ f(w_{j-2})\ | - 2\ ] & j = 3, 5, ..(\frac{r}{2})\ when\ r = 6, 10, 14, ... \\ -(f(w_{j-1}) & j = 2 \\ -[\ |\ f(w_{j-2})\ | - 2\ ] & j = 4, 6, ..(\frac{r}{2})\ when\ r = 4, 8, 12, ... \\ -[\ |\ f(w_{j-2})\ | - 2\ ] & j = 4, 6, ..(\frac{r}{2} - 1)\ when\ r = 6, 10, 14, ... \end{cases}$$

$$f(u_j) = \begin{cases} 4 & j = 1 \\ [\,|\,f(u_{j-2})\,| + 2\,] & j = 3, 5, ..(\frac{r}{2})\ when\ r = 6, 10, 14, ... \\ [\,|\,f(u_{j-2})\,| + 2\,] & j = 3, 5, ..(\frac{r}{2} - 1)\ when\ r = 4, 8, 12, ... \\ -(f(u_{j-1}) & j = 2 \\ -[\,|\,f(u_{j-2})\,| + 2\,] & j = 4, 6, ..(\frac{r}{2})\ when\ r = 4, 8, 12, ... \\ -[\,|\,f(u_{j-2})\,| + 2\,] & j = 4, 6, ..(\frac{r}{2} - 1)\ when\ r = 6, 10, 14, ... \end{cases}$$

2.14. Illustration. A double alternate triangular snake $DA(T_7)$ is absolute mean cordial graph.

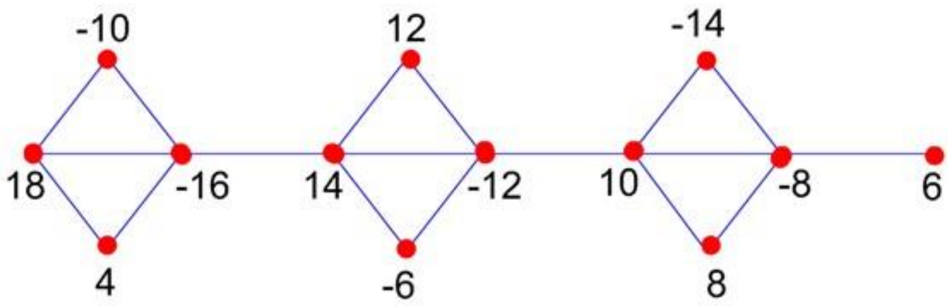

figure 7: A double alternate triangular snake *DA*(T_7) with p = 13 and q = 18 is absolute mean cordial graph.

Obviously the condition $|\ e_f(0) - e_f(1)\ | \leq 1$ is satisfies.
Therefore double alternate triangular snake $DA(T_r)$, $\forall\ r \geq 3$ is absolute mean cordial graph.

2.15. Illustration. Cycle C_5 and alternate quadrilateral snake $AQ(S_3)$ are not absolute mean cordial graph.

2.16. Concluding Remark. The current work has contributed a new labeling is absolute mean cordial labeling. we showed absolute mean cordial labeling of triangular snake T_r, $\forall\ r \geq 3$, cycle C_s, $s \geq 4, s$ 6= 5, friendship graph F_t, helm graph H_m, alternate triangular snake AT_r, $\forall\ r \geq 3$, double triangular snake DT_r, $\forall\ r \geq 3$, double alternate triangular snake $DA(T_r)$, $\forall\ r \geq 3$. The labeling pattern is shown through representations to more readily comprehend the determined outcomes.

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