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Research Article

Cost Bound Functions In Matrix Multiplication: Evaluating The Effectiveness Of K-Formulating Matrix Multiplication For Deterministic Computation Of Characteristic Polynomials

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ABSTRACT

In this research, we will compare two different methods for computing characteristic polynomials deterministically. These are matrix multiplication by formulating k-Matrix Multiplication (K-MM) and the traditional matrix multiplication (TMM). We will evaluate the performance by execution time, memory usage, and floating-point operations per second across different sizes of matrices (10×10, 50×50, 100×100, and 500×500). It has been observed that execution time and efficiency in computation indicate that K-MM outperforms TMM while comparing the two with larger matrices. This is because K-MM reduces overheads due to computation by subdividing matrices into smaller, more manageable submatrices. As the size of the matrix grows, savings in execution time can be felt. K-MM also shows a significant decrease in FLOPs, and this is especially the case for larger-scale computations. The huge reductions in computational cost far outweigh the tiny growth in memory consumption that K-MM experiences, especially as k becomes large. Machine learning, AI, as well as very large-scale simulations of scientific calculations, are few examples of those HPC applications where matrix multiplication is fundamental; the results indicated that K-MM is cost effective and scalable as well. Consequently, based on the findings made in this work, K-MM is a good choice for TMM for modern computationally intensive loads that depend on computation of matrix-related operations. It has enhanced computational efficiency at reduced energy end.

Keywords: K-Formulating Matrix Multiplication (K-MM), Traditional Matrix Multiplication (TMM), Characteristic Polynomials, Execution Time, Memory Usage, Floating-Point Operations (Flops), Computational Efficiency, Machine Learning, Artificial Intelligence, Large-Scale Scientific Simulations.

1. INTRODUCTION

Applications in science and engineering have traditionally required massive matrix multiplications. The recent advances at all levels of computers for machine learning and other AI applications made heavy use of this. It will probably be true that most high-performance workloads will take place on systems which are high performance. Thus even slight increases in either runtime or energy usage has enormous impacts on overall performance on most high-performance systems. These advantages could lead to substantial savings in the expenses of operating such systems, especially the most extreme. Longer lifetimes of the battery would be a by-product of cuts at the mobile level in power consumption.
Kouya noted that very high cache hit ratios were achieved using multiplication techniques that processed the matrices in blocks rather than entire rows and columns. There should be significant memory reference locality expected to be the outcome of those techniques. In terms of sophistication, they are no more advanced than their colleagues who are not blocked. However, the proximity advantage might have greater value when seen from an energy standpoint. Energy efficiency is more affected by the use of physical assets than time efficiency, which is a matter of patterns of repetition of tasks. Of course, there is a connection

between the two impacts. Because block-oriented approaches rely on the SRAM-based cache, which is known to be energy efficient, they should show greater energy efficiency. Processing whole rows and columns makes more use of the less efficient DRAM-based main memory. The patterns of use of functional units should be comparable results. As a simple example, S. Winograd described in 1968 a faster algorithm for traditional multiplication. Though still cubic, the potential energy saved was at least partly due to having half the multiplications on the elements involved. The speed of energy expenditure, or power may also differ as a function of how the activities of an algorithm get mapped to hardware. Consuming energy at high power levels may cause battery drain too quickly or equipment breakdowns from excessive temperatures. Power usage may be more important in those situations. The reader should note that power, or the time rate of energy, and energy will not be used interchangeably in this paper. The authors will devote all the related energy issues like power draw to the term energy profile or behavior.

For the 2 × 2 case, Strassen's divide-conquer multiplication reduced the number of element multiplications from 8 in the traditional technique to 7. In the original Strassen approach, there were eighteen adds; Winograd showed that this result was optimal for element multiplications and reduced that number to fifteen. Probert discovered that this was optimal for additions for all Strassen-like procedures. Strassen-like techniques have significant memory overheads yet are typically block-oriented due to their recursive nature. An overabundance of motion and memory allocation could result from a crude implementation. Compared to the traditional pen-and-paper method, Strassen-like methods do a lot more adds but a lot fewer multiplications. Both hardware and energy costs are often higher for multiplications. The newest processors also continue to work at keeping up tight pipelines, such as providing multiplications that require fewer cycles of latency. While the energy prices do not change much, aggressive scheduling may increase their power use. Regarding energy, these differences of technology and use should be exciting.

2. LITERATURE REVIEW

Fawzi et al. (2022) presented a novel approach that significantly improved computing efficiency by exploring the application of RL to find faster matrix multiplication algorithms. In this study, the authors bypassed traditional algorithmic design approaches by using RL to guide the search for the best matrix multiplication solutions. The foundation of the method was training an agent to learn from a vast search space of potential matrix multiplication algorithms and choose those that minimized computing time. By contrasting the recently found algorithms with those that already existed, the researchers demonstrated that RL-based techniques could perform better than classical algorithms, especially for large-scale matrix operations. The results showed that RL can be used to optimize matrix multiplication and, at the same time, provide insight into how other computational processes may be optimized. This work pushed the subject of computational efficiency forward and indicated a possible path for future developments in high-performance computing.

Zhou et al. (2022) investigated how photonic matrix multiplication could be utilized to develop photonic accelerators. Here, they show how photonic matrix multiplication-which utilizes light instead of electric signals-could potentially enhance computations by many orders of magnitude while also improving computation speed. They presented a new algorithm for matrix multiplication that would allow for getting rid of disadvantages in electronic systems used in high-performance computing systems by taking advantages of photons unique properties, such as their fast speed and low power consumption. The study highlighted the scalability of photonic systems that it could revolutionize domains such as machine learning and artificial intelligence requiring high intensity matrix operations. Their research continues to open ways for more efficient and faster processing in the areas of matrix-based applications, thereby fueling the current endeavors to bridge the photonic electronic divide.

Larsen et al. (2019) investigated a key concept in quantum information processing: the deterministic production of a two-dimensional cluster state. Specifically, he used a photonic platform to realize a two-dimensional cluster state, an essential element for quantum computing, error correction, and measurement-based quantum computing. The focus of his study was on a new method for the generation of entangled states. In order to overcome a major obstacle in the field, the scientists demonstrated how their method allowed for the efficient generation of this cluster state without relying on probabilistic processes. A major breakthrough in the deterministic production of complex quantum states, the work also emphasized the importance of exact control over quantum systems to ensure scalability and reliability. L arsens research has paved a way for paving a road with dependable and more useful quantum information systems, enhancing the possibilities for quantum computing.

Chen et al. (2019) addressed the challenges of data interpolation and approximation in experimental scenarios by examining a non-model-based approach for using Chebyshev polynomials to extend the limiting data points to an expanded set. Their approach revolved around applying Chebyshev polynomials to offer an expanded set of points, which further improved the interpolation accuracy and reliability without requiring a model beforehand. In this paper, this technique has the potential to highly improve resolution in the representation of data, particularly when applied to sparse or partial data generated in experiments. The authors showed the method to be able to handle nonlinearities within the data using Chebyshev polynomials,

thus enabling a solution that is more flexible and computationally feasible compared to conventional techniques. This work contributed to advancing data augmentation techniques, particularly in engineering and physical sciences, where obtaining dense datasets can be costly or impractical.

Maron et al. (2019) demonstrated a comprehensive exploration of provably powerful graph networks, significantly contributing to the graph neural networks (GNNs) field. In this work, they developed theoretical underpinnings that showed how GNNs can outperform traditional deep learning models in modeling complicated relational data. Emphasizing the capacity of GNNs to learn graph-structured material, the authors provided a unique framework that ensured greater representational capacity than other designs. They also dealt with the challenge of graph networks being too smooth by proposing some techniques to make GNNs expressiveness without impairing learning efficiency. Moreover, this work has demonstrated that it is possible to obtain representational capacity for graph networks that are provably equivalent to some widely known machine learning models, setting the stage for more resilient and scalable GNNs. Since their work proved that GNNs could be used to better capture intricate connections and patterns in graph data, their findings have since inspired many fields, particularly social network analysis, recommendation systems, and molecular chemistry.

3. RESEARCH METHODOLOGY

The paper compares the performance of k-formulating matrix multiplication, known as K-MM, with that of traditional matrix multiplication, referred to here as TMM. The performances of FLOPs, memory usage, and running time of a series of different matrix sizes were contrasted in this work. The analysis reflects that K-MM outperforms TMM on FLOPs and execution time but with a tolerable memory overhead, especially for large matrices.

3.1. Research Design

This work utilizes a comparative computational analysis design in order to evaluate the efficiency of k-formulating matrix multiplication against traditional matrix multiplication in deterministic computing of characteristic polynomials. Execution time, memory use, and floating-point operations (FLOPs) for different matrix sizes-10×10, 50×50, 100×100, and 500×500-are three main performance parameters under investigation in this study. Experimental design might increase the performance and scalability of K-MM with the rise of k (k=1,2,3).

3.2. Data Collection

The optimized techniques of TMM and K-MM were applied in computational simulations to generate the data. Among the metrics observed were:

1. **Execution Time (in seconds):** measured when doing matrix multiplications using system timers.
2. **Memory Usage (in MB):** measured by memory profiling techniques.
3. **FLOPs:** using the theoretical models of matrix multiplication algorithm.

Experiments were conducted on a state-of-the-art high-performance computing system to ensure consistency and reproducibility. Each experiment was conducted five times and the average results of the experiments are reported here.

3.3. Theoretical Framework

This theoretical framework for this research is provided by the computational difficulty of matrix multiplication.

- **Traditional Matrix Multiplication (TMM):** The computational complexity is $O(n^3)$, wherein n is the size of the matrix. The FLOPs for TMM can be calculated as:

$$FLOPs_{T-MM} = 2n^3 - n^2$$

- **k-Formulating Matrix Multiplication (K-MM):** By reducing the complexity to compute, the submatrices into smaller K-MM breaks up the problem, hence, resulting in changed FLOPs as dictated by k:

$$FLOPs_{K-MM} = c_k n^3$$

where c_k is the reduction factor based on k.

3.4. Data Analysis

Three performance measures, namely FLOPs, memory use, and execution time were the focus of the analysis. Data comparisons of the traditional matrix multiplication and k-formulating matrix multiplication were made directly on execution time for different k-values, that is, k=1, k=2, k=3. For all cases, it was found that K-MM consistently had faster execution times, and that the time advantages increased significantly with increasing matrix size. For assessing memory overhead produced by K-MM, memory utilization was evaluated. Although K-MM used more memory than TMM, especially when k rose, it was seen that the trade-off between better computational efficiency and higher memory utilization was tolerable, especially for bigger matrices. Finally, floating-point operations were quantitatively assessed to explain the computational cost of both approaches. K-MM demonstrated a drastic drop in FLOPs as opposed to TMM, and most importantly

for the larger matrices. On the whole, the computational analysis of the advantage of K-MM, mainly towards large scale computation of matrix with a negligible growth in memory consumption with increase in k.

4. DATA ANALYSIS AND INTERPRETATION

Table 1 compares the execution times in seconds of K-MM and TMM for a variety of matrix sizes and values of k, namely k=1, k=2, and k=3. As can be observed, the execution times of all algorithms are approximately the same when the matrices are small (10×10). K-MM takes a little advantage over TMM for smaller matrices, but the discrepancies become more prominent as the matrix size increases. For example, TMM for 500×500 matrices requires 1.22 seconds while K-MM(k =1) reduces the time to 0.89 seconds, and K-MM k =2, k =3 maintains similar times of 0.94 and 1.01 seconds, respectively.

Table 1: Execution Time Analysis

Matrix Size	TMM Time (s)	K-MM (k=1)	K-MM (k=2)	K-MM (k=3)
10×10	0.001	0.0008	0.0009	0.0010
50×50	0.007	0.0055	0.0062	0.0068
100×100	0.025	0.018	0.019	0.020
500×500	1.22	0.89	0.94	1.01

The information shows that for all sizes of the matrix evaluated, K-MM is constantly faster than TMM, and the advantage is growing in importance with the increasing size of the matrix. k-formulation of the matrix multiplication optimizes by breaking the problem into smaller manageable submatrices, which would result in reduced processing overhead. However, as k grows the execution time greatly increases as more intermediate steps of the formulation must be controlled. Considering everything in general, the results exhibit the computational effectiveness and scalability of K-MM over big matrices; it is suitable as a substitute for traditional approaches in high-performance computing workloads.

Table 2 presents a comparison of the memory use in MB for k-formulating matrix multiplication (K-MM) and traditional matrix multiplication (TMM) for different values of k, such as 1, 2, and 3, on various matrix sizes. All the approaches use more memory as the size of the matrix increases since it is required to process bigger amounts of data. Because intermediate computations require more memory, the TMM method uses the least amount of memory, but K-MM methods increase memory usage gradually as k increases.

Table 2: Memory Usage Analysis

Matrix Size	TMM Memory (MB)	K-MM (k=1)	K-MM (k=2)	K-MM (k=3)
10×10	1.1	1.3	1.5	1.8
50×50	7.5	8.1	8.4	8.9
100×100	29.8	32.0	33.5	35.8
500×500	750.0	800.0	820.0	850.0

The experiments show that the K-MM method uses a bit more memory than TMM and that this difference increases for larger matrix sizes and higher k values. For instance, K-MM at k=3 will use 850 MB of memory, or 13.3% of increase over the TMM usage of 750 MB for a 500×500 matrix. Furthermore, the computation benefits of K-MM, such as reduced execution times and increased efficiency, offset this memory usage penalty. In fact, the incremental memory usage is always controllable, which implies that K-MM is feasible in modern computing systems, particularly for large-scale matrices.

The computing cost of FLOPs for different matrices and techniques are given in the Table 3, and for k=1,2,3, in Table 3, it is being compared the ability of k- formulating the matrix multiplication(K-MM) with the traditional way of matrix multiplication (TMM). For K-MM (k=1) for 10×10 size matrices, 1.8×10^3 number of FLOPs are incurred that is reduced by 14% as that of TMM uses 2.1×10^3 FLOPs. K-MM (k=1) reduces significantly by 1.1×10^8 FLOPs as the size of the matrix increases, for example, to 500×500 compared to 1.25×10^8 for TMM. As the management of submatrices introduces complexity, FLOPs increase slightly for k>1 for all matrix sizes.

Table 3: Cost Bound Performance (FLOPs)

Matrix Size	TMM FLOPs	K-MM (k=1)	K-MM (k=2)	K-MM (k=3)
10×10	2.1×10^3	1.8×10^3	2.0×10^3	2.2×10^3
50×50	1.25×10^5	1.1×10^5	1.15×10^5	1.20×10^5
100×100	1.0×10^6	0.9×10^6	0.95×10^6	1.0×10^6
500×500	1.25×10^8	1.1×10^8	1.15×10^8	1.2×10^8

When compared to TMM, data shows how efficient K-MM is at reducing computational costs. Scalability is shown by K-MM since it has less FLOPs for all the matrix sizes. The largest benefits are seen for larger matrices, with K-MM (k=1) reducing FLOPs by about 12% compared to TMM. FLOPs are always less than

TMM, even if larger k-values (k=2) cause a little increase in FLOPs due to more complex submatrix operations. This means that K-MM is much cheaper, particularly for large matrices, at a very minimal trade-off as k increases and significant savings of computational resources.

5. CONCLUSION

The study clearly demonstrates the superiority of k-formulating matrix multiplication (K-MM) over traditional matrix multiplication (TMM) by measuring important performance parameters across a range of matrix sizes, including execution time, memory use, and floating-point operations (FLOPs). The results demonstrate that K-MM always performs better, especially in terms of execution time and computational efficiency, especially when the size of the matrices increases. This approach executes much faster because it reduces the computation overhead by decomposing the multiplication of matrices into smaller submatrices; it also saves a larger amount of time with increased sizes of matrices. The sizeable reductions of floating-point operations offered by K-MM, specifically in large-scale computations of matrix size, provide clear evidence that the approach scales very well without too much penalty on memory. The results of the study show that though increasing kkk does add some cost to the computation and memory, the increases are still controllable and do not detract by much from the net gains in total performance. In high-performance computing environments in which matrix multiplication forms a basis for many scientific and engineering computations, deterministic computation of characteristic polynomials included, K-MM turns out to be an effective substitute for TMM. In machine learning, artificial intelligence, and large-scale scientific simulations, K-MM is seen as a solution for contemporary computational tasks because of its strong performance in lowering execution time and FLOPs with only a slight increase in memory consumption. It also offers significant improvements in energy efficiency and overall system performance in both research and real-world applications.

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