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Research Article

Time Optimization Transportation Problem with Minimum Cost Penalty

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ABSTRACT

Time-optimized transportation the problem deals with cost as well as time minimization for the transportation of the goods from the source. Recognizing the need for a more versatile approach that considers both time and cost optimization objectives, this paper introduces a novel TOMCP algorithm. The proposed algorithm aims to provide an optimum solution for the transportation problem by concurrently minimizing both time and cost variables. By bridging the gap between traditional methods, the research contributes to a more comprehensive and adaptable framework for addressing transportation problems in real-world scenarios.

Keywords: Transportation Problem, Time-cost minimization Problem, Linear Programming.

1. Introduction

The transportation problem is a special case of the linear programming problem, which was first introduced by Hitchcock in 1941. Cost minimization for transportation of goods from source to destination was derived by many of the authors later on. Goods transportation in real life is not only cost-oriented; sometimes, for some deteriorating products and in some emergency situations, it is necessary to deliver the goods in the shortest time. The time-cost minimization problem arose from that kind of situation. Initially, time-cost minimization for the transportation problems was explained by Hammer in 1969. Several extensions of the model for cost minimization are presented in his paper. Time minimization problems were discussed by Kanti Swarup and J.K. Sharma in 1977. Also, these methods are only to minimize the time in transportation problems. Minimization of time and cost in the single algorithm is effective for any transportation problem, but it is challenging because of two different matrices. In this, the TOMCP (Time Optimization with minimum Cost Penalty) algorithm transportation time improves over cost reduction by implementing the partition technique.

2. Problem formulation

The objective is to reduce the total cost and travel time, subject to constraints.
This bi-objective problem's mathematical formulation can be shown as follows:
The unit cost, time, quantity available at the origin, and demand are all known, assuming the existence of sources and destinations. A further assumption is that the product can be carried from any point of origin to any point of destination, and the transportation time is not contingent on the quantity of product transported.

Let us consider the following:

- Amount of time required to transport a product from its i^{th} origin to its j^{th} destination= t_{ij};
- Cost of transporting a unit of product from its i^{th} origin to its j^{th} destination= c_{ij};
- Quantity of product units available at the origin = a_i;
- Amount of demand at the destination = b_i;
- Total number of product units transported from its origin to its destination= x_{ij}.
- Minimized Total Time= T^*

- Minimum Time = t^*
- Minimized Total Cost = Z^*
- Set of basic variables from time matrix = γ_t
- Set of basic variables from cost matrix = γ_c

2.1. Definitions:

2.1.1. Total minimized time: The total minimized time is the cumulative duration required to complete each job individually at the final stage of the transportation problem. This method yields the most effective result when task repetition has been scheduled after the completion of each job at distinct routes.

2.1.2. Minimum time: The minimum time is the maximum time for all individual assignments at the final stage of the transportation problem.

Find

$$x_{ij} \geq 0(i = 1,2,...,m;\ j = 1,2,...,n)$$

Minimize Total cost function

$$Z^* = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Minimize Time function $$t^* = \underset{\{(i,j) \in x_{ij} > 0\}}{MAX} t_{ij}$$

Minimize Total time function

$$T^* = \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}y_{ij} \quad Where,\ y_{ij} = \begin{cases} 1, if\ x_{ij} > 0 \\ 0, if\ x_{ij} = 0 \end{cases}$$

Subject to:

$$\sum_{i=1}^{m} x_{ij} = b_i\ , \sum_{j=1}^{n} x_{ij} = a_j$$

$$x_{ij} \geq 0(i = 1,2,...,m;\ j = 1,2,...,n) \qquad Where, \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

This problem can be solved in the following way:
(i) First, figure out the optimum cost irrespective of the passage of time.
(ii) Next, we reduce the time attribute with respect to the lowest cost obtained in the previous outcome.

Step (ii) is repeated until an infeasible solution is obtained.
At first iteration, let Z_0^* be the minimum total cost and $T_0^*$and ${t_0}^*$ are be the optimal total time and minimum time of the problem with respect to the minimum cost matrix. (Z_0^*, T_0^*, t_0^*) is called the time-cost pair at the first iteration.
Using the re-optimization procedure, let the solution be infeasible after the m^{th} iteration. Therefore, we get the following complete set of time-cost pairs:

$$(Z_0^*, T_0^*, t_0^*), (Z_1^*, T_1^*, t_1^*), (Z_2^*, T_2^*, t_2^*), \ldots (Z_{m-1}^*, T_{m-1}^*, t_{m-1}^*)$$

Find the optimum pair among the above time-cost pairs by TOMCP as follows.

3. TOMCP- Algorithm

Step 1.
Obtain the optimal cost by applying any method pertaining to cost variables of the transportation problem and determine the pair(Z_0^*, T_0^*, t_0^*). Also, differentiate the set of basic variables (from the time matrix) and (from the cost matrix).

Step 2.
Let us divide the set of non-basic variables γ_t and $\gamma_c$of time and cost into L_1, L_2, L_3 … and K_1, K_2, K_3 … respectively partitions as follows:
2.1. L_1 Consist location $(h, k)$of minimum $t_{ij}$from γ_t, next minimum values are in L_2, L_3, and so on.
2.2. Likewise, divide the set γ_c using the minimum of c_{ij} from K. Place the minimum in the partition K_1, the next minimum in K_2, and so forth.

Step 3.
Determine

$$\Delta = \{\delta_{ij} = (|q-p|+1)p \text{ , } Where, (i,j) \in L_p \text{ } and \text{ } K_q \text{ } both.\}$$

Step 4.
Determine $\mu_{mn} = \min_{(i,j)\in L_p \& K_q} \{\delta_{ij} = (|q-p|+1)p\}$

Now introduce cost variables C_{mn} respective to μ_{mn} in basic variables by using the common transportation problem method (loop). If tied between two μ_{mn},then select μ_{mn} with minimum cost.

Step 5.
Re-cost M to variable removed from basic variables $\gamma_c$and determine(Z_1^*, T_1^*, t_1^*). Where, M is sufficiently large positive integer.

Step 6.
If $Z_m^* < M$, then repeat steps 2 to 5 until an infeasible solution is obtained.

Step 7.
Infeasible solution obtained, if $Z_m^* \geq M$ for some integer m.
$\{(Z_0^*, T_0^*, t_0^*), (Z_1^*, T_1^*, t_1^*), (Z_2^*, T_2^*, t_2^*), \ldots, (Z_{m-1}^*, T_{m-1}^*, t_{m-1}^*)\}$is set of feasible solutions.

Step 8(Optimality Test).
To determine the optimum solution from a set of feasible solutions, apply the following steps:
8.1. Find the weighted sum $w_t$and w_c of T_i^* and C_i^*.
8.2. Find decision variable $\tau_i = 1/w_t \cdot T_i^*$ and $\zeta_i = 1/w_c \cdot C_i^*$
8.3. Represent τ_i and ζ_i in graphic form and determine the intersection as an optimum solution pair.
8.4. Determine the optimality variable $\in_i = \tau_i + \zeta_i$ for each pair; Minimum value of $\in_i$ gives optimum pair.

4. Numerical example

We imposed TOMCP algorithm on the following bi-criterion transportation problems where the units of cost and time are taken in one standard scale.
Table 1 gives the values of variable cost C_{ij} $(i = 1,2,3;\ j = 1,2,3)$ and Table 2 gives the values of time t_{ij} $(i = 1,2,3;\ j = 1,2,3)$.

4.1. *Problem:-I*

Origin (i)/ Destination (j)	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	25	45	12	56	26
O_2	34	62	34	52	21
O_3	75	43	58	74	29
O_4	45	84	65	25	24
Demand (b_j)	14	26	35	25	100

Table:-1.1

Origin (i)/ Destination (j)	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	2	4	9	2	26
O_2	9	6	5	3	21
O_3	9	8	6	7	29
O_4	7	5	9	3	24
Demand (b_j)	14	26	35	25	100

Table:-1.2

Feasible solutions found after each iteration are as follows,

$$(Z_0^*, T_0^*, t_0^*) = (2934,43,9), \quad (Z_1^*, T_1^*, t_1^*) = (3116,36,9),$$
$$(Z_2^*, T_2^*, t_2^*) = (3692,31,7), \quad (Z_3^*, T_3^*, t_3^*) = (3672,26,6),$$
$$(Z_4^*, T_4^*, t_4^*) = (3694,27,6), \quad (Z_5^*, T_5^*, t_5^*) = (M,27,6)$$

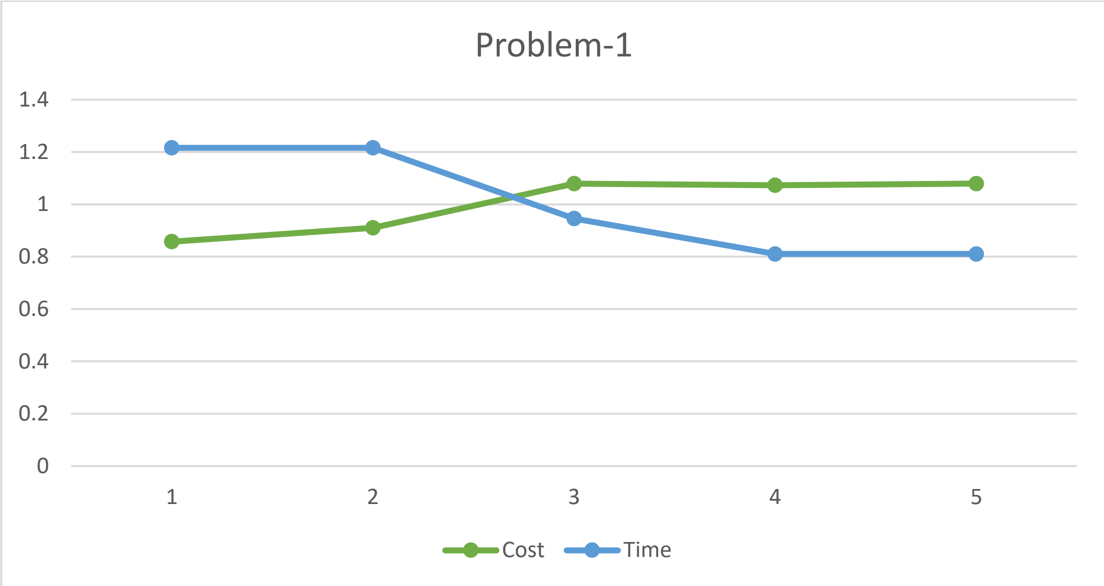

Figure-1.1

Now by applying the optimality test,
i) Graphical solution
Form figure 1.1 Optimum result is for $k = 3$

Then, $(Z_2^*, T_2^*, t_2^*) = (3692,31,7)$ is optimum pair.
ii) Optimum result by decision variable,
Minimum of $\in_k$ is $\in_4$
$(Z_3^*, T_3^*, t_3^*) = (3672,26,6)$ is optimum pair.
Which is more suitable for this problem.

4.2. *Problem: -II*

Origin (i)/ Destination (j)	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	81	91	28	96	35
O_2	91	63	55	16	45
O_3	13	10	96	97	20
Demand (b_j)	20	30	40	10	100

Table-1.1

Origin (i)/ Destination (j)	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	679	392	706	446	35
O_2	758	655	532	397	45
O_3	743	171	277	823	20
Demand (b_j)	20	30	40	10	100

Table-1.2

Feasible solutions found after each iteration are as follows,
$(Z_0^*, T_0^*, t_0^*) = (46000,96,349)$, $(Z_1^*, T_1^*, t_1^*) = (47040,91,281)$,
$(Z_2^*, T_2^*, t_2^*) = (51410,81,253)$, $(Z_3^*, T_3^*, t_3^*) = (M, 91,281)$
Now by Applying the optimality test,
i) Graphical solution
From figure 1.1 optimum result is for $k = 2$

then, $(Z_2^*, T_2^*, t_2^*) = (3692,31,7)$ is optimum pair.
ii) Optimum result by decision variable, Minimum of $\in_k$ is $\in_4$
$(Z_3^*, T_3^*, t_3^*) = (3672,26,6)$ is optimum pair.
Which is more suitable for this problem.

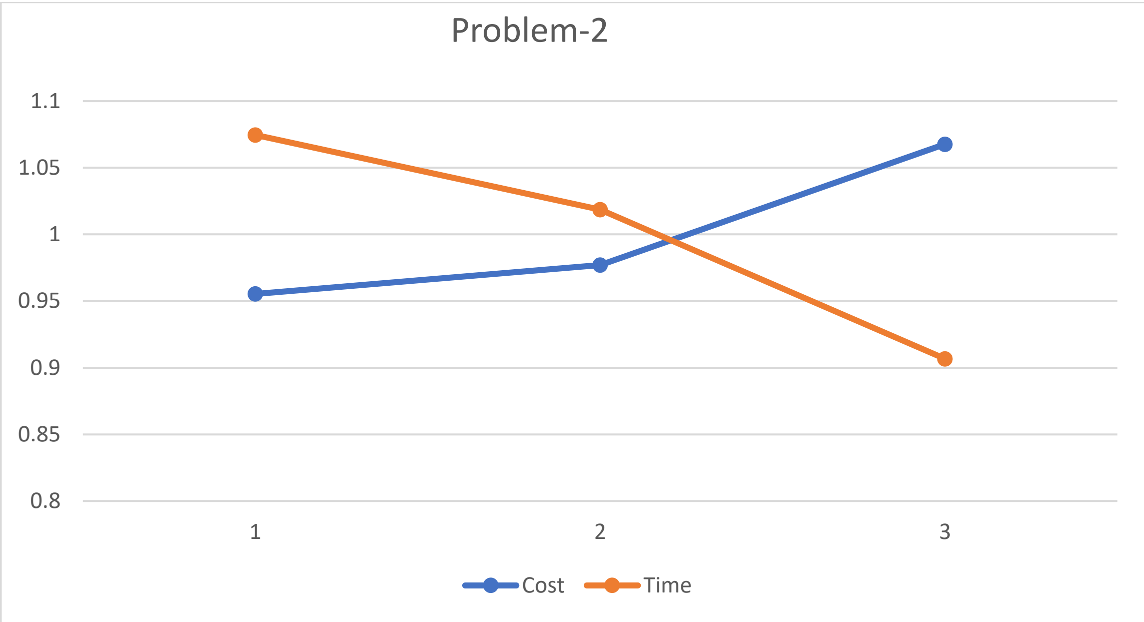

Figure-1.2

5. Result discussion

The time-cost transportation problem involves the integration of two distinct matrices. We have applied the modification method given by J.K. Sharma to the cost matrix, identifying the optimal cost solely from the cost matrix. Allocations designated for cost minimization will also be evaluated within the time matrix; however, it may not yield the optimal solution for the problem. Using iterations, the TOMCP algorithm is applied step-by-step to both matrices, as previously discussed. The transportation cost found in the first stage escalated with each iteration. To end the iterative algorithm sufficiently, a large positive integer M was added instead of removing the basic cost variable. After some iteration, time-cost pairs identified in each iteration produce the feasible solution set. To identify the optimum solution from that optimality test used, which gives the best optimum result from a set of feasible solutions. The optimism test was applied in two different ways, graphically as well as by the decision variable. The solution found by decision variables is more optimal, as we can see in the numerical example discussed, than the graphical method. But, from graphical representation, we can conclude that time will decay with an increment of the cost variable, which is more useful when some perishable items or emergency supplies are from source to destination.

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