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Research Article

Robust Fault Detection in Rotating Machinery under Variable Load Conditions Using Integrated Multiscale and Multiway PCA

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ABSTRACT
Various methodologies based on frequency domain and multivariate statistical analysis are employed for the interpretation of vibration data. However, the impact of load fluctuation on these diagnostic techniques has not been thoroughly delineated. This study investigates the impact of loading variations on vibration response and proposes that a combined wavelet and multiway principal component analysis (MPCA) methodology can enhance the sensitivity of vibration-based anomaly detection techniques for rotating machinery. The methodology utilizes multivariate vibration signals acquired from several sensor locations and under diverse load situations, resulting in a three-dimensional dataset. It is employed to denoise PCA on wavelet-transformed data. The dataset is subsequently converted into a two-dimensional matrix using a novel unfolding technique, wherein each individual batch recorded under a certain load is regarded as an observation for further PCA analysis. This approach facilitates the analysis of vibration signals in relation to baseline data obtained from standard operational conditions under varying applied loads, therefore differentiating load-induced variability from genuine mechanical flaws. Numerous simulation studies on bearing defect detection have been conducted to validate the suggested methodology.
Keywords: Condition Monitoring, Multiscale Principal Component Analysis (MSPCA), Vibration Analysis, Load Variability, Bearing Fault Detection

1 Introduction

Compressors and turbines in rotating machinery are vital in process industries. These systems frequently incorporate components such as bearings and gearboxes, which are susceptible to mechanical problems. These failures result in unanticipated equipment downtime, jeopardize staff safety, and adversely affect the company's profitability. The early failure detection of rolling components has been an active study field for decades, motivated by the necessity to enhance operational safety and dependability. Condition monitoring, particularly vibration analysis, is a very effective diagnostic instrument among contemporary defect detection approaches. Single-axis or tri-axial accelerometers are positioned around the bearings to detect vibrations resulting from mechanical displacements. Signals are collected in the time domain via data gathering devices, thereafter transformed into the frequency or time-frequency domains to identify indicators of failures or abnormalities in the bearing components [4].

2 Literature Review

The conventional procedure involves the installation of many sensors at various axial locations to capture distinct axial regions of the machines and a broad spectrum of vibration characteristics. The high-dimensional datasets produced need the utilization of multivariate statistical methods to exploit the redundancy and richness of the sensor data. Dimensionality reduction techniques are advantageous as they simplify measurement data from a high-dimensional vector to a lower-dimensional space, facilitating the expression and visualization of fault features, hence enhancing monitoring feasibility [28].

Subsequently, to achieve successful feature extraction, an improved approach for analyzing multiscale signal denoising is employed. A notable technique that has arisen is known as Multiscale Principal Component Analysis (MSPCA), initially presented by Bakshi [29]. This document presents the implementation of a novel and robust signal processing technique that integrates wavelet transformation with Principal Component Analysis (PCA) to extract the most significant features from heavily contaminated signals, resulting in a reliable method for fault detection in challenging industrial environments.

Nonetheless, particularly in recent developments in this field, researchers have used alternative complex multivariate methodologies. Žvokelj et al. [33] utilized kernel PCA and empirical mode decomposition for the defect identification of large-scale, low-speed bearings. The primary signal components were extracted via their methods, and fault identification was accomplished by T^2 plots derived from the kernel PCA model. Similarly, Jack and Nandi [22] employed a hybrid approach combining Support Vector Machines (SVMs) and Artificial Neural Networks (ANNs) to identify several flaws, including inner and outer race, cage, and rolling components, utilizing vibration data acquired from a controlled test rig. Statistical properties derived from moments and cumulants were computed, optimal inputs were identified with a genetic algorithm (GA), and a support vector machine (SVM) with a radial basis function (RBF) kernel was employed. In a separate research, Lei et al. [25] introduced a clustering-based diagnostic approach that utilized an enhanced Fuzzy C-Means (FCM) algorithm after feature selection and weighting processes to assess bearing conditions at different fault severities.

Although these approaches are recognized for their efficacy, they often overlook a crucial element in actual fault diagnostic systems: the impact of dynamically fluctuating operational loads. Dynamic industrial environments display fluctuations in load, frequently resulting from the stochastic supply of raw materials or variations in equipment quality. These adjustments might significantly alter the vibration signature of the equipment, resulting in false alarms or concealing actual failures if not appropriately normalized. For example, Figure 1 illustrates data from sensors affixed to a natural gas compressor, indicating radial vibration as the average plant load decreased from 90% (September–December 2024) to 85% (April–September 2023), accompanied by a reduction in displacement amplitude from 17–23 mm to 15 microns. Significantly, no mechanical failure contributed to the alterations in [7], [35].

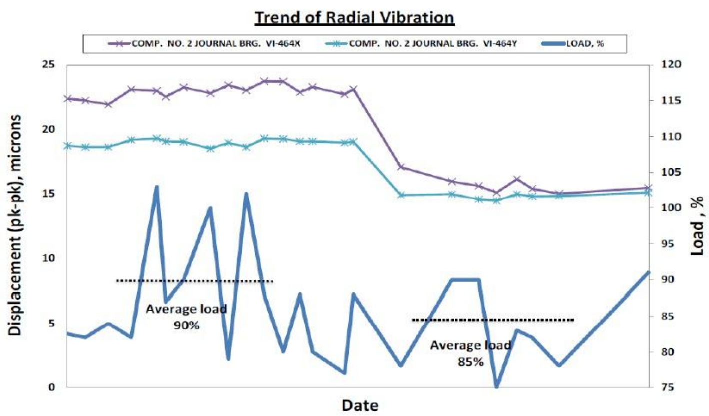

Fig. 1 Magnitude of vibration of a fertilizer industry natural gas compressor as a function of load change

Such discoveries underscore the necessity for diagnostic methods capable of distinguishing between genuine mechanical failures and load situations. To achieve this objective, we introduce a comprehensive fault identification approach that utilizes the denoising capabilities of MSPCA in conjunction with the batch analysis proficiency of Multiway PCA (MPCA) [21]. MPCA, formerly utilized for batch process monitoring, enables the analysis of conditions across several operational states of production. The utilization of wavelet-filtered features in the MPCA model enhances accuracy in fault detection under fluctuating loads, hence minimizing the possibility of misinterpretation due to process fluctuations.

This document is structured as follows: Section 3 delineates the data preparation techniques employed to enhance the quality of vibration signals and facilitate feature extraction. Section 4 provides a more detailed description of the proposed technique, including the integration of multiscale and multiway PCA. Section 5 evaluates the efficacy of the suggested method in detecting bearing defects under varying load situations and conducts a comparison study of the created diagnostic methodology against current methodologies utilizing simulated data. Section 6 closes the study, highlighting the substantial contributions and consequences of the research.

3 Data Acquisition and Preprocessing

The acquired vibration signals are quantified in displacement (in microns) and collected using accelerometers distributed around the bearings. The vibration analysis technique, encompassing transformation into both frequency and temporal domains, is illustrated in Figure 2.

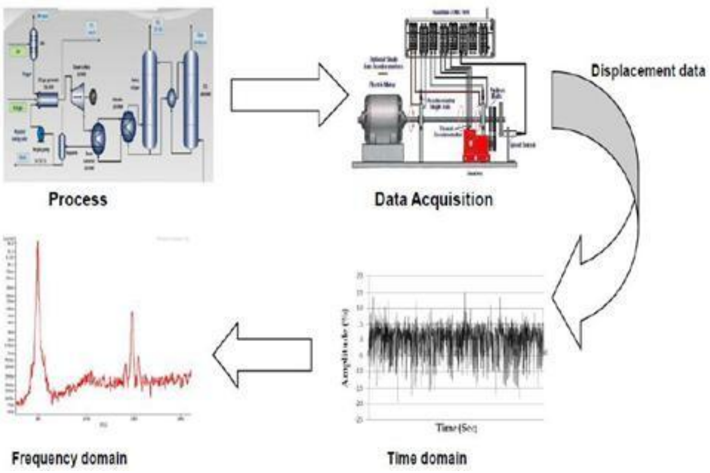

Fig. 2 Vibration analysis in detail [4]

4 Methodology

The suggested methodology comprises two sequential steps. Phase I: The Mixed Sky Helper use MS PCA-based MS denoising to filter bright signals, optimizing the matrix rating advantage. This enhances the signal-to-noise ratio and preserves essential fault characteristics. The second step employs Multiway Principal Component Analysis (MPCA) to address the functional dependencies under dynamic operating conditions, including variable loads and RPM. Collectively, these two phases provide robust defect detection in intricate environments. Figure 3 presents a schematic illustration of the comprehensive process.

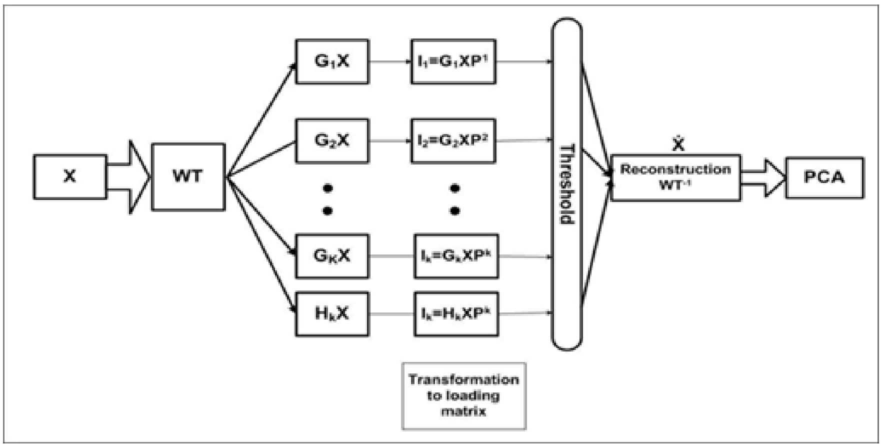

Fig. 3 A schematic representation of the complete methodology

4.1 MSPCA-Based Denoising

MSPCA integrates the wavelet analysis approach with Principal Component Analysis (PCA). The Multiscale Principal Component Analysis (MSPCA) approach was initially introduced by Bakshi in [29], combining the benefits of wavelet-based filtering with the dimensionality reduction capabilities of PCA. Wavelet analysis, a proficient time-frequency method, is especially beneficial for analyzing non-stationary signals by revealing concealed time-frequency components, sometimes referred to as signal fingerprints, inside noisy data [17]. The Discrete Wavelet Transform (DWT) is advantageous for vibration signal analysis due to its straightforward calculation and its ability to concurrently localize signals in both time and frequency domains.

The output signal is downsampled by a factor of two at each level of the wavelet-based filtering procedure [29]. This decimation step effectively reduces the dimensionality of the data while preserving critical elements of the signal information. This approach is illustrated in Figure 4, derived from [17].

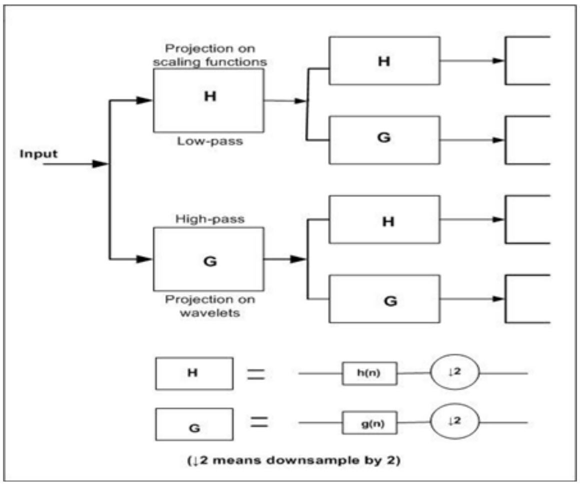

Fig. 4 Wavelet decomposition [19]

Filtering is conducted in the domain of decomposed wavelet coefficients. By selecting preserving the essential coefficients at all decomposition levels, the signal is rebuilt using the inverse wavelet transform, therefore recovering only significant fault-related features in the denoised signal.

4.2 Multiway PCA (MPCA)

Multiway Principal Component Analysis (MPCA) is an advanced variant of conventional PCA utilized for the analysis of three-dimensional arrays. In condition monitoring, data are often gathered in batches across time, during which the system may operate under varying loading circumstances. The approach known as MPCA is employed to distinguish inter-batch variation by using patterns, hence rendering the differences among these data sets readily interpretable. The baseline sound operating envelope is established using vibration signals captured under different load levels in the absence of mechanical problems. This architecture allows for the classification of incoming data by comparing each new batch to the previously determined reference, facilitating the identification of normal or incorrect behavior (as demonstrated in Figure 5) [21]. The vibration signal datasets gathered under various situations may be organized into a three-dimensional structured data matrix.

A data batch corresponding to a certain load state is regarded as a distinct entity inside the MPCA framework. The three-dimensional data matrix may be divided into a summation of outer products of score vectors and loading matrices, accompanied by a residual matrix. In decomposition, all trends, seasonality, and residuals are determined by reducing residuals using a least squares optimization technique, so capturing the most significant impacts on the system while eliminating noise and extraneous fluctuations.

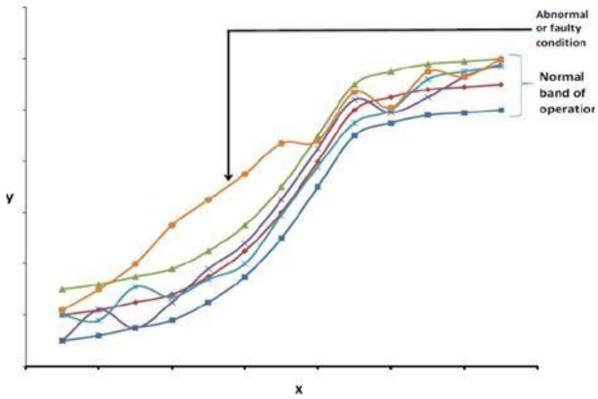

Fig. 5 Multiway PCA methodology [21]

This decomposes the data into low-dimensional score spaces to include variation across both variables and time. At each time point, these score spaces effectively represent the variability of system behaviors under diverse operational settings. Each loading matrix represents the most significant temporal variation of the variables in relation to the mean trajectory across all loading situations. Consequently, in addition to assessing the extent of each variable's deviation from its mean, MPCA considers the correlations among the various variables, providing a comprehensive overview of the system's dynamic behavior [21].

4.3 Analysis of vibration data with the multiscale-MPCAmethodology

The matrix represents batches of individual vibration signals, each corresponding to measurements from sensors positioned around the bearing at a certain axial point. The datasets in these batches may not have been collected under identical operational conditions, as the load and RPM of the machine might vary between datasets. Consequently, we utilize Wavelet Transformation (WT) to breakdown each variable into several frequency ranges. Denoising of the signals is executed using uniscale Principal Component Analysis (PCA) on the coefficients corresponding to each scale. The signal may be reconstructed by multiplying each frequency scale by its respective relevance factor and aggregating the results. The data undergoes preprocessing prior to the application of PCA to the unfolded data matrix. The preprocessed matrix is further analyzed using PCA, followed by the observation of the resultant T^2 statistics or Q-statistics. The T^2 and SPE (Squared Prediction Error) plots illustrate each data batch represented as a plotted point. A faultless dataset may be presumed to be within established parameters, however a dataset exhibiting considerable fluctuation will exceed the threshold and be classified as anomalous. The fresh dataset is amalgamated with previously obtained datasets as the former are discarded. When the T^2 or Q statistic exceeds the threshold, it signifies that the data batch is anomalous, indicating a malfunction in the equipment.

4.4 Combined Multiscale-MPCA Framework

A variety of models addressing flaws in the vibrations of rolling bearing elements were examined in the literature. We utilized the research of Cong et al. to model the bearing defect. The simulation utilizes a multi-body model of the rotor-bearing system. This model includes the rotor, supporting bearings, and the motor crankshaft. Rotor dynamic forces are meticulously modeled here, taking into account the direct impact of the bearing load. Fault vibration signals may be simulated by examining the bearing load and implementing it inside an impulse signal model. The bearing load was partitioned into two components: a constant load, including the system's gravitational force, and an alternating load, generated by inertial forces. In the last component, all forces exerted on the bearing were computed individually for the components along two orthogonal directions.

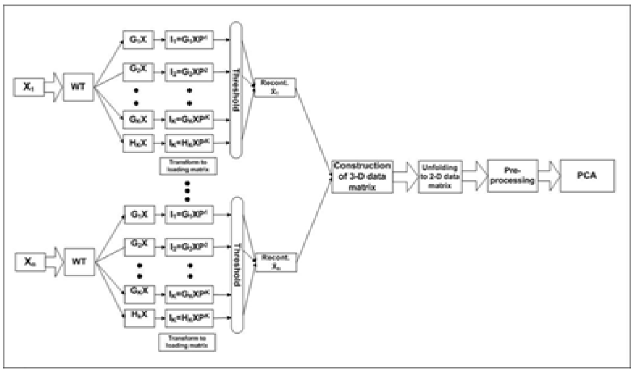

Fig. 6 Multiscale MPCA model

5. Simulation Study

Table 1 presents the numerical parameters utilized in the simulation. The signal was simulated from 0.0s to 0.4s with a sample period of 0.0001 seconds, resulting in a sequence of discrete temporal data points for subsequent analysis. Figure 3.8a illustrates the standard vibration signal characterized by white Gaussian noise, devoid of any faults. Figure 3.8b illustrates the simulated vibration signal exhibiting a flaw, which has been contaminated with Gaussian noise at a signal-to-noise ratio (SNR) of -15 dB. The signals underwent preprocessing by scaling, mean centering, and unit variance normalization. The disparity between the normal and defective signal is not readily discernible to the naked eye, as seen in Figure 3.8; hence, these signal changes resulting from a fault are not perceptible amidst the noise.

Table 1 Simulation parameters

Constraints	Range
Alternate load	0.500
Constant load	2.50-8.50
Time period	1e−5
Decay parameter	600
SNR for Gaussian white noise	-15 dB
Fault amplitude	2.50-4.50
Jitters	0.00005
Natural frequency	(2 × π × T)

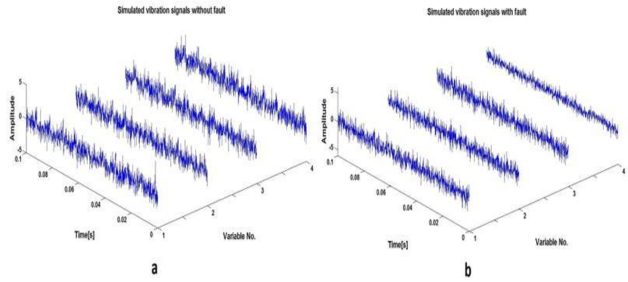

Fig. 7 Normal vs Faulty simulated signals

3 Results and Discussion

The conventional MSPCA exhibited erroneous detections due to its sensitivity to load change during first MS detections. The proposed technique accurately recognized only genuine defects. This research examines the impact of load variation on MS-PCA performance. Figure 8a The T statistics of a steady load at T2. It is readily evident that all values remain inside the control boundaries, indicating no defects have been introduced. The fault is generated. Conversely, in Figure 8b, the T^2 statistics are presented for a different loading situation, and the T^2 value distinctly above the control limit, signifying that the data is erroneous. However, it is important to note that the problem was not included into the data; just the load associated with the phase was altered. This distinctly illustrates a limitation of the current MS-PCA approach, namely, its failure to account for the effects of load variations. Consequently, the technique for interpreting vibration data under various load situations requires enhancement.

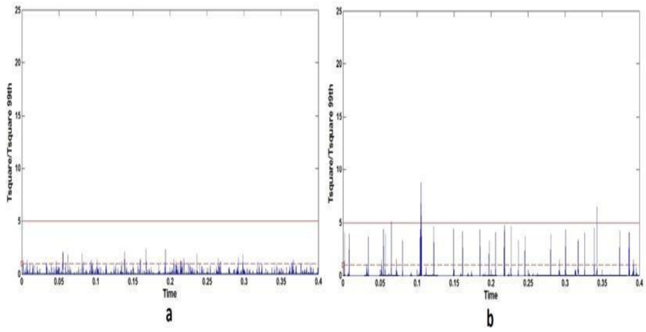

Fig. 8 (a) Normal vibration T 2 statistics and (b) normal vibration T 2 statistics with load variation are the findings of the MS-PCA for the influence of load change

The preceding section indicated that vibration signals were generated both with and without faults under varied load levels. We used the suggested technique to these synthetic signals via signal decomposition via a reverse biorthogonal (rbio 5.5) wavelet packet [28]. Decomposition level 11 was determined to be best for extracting the vibration signature. Only one PC was utilized at each level, as the signals represent many realizations of the identical vibration measurement. The reshaping parameter, μ, was increased to 10,000, as a greater factor facilitates the implementation of a "limit" that compels values below the threshold to converge towards zero, hence enhancing the separation in the data extraction process.

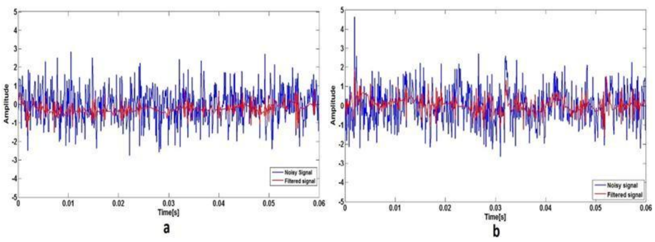

Fig. 9 (a) An unfiltered and unprocessed vibration signal with an amplitude of 2.5 and (b) an unprocessed and unprocessed vibration signal with an identical but different load A magnitude of 7.5

Figure 9 depicts the vibration signals acquired at two distinct load levels: 2.5 kg in Figure 9a and 7.5 kg in Figure 9b, with data taken at constant load levels. Additionally, vibration signals for seven separate loading situations were simulated and organized into a three-dimensional data matrix. The matrix was subsequently transformed into a two-dimensional matrix, as previously mentioned, with each dimension measuring 16,384 by 7. The data in the matrix were transformed into absolute values and normalized using

Multiplicative Signal Correction (MSC) and autoscaling. A significant component of this methodology is preprocessing, which eliminates non-linear trends and standardizes all variables to a uniform scale.

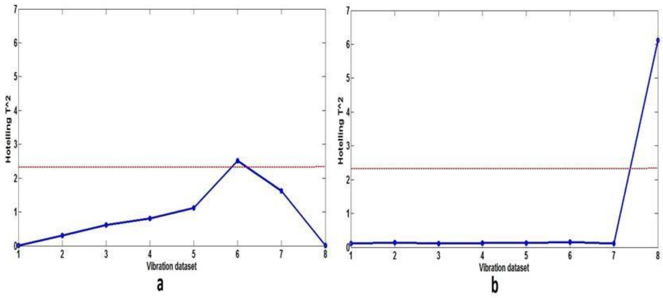

Fig. 10 Preprocessing affects MSPCA-MPCA approach (a) T 2 statistics of defective vibration with load variation without preprocessing and (b) T 2 statistics of defective vibration with load variation after preprocessing

Figure 10a illustrates the outcome without preprocessing; notably, the 8th dataset from the defective bearing is not discernible in the T^2 plot. Conversely, figure 10b illustrates the outcome post-preprocessing, with the 8th element designated as defective due to its above the T^2 threshold. T^2 statistics were computed for each data batch, and principal component analysis (PCA) was conducted on the two-way data matrix following data preprocessing. Only one principal component was employed in the Multiway-PCA phase, as the four accelerometers utilized in this investigation exhibit significant correlation and capture analogous vibration signals. In this instance, a single principal component explains for the majority of the variability in the data.

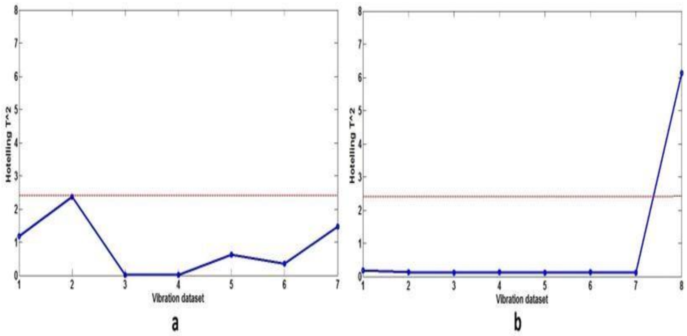

Fig. 11 The suggested Multiscale-MPCA technique for fault detection (a) The normal vibration T 2 statistics with load variation and (b) the defective vibration T 2 statistics with load variation (fault at load 3.5)

The comprehensive results from the suggested approach for fault identification, utilizing the Multiscale-MPCA analysis, are depicted in Figure 11. The various loads are indicated on the x-axis for each sample. Under normal conditions: The T^2 statistics depicted in Figure 11a exhibit fluctuations attributable to load variations; nevertheless, they consistently remain below the detection threshold, with values approximately less than 2.5. However, when a flawed data set is analyzed, as seen in Figure 11b, the T^2 value significantly exceeds the control limit, indicating a defect in the dataset.

The suggested approach utilized seven datasets including various fault magnitudes and situations, achieving 100% accuracy for bearing faults under differing load conditions.

Moreover, all defective instances were accurately recognized across all seven load circumstances with a 100% success rate. The method effectively minimized false positives by identifying load-induced variation.

5 Conclusion

This paper proposes a novel multivariate statistical technique—combining Multiscale Principal Component Analysis (MSPCA) and Multiway Principal Component Analysis (MPCA)—to detect outliers in rolling element bearings under different load situations. This method use Multiscale PCA to identify significant vibration patterns from very noisy data, enabling the detection and differentiation of problems. Conversely, MPCA regards each collection of multivariate data acquired under specific load circumstances as an object, facilitating the comparison of fresh vibration data with several sets gathered under varying loading conditions. An enhanced technique for bearing failure signal processing is utilized to replicate different load scenarios by activating a constant load on the rotor. The simulation experiments provide superior fault identification performance compared to other methods, such as MSPCA, which none of these techniques can reliably identify. This study presents an integrated MSPCA-MPCA method for identifying problems in rotating machinery under fluctuating loads. The MSPCA mitigates noise by isolating signal characteristics and excluding noise, whereas the MPCA is designed to assess behavior within operational unpredictability. In comparison to conventional approaches, the strategy demonstrated enhanced specificity in simulations, markedly decreasing false positives during load shifts.

6 References

[1] Andrew K.S. Jardine, Daming Lin, and Dragan Banjevic. A review on machinery di- agnostics and prognostics implementing condition-based maintenance. Mechanical Systems and Signal Processing, 20(7):1483 – 1510, 2006.
[2] Matej Žvokelj, Samo Zupan, and Ivan Prebil. Multivariate and multiscale monitoring of large-size low-speed bearings using ensemble empirical mode decomposition method combined with principal component analysis. Mechanical Systems and Signal Processing, 24(4):1049 – 1067, 2010.
[3] Bhavik R. Bakshi. Multiscale PCA with application to multivariate statistical process monitoring. AIChE Journal, 44(7):1596–1610, 1998.
[4] Matej Žvokelj, Samo Zupan, and Ivan Prebil. Non-linear multivariate and multiscale monitoring and signal denoising strategy using kernel principal component analysis combined with ensemble empirical mode decomposition method. Mechanical Systems and Signal Processing, 25(7):2631 – 2653, 2011.
[5] L.B. Jack and A.K. Nandi. Fault detection using support vector machines and artificial neural networks, augmented by genetic algorithms. Mechanical Systems and Signal Processing, 16(2-3):373–390, 2002. cited By (since 1996)169.
[6] Yaguo Lei, Zhengjia He, Yanyang Zi, and Xuefeng Chen. New clustering algorithm- based fault diagnosis using compensation distance evaluation technique. Mechanical Systems and Signal Processing, 22(2):419 – 435, 2008.
[7] P. Borghesani, R. Ricci, S. Chatterton, and P. Pennacchi. A new procedure for using envelope analysis for rolling element bearing diagnostics in variable operating conditions. Mechanical Systems and Signal Processing, 38(1):23 – 35, 2013.
[8] Radoslaw Zimroz, Walter Bartelmus, Tomasz Barszcz, and Jacek Urbanek. Diagnostics of bearings in presence of strong operating conditions non-stationarity—a procedure of load-dependent features processing with application to wind turbine bearings. Mechanical Systems and Signal Processing, 46(1):16 – 27, 2014.
[9] Johan A. Westerhuis, Theodora Kourti, and John F. MacGregor. Comparing alternative approaches for multivariate statistical analysis of batch process data. Journal of Chemometrics, 13(3-4):397–413, 1999.
[10] Harald Martens and Edward Stark. Extended multiplicative signal correction and spectral interference subtraction: New preprocessing methods for near infrared spectroscopy. Journal of Pharmaceutical and Biomedical Analysis, 9(8):625 – 635, 1991.
[11] Eigenvector Research Incorporated. Advanced pre-processing: Normalization from eigenvector research inc. web content, July 2011.
[12] Rodolphe L. Mortard & Babu Joseph, editor. Wavelet Applications in Chemical Engineering. Kluwer Academic Publishers, 1st edition, 1994.
[13] Haitao Guo C. Sidney Burrus, Ramesh A. Gopinath. Introduction to Wavelets and Wavelets Transforms. Number ISBN 0-13-489600-9 in 1. Prentice Hall, Upper Saddle River, New Jersey 07458, 1998.
[14] S.G. Mallat. A theory for multiresolution signal decomposition: the wavelet representation. Pattern Analysis and Machine Intelligence, IEEE Transactions on, 11(7):674–693, 1989.
[15] K.A. Kosanovich, M.J. Piovoso, K. S. Dahl, J.F. Macgregor, and P. Nomikos. Multi-way PCA applied to an industrial batch process. In American Control Conference, 1994, volume 2, pages 1294–1298, 1994.
[16] Nola Tracy, John Young, and Robert Mason. Multivariate control charts for individual observations. Journal of Quality Technology, 24:88–95, April 1992.
[17] J Antoni and RB Randall. A stochastic model for simulation and diagnostics of rolling element bearings with localized faults. Journal of vibration and acoustics, 125(3):282–289, 2003.
[18] Feiyun Cong, Jin Chen, Guangming Dong, and Michael Pecht. Vibration model of rolling element bearings in a rotor-bearing system for fault diagnosis. Journal of Sound and Vibration, 332(8):2081 – 2097, 2013.
[19] P.D. McFadden and J.D. Smith. Model for the vibration produced by a single point defect in a rolling element bearing. Journal of Sound and Vibration, 96(1):69 – 82, 1984.
[20] Ming Guo, Lei Xie, Shu-Qing Wang, and Jian-Ming Zhang. Research on an integrated ICA-SVM based framework for fault diagnosis. In Systems, Man and Cybernet- ICS, 2003. IEEE International Conference on, volume 3, pages 2710–2715 vol.3, 2003.
[21] H. Hotelling. Analysis of a complex of statistical variables into principal components. Journal of Educational Psychology, 24:417–441, 1933.
[22] S. A. Imtiaz, S. L. Shah, R. Patwardhan, H. A. Palizban, and J. Ruppenstein. Detection, diagnosis and root cause analysis of sheet-break in a pulp and paper mill with economic impact analysis. The Canadian Journal of Chemical Engineering, 85(4):512–525, 2007.

[23] J. Edward Jackson. Control procedures for residuals associated with principal component analysis. Technometrics, 21(3):341; 341–349; 349, August 1979. doi:10.2307/1267757.
[24] S. Joe Qin. Statistical process monitoring: basics and beyond. Journal of Chemo- metrics, 17(8-9):480–502, 2003.
[25] James V. Kresta, John F. Macgregor, and Thomas E. Marlin. Multivariate statistical monitoring of process operating performance. The Canadian Journal of Chemical Engineering, 69(1):35–47, 1991.
[26] Gang Li, S. Joe Qin, Yindong Ji, and Donghua Zhou. Reconstruction based fault prognosis for continuous processes. Control Engineering Practice, 18(10):1211 – 1219, 2010.
[27] Li Li and Liangsheng Qu. Machine diagnosis with independent component analysis and envelope analysis. In Industrial Technology, 2002. IEEE ICIT '02. 2002 IEEE International Conference on, volume 2, pages 1360–1364 vol.2, 2002.
[28] David Logan and Joseph Mathew. Using the correlation dimension for vibration fault diagnosis of rolling element bearings-i. basic concepts. Mechanical Systems and Signal Processing, 10(3):241 – 250, 1996.
[29] David Logan and Joseph Mathew. Using the correlation dimension for vibration fault diagnosis of rolling element bearings-i.basic concepts. Mechanical Systems and Signal Processing, 10(3):241 – 250, 1996.
[30] K. Pearson. On lines and planes of closest fit to systems of points in space. Philosophical Magazine, 2(11):559–572, 1901.
[31] A. Simoglou, E.B. Martin, and A.J. Morris. Statistical performance monitoring of dynamic multivariate processes using state space modelling. Computers & Chemical Engineering, 26(6):909; 909; PII S0098–920; 920; 1354(02)00012–1, 2002. doi:10.1016/S0098-1354(02)00012-1.
[32] V.A. Skormin, L.J. Popyack, V. I. Gorodetski, M. L. Araiza, and J. D. Michel. Applications of cluster analysis in diagnostics-related problems. In Aerospace Conference, 1999. Proceedings. 1999 IEEE, volume 3, pages 161–168 vol.3, 1999.
[33] Irem Y. Tumer, Edward M. Huff, Ph. D, and Ph. D. Principal components analysis of triaxial vibration data from helicopter transmissions, 2002.
[34] S. Valle, W. Li, and S.J. Qin. Selection of the number of principal components: The variance of the reconstruction error criterion with a comparison to other methods. Industrial and Engineering Chemistry Research, 38(11):4389–4401, 1999. cited By (since 1996)141.
[35] Svante Wold, Kim Esbensen, and Paul Geladi. Principal component analysis. Chemometrics and Intelligent Laboratory Systems, 2(1–3):37 – 52, 1987.
[36] Ball & roller bearings: Failures , causes and countermeasures. Technical Brochure Cat No. B3001E, JTEKT corporation, Nagoya Head Office 15th Floor, Midland Square, 4-7-1 Meieki, Nakamuraku, Nagoya 450-8515 Japan.
[37] Rolling bearing damage: recognition of damage and bearing inspection. Technical Report WL 82 102/3 EA, Schaeffler group industrial, Schaeffler Technologies GmbH & Co. KG 91074 Herzogenaurach Germany, 2001.